INVESTIGATIONS ON THE TRAFFIC BEHAVIOR OF THE COMMON CONTROL IN SPC SWITCHING SYSTEMS

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ABSTRACT

This paper deals with the investigation of the traffic behavior of the common control of an experimental PCM switching system with stored program control.

An outstanding characteristic feature of the switching system is extensive preprocessing of control information by peripheral control units.

The investigations are done by simulation and calculation, resp. For simulation a model of the entire switching system is developped which includes a submodel for the subscriber behavior.

For calculation a simplified model of the common control is developped and analyzed. This model consists of a single server system with batch input (variable batch size) at equidistant instants and service times according to an Erlangian probability distribution function. Each arrival causes a constant overhead phase in the server.

Numerical results (obtained by simulation) show the influence of the subscriber behavior and of the preprocessing functions on the performance of the system. Furthermore, interesting traffic values for the switching system are given.

1. INTRODUCTION

Electronic switching systems with stored program control (SPC) have achieved worldwide acceptance /1/. Many telephone organizations in the world install SPC switching systems or plan to do it.

Stored program control leads to a new architecture and operational mode of switching systems (central control, preprocessing of peripheral control information, etc.). Besides traffic dimensioning problems of the switching network /2/, a careful dimensioning of the central and peripheral control devices with special regard to their traffic behavior is of great importance /3,4/. This behavior is not only influenced by the interworking of the system components but also by the subscriber behavior.

To study the arising problems of SPC switching systems, an experimental PCM switching system with extensive preprocessing for telephone and data traffic is under development at the Institute of Switching and Data Technics, University of Stuttgart.The aims of this development are the investigation of:

- The influence of new technologies on the architecture of switching systems (e.g. microprocessors for preprocessing control units).
- The influence of the architecture on the operational mode of the system (e.g. on the switching programs).
- The flow of control information and the traffic behavior of the common control and the preprocessing units.
- The integration of data traffic in an integrated switching system and its influence on the control units and the traffic behavior of the system.

This paper deals with the investigation of the traffic behavior of the switching system with special regard to the central control and the flow of control information.

In Chapter 2 the experimental switching system will be briefly described.(Detailed information can be found in /5,6,7/). For the investigation a model of the real system and of the subscriber behavior is developped in Chapter 3. This model is investigated by simulation (Chapter 4) and a simplified version of the model by calculation (Chapter 5).

2. THE EXPERIMENTAL PCM SWITCHING SYSTEM

Fig. 1 shows the basic structure of the experimental switching system. The subscribers are connected with their exchange via concentrators CT. Each concentrator is controlled by a concentrator control CC within the exchange. Concentrator and concentrator control perform in cooperation many of the functions for call establishment autonomously without the central control (switching computer SC). Signalling to and from other exchanges is done by the trunk control. Throughout the switching system PCM transmission and switching is performed.

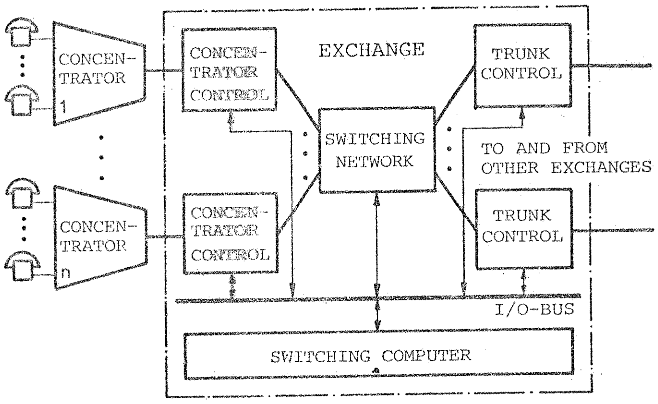


FIG. 1: STRUCTURE OF THE SWITCHING SYSTEM

The central processor supervises the peripheral devices and establishes the connections through the switching network. Information to and from the switching computer is transferred at fixed sampling instants (clock driven). In the following only the function of the components which are necessary for the model will be briefly described.

2.1 The Concentrator (CT)

The concentrator connects the subscribers with the exchange. It performs a traffic concentration of the subscriber lines to the channels of the PCM transmission system and PCM coding and decoding. Furthermore, the concentrator provides digit receivers for multi-frequency coded (MFC) digits.

The concentrator scans periodically all subscriber lines for off-hook and on-hook events (change of the subscriber state). After detecting a change of the subscriber state, this new state is transferred to the concentrator control. Controlled by the CC, the concentrator links a calling subscriber to a PCM channel and to a digit receiver. Dialled digits received by digit receivers are sent to the CC. Disconnection of digit receivers and of PCM channels is initiated by instructions from the CC.

The concentrator is supervised by the CC and all operations are directly activated by the CC. The switching computer has no direct access to the concentrator.

2.2 The Concentrator Control (CC)

The concentrator control processes the messages from the concentrator and the instructions from the switching computer. Two types of messages are distinguished:

- Messages which are sent to the SC without preprocessing (dialled digits, off-/on-hook of a called subscriber)

- Messages which are preprocessed by the CC (off-hook and on-hook of a calling subscriber). The CC allocates PCM channels and digit receivers to calling subscribers. After a successful call attempt (calling subscriber is connected to a PCM channel and to a digit receiver), a message is generated for the SC. Otherwise, if this attempt was unsuccessful, no message is sent to the SC.

The following instructions from the SC are distinguished:

- Connect a PCM channel with a subscriber.
- Disconnect a subscriber from a PCM channel and/or from a digit receiver.
- Connect or disconnect an audible tone.

2.3 The Switching Network and the Trunk Control

Path selection for the switching network is performed by the SC. The necessary information for the switching network to connect or to disconnect an incoming and an outgoing PCM channel is received from the SC.

The trunk control performs the information interchange between the switching computers of different exchanges over common signalling channels of the PCM systems.

2.4 The Switching Computer (SC)

As main tasks the switching computer performs the interpretation of dialled digits, path selection for the switching network and initiates connections and disconnections. Furthermore, metering, charging, maintenance and other administrative functions are done. Besides these functions, the switching computer supervises and coordinates all peripheral devices.

New service features (e.g. abbreviated dialling) are implemented in the SC. Due to the preprocessing functions of the CC, the flow of control information is considerably reduced. Therefore the load of the switching computer is diminished.

The information interchange between the peripheral devices (CC, CT, switching network and trunk control) and the SC is done over an I/O bus cyclically at fixed sampling instants. Only one information (message or instruction) to and from each of the peripheral devices can be transferred at the sampling instant.
Each type of message is processed by an individual program in the SC.

3. MODELLING OF THE SYSTEM

For the investigations of the central control a model

- of the real system and
- of the subscriber behavior

is developped and described in the following sections. This model is analyzed by simulation and a simplified version by calculation.

3.1 Model of the Subscriber Behavior

The actions of the subscribers (going off-hook, going on-hook, dialling digits) denoted in the following as "EVENTS" produce messages to the SC, which form an essential part of the flow of control information. Therefore, the model of the subscriber behavior should comprise these events. The events can be subdivided into

- initial events (off-hook)
- dependent events after one initial event (digits, on-hook).

The times T_{ar} between two initial events are distributed according to a negative exponential distribution function

$$P\left\{T_{ar} \geq t\right\} = e^{-\lambda t}$$

with mean arrival rate λ. The negative exponential pdf (probability distribution function) has been proven to be a good approximation for call attempts /8,9/.

To produce predetermined traffic peaks in order to investigate the transient behavior of the central control, the mean rate λ can be altered as a function of time. (Fig.2).

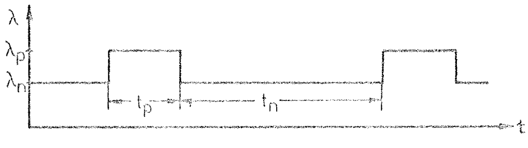


FIG. 2: MEAN ARRIVAL RATE λ AS A FUNCTION OF TIME t

The mean rate λ is changed to λ_p for the time t_p, and to λ_n for the time t_n. To obtain statistically reliable data on the system behavior, this process has to be repeated periodically /10/.

A sequence of one initial event and all its dependent events will be denoted in the following as a service request chain SRC. The service request chains depend on:

- the "original" subscriber behavior (not influenced by the switching system)
- the receipt of busy tone (sent by the switching system) which terminates the call establishment and leads to an on-hook event.

According to different subscriber behavior four types of so called "original" SRC's are distinguished:

- First type: Subscriber goes on-hook without dialling
- Second type: Uncompleted dialling
- Third type: Called subscriber doesn't answer
- Fourth type: Called subscriber answers

These types do not include the reaction of the subscriber to busy tone. Therefore, another type of SRC is introduced:

-Fifth type: Subscriber goes on-hook after busy tone.

This type of SRC appears, if either the called subscriber (B subscriber) is busy or if no idle channel is available. In /10/ measured values for the probabilities of the different types of SRC's are given.

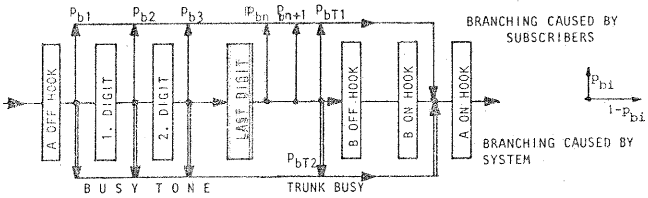


FIG. 3: SERVICE REQUEST CHAINS

Fig.3 shows all possible SRC's with respect to the experimental switching system. The individual SRC of a subscriber is a sequence of events. This sequence is determined by branching probabilities p_{bi} caused by the subscriber without an influence of the switching system and by probabilities for busy tone $p_{bT} = p_{bT1} + p_{bT2}$. ($p_{bT1}$ = probability for busy tone because B subscriber busy; p_{bT2} = probability for busy tone because no idle channel is available (trunk busy)). Values for these probabilities and for the mean times between the events are given in Table 1.

The subscribers are connected with the switching system via n concentrators. Therefore, the subscribers are divided up into n groups, each group belonging to one concentrator. The behavior of such a group is considered to be uncorrelated with the behavior of all other groups. Therefore, the mean arrival rate λ is defined for one concentrator. Within each concentrator an event generator (EG) produces the SRC's individually for each subscriber which is off-hook (Fig.4). Therefore, several SRC's can run in parallel. Initial events (off-hook) are also produced by the EG according to the mean arrival rate λ and the negative exponential distribution function for initial events.

The time intervals between two consecutive dependent events of one subscriber are distributed according to an Erlangian pdf of k-th order. The mean time interval a_i and the parameter k_i of the pdf are dependent on the type of the next event to be generated.

The SRC of a subscriber is terminated if busy tone is sent to the subscriber by the SC.

To study the influence of repeated call attempts on the load of the central processor, it is possible to generate new initial call attempts (events) caused by rejected call attempts. The maximum number of repeated call attempts as well as the mean time between attempts and the pdf are parameters of the model.

3.2 The Model of the Switching System

The hardware model of the switching system consists of two components:

- the model of the peripheral devices(CT, CC)
- the model of the switching computer.

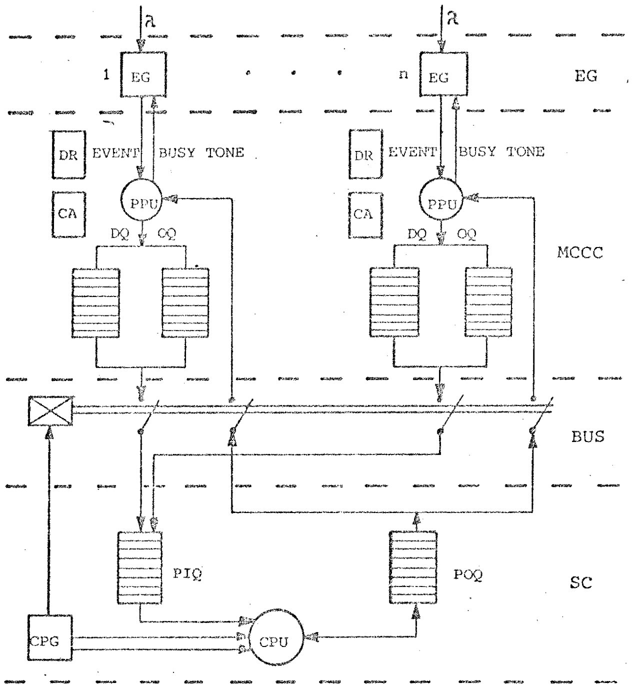
The switching network is only modelled with respect to trunk busy condition. Trunk busy means, that no idle PCM channel to the called subscriber is available. The detection of trunk busy is based on lists, which are used in the model of the concentrator and the switching computer. Therefore, the model of the switching computer processes trunk busy condition.

Furthermore, the trunk control units are not modelled in the basic model described below. That means, that only internal traffic is considered in the model.

3.2.1 Model of the Concentrator and Concentrator Control

The model of the concentrator and the concentrator control MCCC (Fig.4) consists of the peripheral processing unit PPU and two queues:

- the queue for digits (DQ)
- the queue for on-hook and off-hook events (OQ).



BUS	BUS-SYSTEM
CA	LIST OF CHANNELS
CPG	CLOCK PULSE GENERATOR
CPU	CENTRAL PROCESSING UNIT
DQ	QUEUE FOR DIGITS
DR	LIST OF DIGIT RECEIVERS
EG	EVENT GENERATOR
MCCC	CONCENTRATOR/ CONCENTRATOR CONTROL
OQ	QUEUE FOR ON-HOOK AND OFF-HOOK MESSAGES
PIQ	PROCESSOR INPUT QUEUE
POQ	PROCESSOR OUTPUT QUEUE
PPU	PERIPHERAL PROCESSING UNIT
SC	SWITCHING COMPUTER

FIG. 4: SIMULATION MODEL OF THE SWITCHING SYSTEM

Microprograms running in the concentrator control CC are only modelled with respect to their function and not to the run times. These run times are only a fraction of the time between two sampling instants and cause no additional delay on the processing of subscriber events in the concentrator control.

The PPU performs the following tasks:

- Allocation of PCM channels and digit receivers to calling subscribers (A subscribers) according to a list. (DR for digit receivers, CA for PCM channels).
- Rejection of call attempts, if no PCM channel or digit receiver is idle. (Without sending a message to the SC).
- Allocation of PCM channels to called subscribers (B subscribers), activated by an instruction from the SC, and sending of a message (acknowledgement) to the SC, whether the channel could be switched to the subscriber or not.
- Terminating of the original SRC of a subscriber as a reaction on a busy tone instruction from the SC, caused by trunk busy or subscriber busy. The next subscriber event is then an on-hook event.

The input to the concentrator is produced by the EG. Initial subscriber events (call requests) which are not rejected are filed into the OQ (queue for on-hook,off-hook). Digits are filed into the DQ (queue for digits) and have priority over off-hook, on-hook events.
The model of the periphery consists of n MCCC according to the number of concentrator controls which work in parallel.

3.2.2 Model of the Switching Computer SC

The switching computer is only modelled with respect to its switching functions. All other functions, such as maintenance, error detection and administrative operations are neglected. It consists of the server (CPU) and two waiting queues,
- the processor input queue PIQ and
- the processor output queue POQ.

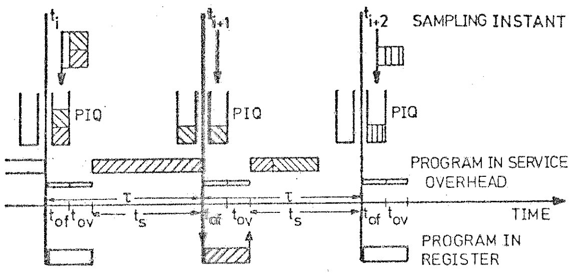
The PIQ is for messages from the periphery and the POQ for instructions to the periphery.

At fixed sampling instants the information interchange is initiated by the clock pulse generator CPG. Only one message from and one instruction to each concentrator control (MCCC) and to the switching network can be interchanged. Therefore batches (groups) of messages from the MCCC with a maximum batch size of n according to the number of MCCC's arrive at the PIQ. This interchange causes a constant processing time (overhead) in the CPU.

The switching programs are modelled with respect to the instructions for the concentrator control (MCCC) and to the run times. Each type of message (e.g.digit) activates a program of its own with individual run time. Dependent on the type of message an instruction is generated and filed into the POQ (measured values of the run times of switching programs in the experimental switching system are given in Chapter 4).

To be able to model SC's with different types of CPU a so-called run time factor RTF is introduced. The RTF determines the processing speed of the CPU related to the processing speed of the CPU of the experimental switching system. For instance a RTF =25 means, that the CPU is 25 times slower than the reference CPU.

The occupation of the CPU and the PIQ will be shortly discussed by an example (Fig.5).



t_{of}	FIXED OVERHEAD PERIOD
t_{ov}	VARIABLE OVERHEAD PERIOD
t_s	TIME FOR PROGRAM SERVICE
τ	SAMPLING PERIOD
PIQ	PROCESSOR INPUT QUEUE

FIG. 5: OCCUPATION DIAGRAM FOR THE CPU

At time t_i-o (shortly before the instant t_i) the PIQ and the CPU are empty. At the sampling instant t_i two messages arrive (batch size =2) and are filed in the PIQ. The first message is served after the fixed overhead period t_{of} and the variable overhead period t_{ov}. Its service is interrupted at the next sampling instant t_{i+1}. The state of the interrupted service is stored in an internal register of the CPU and the service is resumed after the overhead periods. After finishing its service, a waiting message in the PIQ is served.

The variable overhead is a function of the state of the system.

Messages are filed into the PIQ in order of arrival and served according to a FIFO strategy.

3.3 Flow and Processing of Control Information

Different messages from the MCCC to the switching computer SC and instructions from the SC to the MCCC are interchanged. The interchange is driven by the different subscriber events. The sequence of subscriber events and the resulting flow of control information as well as the processing in the SC are depicted in Fig.6.

At the left hand side of Fig.6 the SRC generated by an EG is shown. In the midst of Fig.6 the processing of instructions and the generating of messages by the MCCC and, on the right hand side the processing of messages and generating of instructions by the SC are shown. In Fig.6 only one MCCC and one EG is shown. In the model however, a variety of such SRC's run in parallel in each MCCC. Messages and instructions are only interchanged at fixed sampling instants. As an example the serving of a SRC will be briefly described.

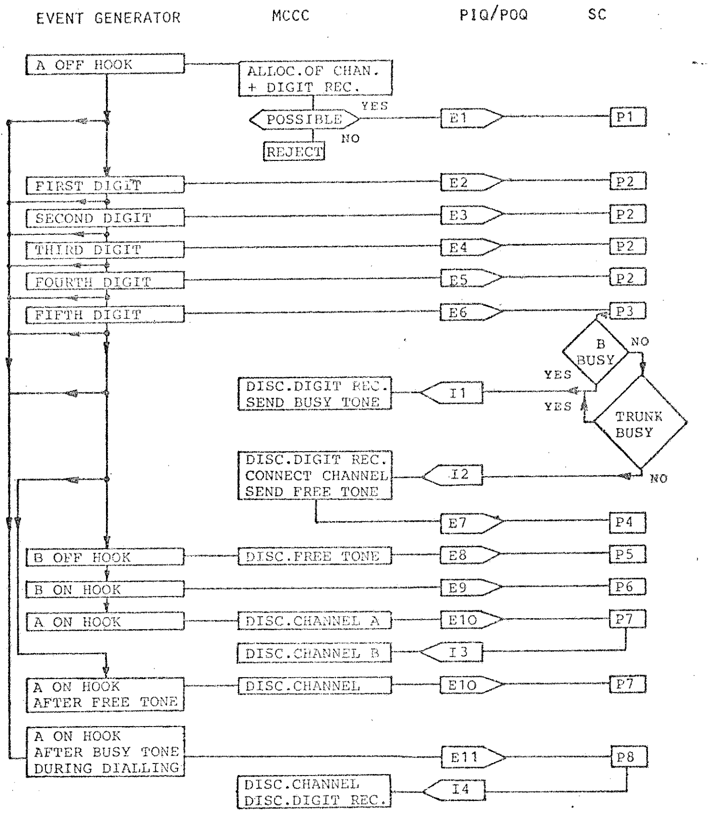


FIG. 6: FLOW OF CONTROL INFORMATION

SUBSCRIBER EVENTS	MESSAGE	ATI	PB		PROGRAMS			
	ABBR.	sec	ABBR.	%	P_i	h_i	c_i	k_i
A subscriber off-hook	E1	-	P_{b1}	10	1	0.55	0.14	51
1. digit	E2	2.5	P_{b2}	2	2	0.45	0.65	2
2. digit	E3	0.7	P_{b3}	2	2	0.45	0.65	2
3. digit	E4	0.7	P_{b4}	2	2	0.45	0.65	2
4. digit	E5	0.7	P_{b5}	2	2	0.45	0.65	2
5. digit (complete dialling)	E6	0.7	P_{bn} P_{bn+1}	2 10	3	3.33	0.35	3
Acknowledgement	E7	-			4	1.70	0.08	156
B off-hook	E8	15			5	0.60	0.18	30
B on-hook after conn.	E9	145			6	1.10	0.99	1
A on-hook after conn.	E1o	1			7	2.98	0.32	9
A on-hook after free t.		40			7	2.98	0.32	9
A on hook after busy t.	E11	4			8	0.63	0.49	4
A on hook during dial.		3.5			8	0.63	0.49	4
	mean h, variance coefficient c					1.06	0.96	

ATI	AVERAGE TIME INTERVAL BETWEEN THIS EVENT AND ITS PRECEEDING EVENT (IN SECONDS)
PB	PROBABILITY FOR BRANCHING AFTER THIS EVENT
h_i	MEASURED MEAN RUN TIMES OF THE SWITCHING PROGRAMS (RELATED TO 1000 CYCLE TIMES OF THE REFERENCE CPU)
c_i	VARIANCE COEFFICIENT OF THE PROGRAM RUN TIMES
k_i	PARAMETER OF THE ERLANGIAN PDF

TABLE 1: MEASURED DATA

After an off-hook event, the MCCC tries to allocate a PCM channel and a digit receiver. If this is possible a message E1 (A subs. off-hook) is sent to the SC, otherwise the call is rejected without a message.

Dialled digits are sent directly without preprocessing in the MCCC to the SC (E2-E6, digits 1-5). The length of the subscriber numbers in the model is 5 digits. After the last digit (complete dialling) the SC determines (with probability) if the B subscriber is busy. In this case the SC sends the instruction I1 (connect busy tone). In the other case a free channel to the B subscriber is hunted. After successful hunting, the instruction I2 (connect channel) is produced. Otherwise the instruction I1 is generated.

The MCCC connects the channel to the B subscriber, disconnects the digit receiver, and sends the message E7 (acknowledgement). (In reality the message "internal blocking" in the concentrating network within the CC may be generated. Because of the neglectable probability of blocking this message is not modelled.)

If the B subscriber goes off-hook, free tone is disconnected and the message E8 is sent. End of connection generates two messages E9 and E10 for B on-hook and A on-hook. A on-hook causes the disconnection of the A subscriber and the B subscriber from the channel. Therefore the SC sends the instruction I3 (disconnect channel).

Other types of SRC are served in a similar way.

4. SIMULATION AND RESULTS

The model described in Chapter 3 has been investigated by simulation. For this purpose a simulation program in ALGOL has been written. The investigations have to be done due to event by event simulation /11/. To obtain reliable results the input parameters, which together with the structure and the operational mode determine the flow of control information, have to be realistic. Therefore, measured data from the literature /10/ and measured data from the experimental switching system have to be used (Table 1).

Results of the analysis are characteristic traffic values for the performance of the system as

- mean load of the central control (Y_{CPU})
- mean waiting time in the PIQ (w)
- mean length of the PIQ ($\bar{\Omega}$)
- maximum length of the PIQ ($\hat{\Omega}$)
- waiting probability in the PIQ (W).

Furthermore the time dependent behavior of the traffic values is given.

4.1 Input Data

Table 1 shows the average length of the different time intervals (ATI) between two successive "dependent" events, measured in the telephone network of the German PTT. These time intervals are assumed to be negative exponentially distributed. The probabilities for branching (PB) for the SRC's (c.f. Fig.3) are also given in Table 1. These values are derived from measured values /10/. To obtain values for the program run times of the experimental switching system measurements on the real system had to be carried out. For this purpose a sequence of control informations (messages) was generated, and the run times of the related programs were measured by means of a time monitor. The mean values for the different programs h_i as well as the variance coefficient c_i are given in Table 1. The pdf for the run times is assumed to be an Erlangian pdf of k-th order. The values for k are derived from the equation $k = (1/c)^2$ for values $k \geq 1$.

The number of concentrators and concentrator controls is n=16.

The mean run time of all different programs for a normal mix of SRC's (p_{bT1} = 17%, other probabilities according to Table 1) is also given in Table 1.

Parameters which were altered for simulation are depicted in the diagrams.

4.2 Results

The results obtained by simulation are listed and discussed in the following. The 95% confidence interval ($\overline{\underline{\mathrm{I}}}$) for all measured values will be given in the diagrams.

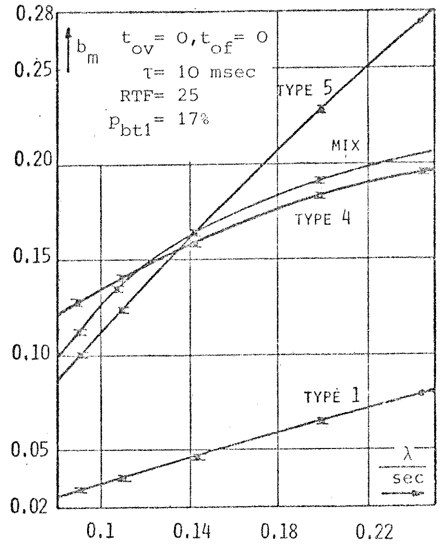


DIAGRAM 1: MEAN BATCH SIZE b_m VS. λ

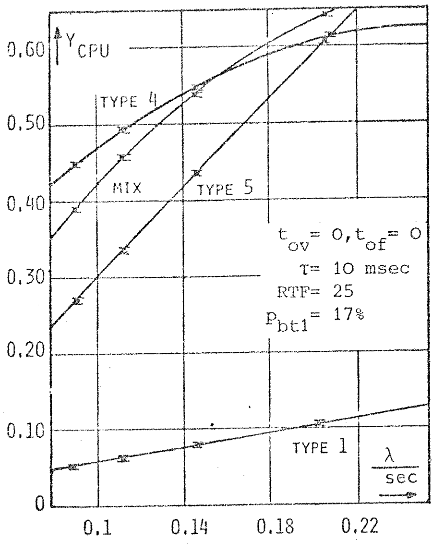


DIAGRAM 2: LOAD OF THE CPU Y_{CPU} VS. λ

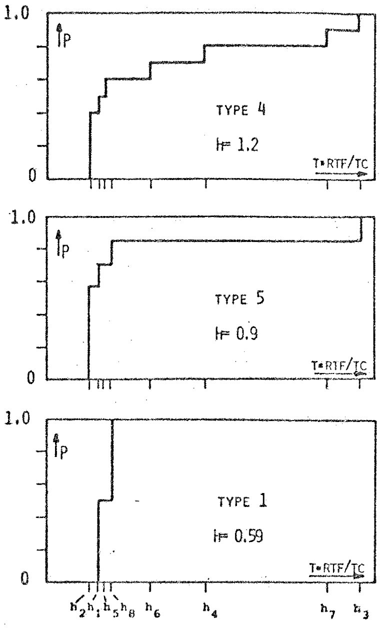


DIAGRAM 3: PDF'S OF THE PROGRAM RUN TIMES FOR DIFFERENT TYPES OF SUBSCRIBER BEHAVIOR

As already mentioned, the subscriber behavior has a great impact on the system behavior. The Diagrams 1-3 show the influence of different subscriber behavior on the mean batch size, on the pdf of the program run times and on the load of the CPU. Four different types of subscriber behavior are distinguished:

- Only type 1 (p_{b1}=1.0)
- Only type 4 (p_{bi}=0, bi=1..bn+1, p_{bT1}=0)
- Only type 5 (p_{bT1}=1.0)
- Normal mix.

(Branching probabilities according to Fig.3 and Table 1)

Diagram 1 shows the mean batch size b_m as a function of the mean arrival rate λ of calls. The curves for type 1 and 5 are nearly a linear function of λ. This is due to the fact, that the number of rejected calls (because no channel or digit receiver is idle) is neglectable. The curves for type 4 and the mixed type increase also monotonously with λ but not linear, because in the depicted range of λ the number of rejected calls increases remarkably with λ. With increasing λ the mean batch size tends to a limiting value. This limiting value is given by the fact, that the number of accepted calls is given for higher values of λ only by the termination rate of calls. (Rejected calls produce no messages).

Diagram 2 shows the load Y_{CPU} of the CPU of the switching computer SC as a function of λ. The curves have the same tendency as the curves in Diagram 1. Because of the relation $Y_{CPU} = b_m \cdot h$ (where h is the mean program run time for all different messages generated by one type of SRC or by the normal mix of SRC's, resp.), the curves show that the mean run time h is approximately independent of the mean arrival rate λ. The subscriber behavior, represented by one type of SRC or by the normal mix of SRC's, of course has a great influence on the mean program run times h.

The effect of a type of SRC or of the normal mix of SRC's on the program run times for the messages is also outlined in Diagram 3. This diagram shows the pdf of the program run times $P\{T_H \leq t\}$ for three types of SRC's. The curves show that the pdf's are strongly influenced by the subscriber behavior and therefore also the mean values. The pdf of the run times for the normal mix is not shown in this diagram, because it is nearly identical to that of type 4.

The traffic behavior is not only influenced by the subscribers but also by the system architecture and the processing speed of the CPU. This influence can be extracted from the following diagrams 4-7.

Diagram 4 shows the carried load per PCM channel and the load Y_{CPU} as a function of the mean arrival rate λ. The curve for ϱ increases monotonously with λ. This is due to the fact, that no call establishment is terminated by overload of the CPU, but only by trunk busy condition. The load of the CPU Y_{CPU} is limited by the fact, that calls, rejected by the concentrator require no CPU time.

Diagram 5 shows the mean waiting time w of all messages in the PIQ as a function of λ for two types of CPU (RTF =25,75) with and without overhead. The mean batch size increases monotonously to an upper bound. Therefore the mean waiting times increase with λ but are also limited, if the processing speed of the CPU is sufficient. A CPU with a RTF of 75 is totally overloaded for higher values of λ. To avoid overload with increasing λ the RTF has to be chosen carefully.

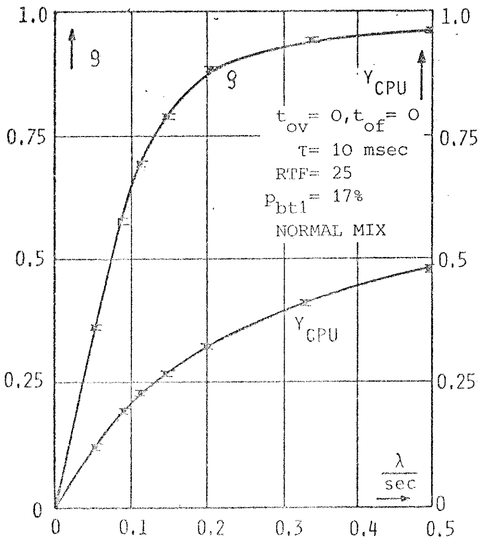


DIAGRAM 4: CARRIED LOAD ϱ ON THE PCM CHANNELS AND LOAD ON THE CPU Y_{CPU} VS. λ

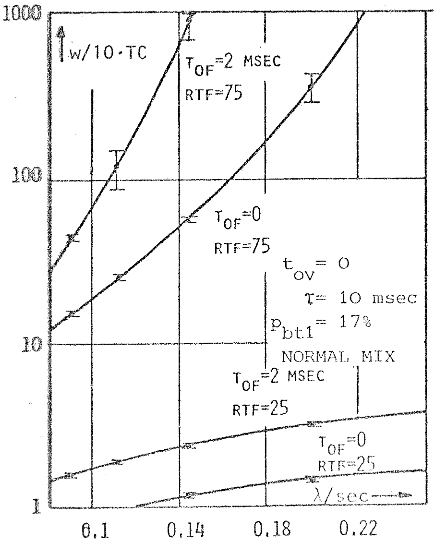


DIAGRAM 5: MEAN WAITING TIME w VS λ (TC = 1000 CYCLE TIMES OF THE REFERENCE CPU)

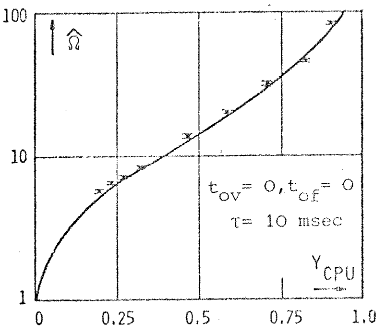
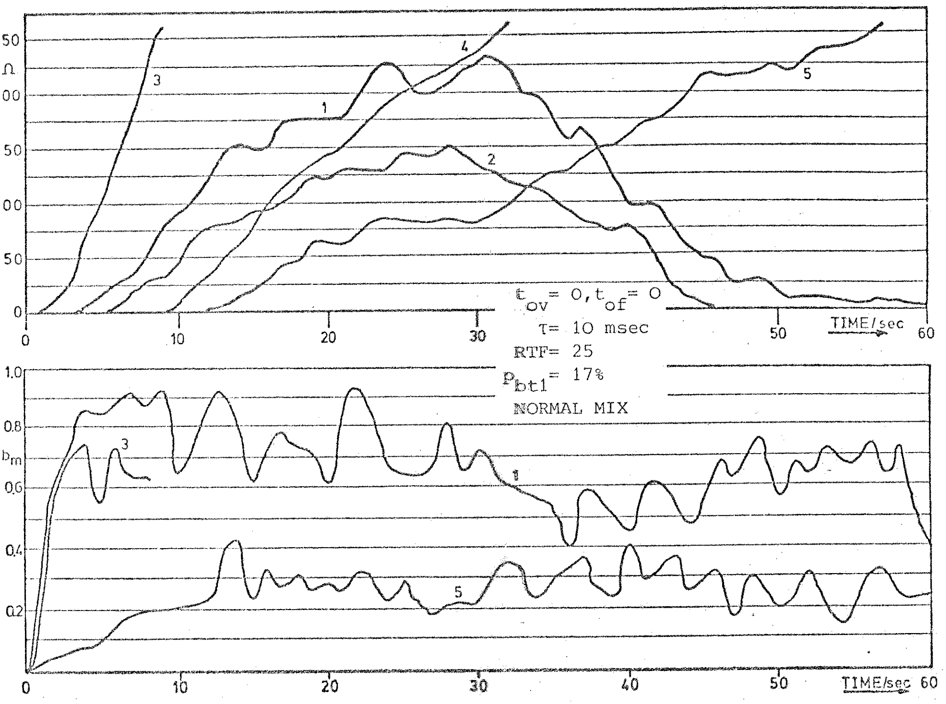


DIAGRAM 6: MAXIMUM LENGTH $\hat{\Omega}$ OF THE PIQ VS. LOAD ON THE CPU Y_{CPU}



curve	λ_p/sec	RTF
1	2	25
2	1	25
3	2	75
4	0.3	75
5	0.25	75

DIAGRAM 7: LENGTH OF THE PIQ AND MEAN BATCH SIZE b_m VS. TIME

Diagram 6 shows the maximum length of the PIQ as a function of Y_{CPU}. This diagram gives an indication for the necessary length of the PIQ if the load on the CPU is known and the overflow probability of the PIQ should be neglectable.

The above presented diagrams have shown mean values for interesting parameters of the system. To study the time dependent behavior of switching systems, these values are not sufficient. Therefore, the characteristic values (in Diagram 7 the mean queue length of the PIQ and the mean batch size) have been plotted as a function of time. The diagrams show curves for different values of λ and RTF.

The parameters for the arrival process are according to Fig.2. At time t=0 the system is empty (λ_n=0). At time t=+0 the mean arrival rate increases to λ_p. Curve 1 shows for instance that a mean arrival rate of λ_p=2·1/sec can be processed by the CPU (RTF=25) without loss in the PIQ, if the length of the PIQ is not less than 250. With growing time the number of processed service requests (messages) becomes greater than the number of incoming service requests. This is due to the fact, that the limitation effect of the concentrator (rejected calls don't require CPU time) limits the number of incoming service requests. The length of the PIQ diminishes therefore. The time until the PIQ has a stationary length (with respect to λ_p) can also be extracted from the diagram.

The rapid increase of curve 3 makes clear, that besides the above mentioned limitation effect additional load control procedures have to be implemented in the SC to avoid overload situations.

Furthermore the batch size b_m is plotted as a function of time in Diagram 7.

5. CALCULATION AND NUMERICAL RESULTS

5.1 Model of the Switching Computer

The model of the switching computer used for mathematical analysis is shown in Fig.7.

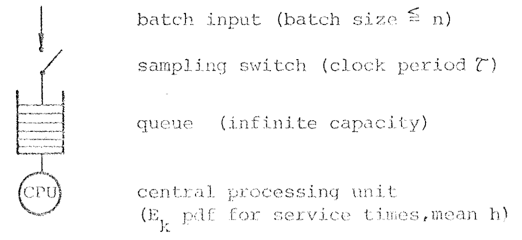


FIG. 7: MODEL OF THE SWITCHING COMPUTER

The model consists of a single server (CPU) with a queue corresponding to the PIQ in Fig.4 The input process from the switching periphery is modelled by batch arrivals of messages (in the following denoted as requests) at equidistant sampling instants (period τ). The batch size is an independent random variable G. The probabilities P{G=i}=g(i) with i=0,1...,n for the batch size are assumed to be independent of the sequence of sampling instants and of the service times of the requests.

The service times of the requests are distributed according to an Erlangian pdf of the k-th order (E_k). The queue discipline is service in order of arrival between batches and first-in, first-out within the members of a batch. The service process is according to section 3.2.2 (Fig.5) but with variable overhead t_{ov} =0.

5.2 Calculation

Characteristic traffic values such as the mean waiting time w and the probability of waiting W can be determined via the probabilities of state. Therefore in Section 5.2.1 the probabilities of state are derived and the characteristic traffic values are determined in Section 5.2.2 and 5.2.3, resp.

5.2.1 Probabilities of State

The probabilities of state here are determined for the instants just before the sampling instants by means of the Imbedded Markov Chain /12,13/.

The state of the system is described by the random variable $X(t_i')$, which represents the number of requests in the system (waiting or in service) just before the sampling instant t_i, where $t_i' = t_i-0$.

In the literature /13,14/ it has been shown that the service process in a $GI|E_k|1$ system (mean service time h) is equivalent to the service process in a system, where instead of single arrivals batches of fixed size k arrive. Each member of a batch requires a negative exponentially distributed service time with mean $h' = h/k$.

This equivalence is based on the fact, that in case of negative exponentially distributed service times, the total time to serve a whole batch can be described by the E_k pdf.

For the calculation of the model according to Fig.7, a subsidiary model (SM) is introduced, which is characterized by

- constant interarrival times between batches (c.f. 3.2.2)
- the random batch size $G' = k \cdot G$, which means that if in the original model (OM) a batch of size i arrives in the SM a batch of size k·i arrives
- negative exponentially distributed service times with mean $h' = h/k$
- the random variable $X'(t_i')$, which represents the number of requests in the system at the instant t_i'

The probabilities $P\{G'=j\} = g'(j)$ are given by

$$g'(j) = g(i) \qquad j = k\cdot i \tag{1a}$$

$$g'(j) = 0 \qquad j \neq k\cdot i \tag{1b}$$

with maximum batch size $m = n\cdot k$ (1c)

Because of the overhead t_{of}, the requests in the SM can only be served during the service period $t_s = \tau - t_{of}$. The number of requests which have been completely served during this period are given by the random variable R. Because of the negative exponentially distributed service times, the probabilities $P\{R=s\} = r(s)$ are given by

$$r(s) = \begin{cases} e^{-a}\cdot\frac{a^s}{s!} & s \geqq 0 \\ 0 & s < 0 \end{cases} \qquad \text{with} \quad a = t_s\cdot\frac{k}{h} \tag{2a}$$

$$r_m(s) = \sum_{i=s}^{\infty} e^{-a}\cdot\frac{a^i}{i!} = 1-e^{-a}\cdot\sum_{i=0}^{s-1}\frac{a^i}{i!} \tag{2b}$$

Equation (2a) holds, if the server remains busy during the service period t_s, and Equation (2b) holds, if the server becomes idle during t_s.

The transition probability $P\{X'(t'_{i+1})=v \mid X'(t'_i)=u\} = p(u,v)$ is independent of the sampling instants t'_i (i=0,1,...). It is given by

$$p(u,v) = \begin{cases} \sum_{\nu=0}^{m} g'(\nu)\cdot r(u+\nu-v) & 0 < v \leqq u+m \\ 0 & v > u+m \end{cases} \tag{3a}$$

$$p(u,0) = \sum_{\nu=0}^{m} g'(\nu)\cdot r_m(u+\nu) \qquad v = 0 \tag{3b}$$

The steady state probabilities $p'(j) = \lim_{i\to\infty} P\{X'(t'_i) = j\}$ now are determined by

$$p'(j) = \sum_{i=0}^{\infty} p'(i)\cdot p(i,j) \qquad j = 0,1,\ldots \tag{4a}$$

$$\sum_{i=0}^{\infty} p'(i) = 1 \tag{4b}$$

To solve this system of linear equations the generating function

$$Gx(z) = \sum_{x=0}^{\infty} p'(x)\cdot z^x \tag{5}$$

is introduced. Schwaertzel /15/ showed, that the generating function Gx(z) is given by

$$Gx(z) = \sum_{\nu=0}^{m} \frac{1-\delta_\nu}{1-z\delta_\nu} \tag{6}$$

where δ_ν ($\nu=1,\ldots,m$) are the m roots of the Equation:

$$\delta^m = \sum_{\nu=0}^{m} g'(m-\nu)\cdot\delta^\nu\cdot e^{-t_s(1-\nu)} \tag{7}$$

In /15/ formulas for the roots of Equation (7) are given. The probabilities p'(j) are given by

$$p'(j) = \frac{1}{j!}\cdot\frac{d^j}{dz^j}\,Gx(z) \tag{8}$$

For practical computation, Schwaertzel has given recurrence formulas for the probabilities p'(j).

Now, the probabilities of state for the original model OM can be calculated by the following considerations:

The arrival of one request in the OM causes the arrival of one batch of size k in the SM. A request in the OM is ready with service only if in the SM all k requests of the same batch are served.

Therefore e.g. the probability p(1) is given by

$$p(1) = \sum_{i=1}^{k} p'(i) \tag{9a}$$

In general, the probabilities p(j) for the OM are given by

$$p(j) = \sum_{i=(j-1)k+1}^{j\cdot k} p'(i) \qquad p(0) = p'(0) \tag{9b}$$

5.2.2 Mean Waiting Time w of a request

To determine the mean waiting time w of a request in the OM (with overhead), at first, the mean waiting time w^* of a request in a system without overhead is considered.

According to /16,17/, the mean waiting time

$$w^* = w_1 + w_2 \tag{10}$$

is composed of two components

- the mean waiting time w_1 of the first member of an arriving batch
- the mean waiting time w_2 of the residual members of the batch

The mean waiting time w_1 in the OM corresponds to the mean total time to serve all requests, which are in the OM (waiting or in service) just before the sampling instant. This time is identical with the mean total time to serve all requests which are in the SM just before the sampling instant. Therefore, w_1 is given by

$$w_1 = E[X']\cdot h' = h'\cdot\frac{d}{dz}\,Gx(z)\Big|_{z=1} \tag{11}$$

The mean waiting time w_2 is given according to /16,17/ by

$$w_2 = \frac{1}{2}\cdot\left[\frac{VAR[G]}{E[G]} + E[G] - 1\right]\cdot h \tag{12}$$

where $E[G] = \sum_{i=0}^{n} i\cdot g(i)$ and $VAR[G] = \sum_{i=0}^{n} (i-E[G])^2\cdot g(i)$

The mean waiting time w in the OM with overhead can be determined according to /18/ by means of the mean queue length.

Fig.8 shows as an example the queue length Ω (number of requests waiting in the queue) during a sampling period.

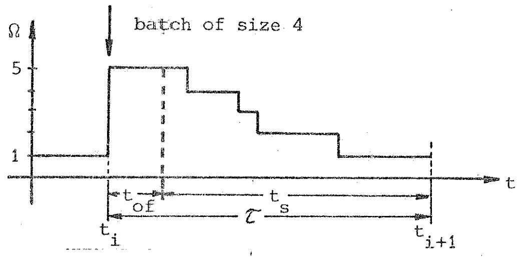


FIG. 8: NUMBER OF REQUESTS IN THE QUEUE

The service process during the period t_s in a system with overhead is equivalent to the service process in a system without overhead but with the sampling period $\tau^* = t_s$, because during the overhead period t_{of} no requests are served. Therefore, the mean queue length Ω_s during the service period in a system with overhead is equal to the mean queue length in a system without overhead but with the sampling period $\tau^* = t_s$. The mean queue length Ω_s is

$$\Omega_s = \frac{E[G]}{t_s}\cdot w^* \tag{13}$$

with w^* according to Equation (10).

The mean queue length Ω_{of} during the overhead period t_{of} corresponds to the sum of the mean number of requests in the queue Ω_b just before the sampling instant and of the mean number of arriving requests at the sampling instant.

$$\Omega_{of} = \Omega_b + E[G] \tag{14}$$

Ω_b can be determined by means of Equation (9)

$$\Omega_b = \sum_{i=1}^{\infty} i\cdot p(i+1) = \sum_{i=1}^{\infty} i\cdot\sum_{j=ik+1}^{(i+1)k} p'(j) \tag{15}$$

with p'(j) according to Equation (8).

The mean queue length during the sampling period is

$$\overline{\Omega} = \frac{t_s}{\tau}\,\Omega_s + \frac{t_{of}}{\tau}\,\Omega_{of} \tag{16}$$

With the general relation $\Omega = \lambda\cdot w$, the mean waiting time w is

$$w = \frac{\overline{\Omega}}{\lambda} = w^* + \left[\frac{\Omega_b}{E[G]} + 1\right]\cdot t_{of} \tag{17}$$

with w^* from Equation (10) and Ω_b from Equation (15).

5.2.3 Waiting Probability W

The waiting probability W here is defined as the probability, that an arriving request has to wait longer than the time t_{of}, because the server is busy after the overhead period t_{of}.

To determine the waiting probability W, at first, the probability 1-W that an arriving request has to wait the time t_{of}, is determined according /17/ by

$$1-W = \frac{p(0)\cdot(1-g(0))}{E[G]} \qquad (18)$$

The probability W is then given by

$$W = 1 - \frac{1-g(0)}{E[G]} \prod_{\nu=1}^{m} (1-\delta_\nu) \qquad (19)$$

where δ_ν are the m roots of Eq.(7).

5.3 Results

The Diagrams 8 and 9 show calculated values for the mean waiting time w and for the waiting probability W in comparison with values obtained by simulation. For calculation the batch size was assumed to be distributed according to a binomial distribution function with maximum batch size equal to 16. (This assumption was justified by simulation results). The differences between the calculated results and the results obtained by simulation in Diagram 9 bases on the assumption of independence of the batch sizes of consecutive sampling instants, which in reality (simulation) doesn't exist. This dependence, however, has only an influence for small values of the sampling period τ (Diagram 8).

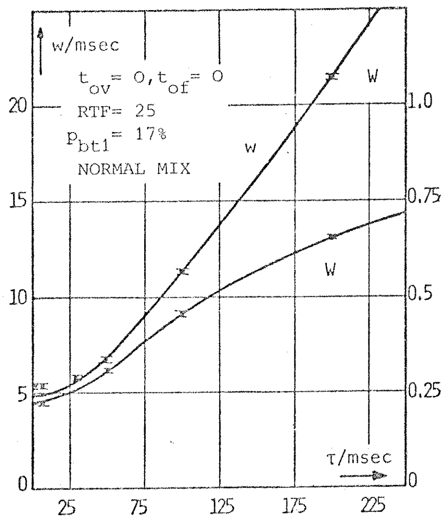


DIAGRAM 8: MEAN WAITING TIME w AND WAITING PROBABILITY W VS. τ

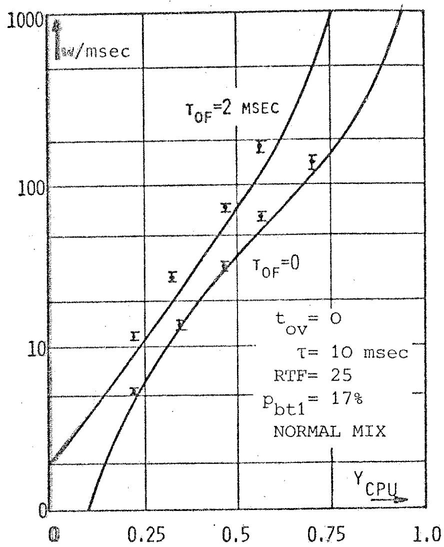


DIAGRAM 9: MEAN WAITING TIME w VS. Y_{CPU}

6. CONCLUSION

The traffic behavior of an experimental switching system with special regard to the central control has been investigated. For this investigations a model of the real system was developped. To obtain characteristic traffic values of the real system, the model was analyzed by simulation and a simplified version of the model was analyzed by calculation.

The investigations have shown the influence of the architecture of the system as well as of the subscriber behavior on the performance of the system.

The preprocessing units of the switching system enhance remarkably the performance of the central control (CPU) and provide a good load limitation effect for the CPU.

Further studies will be carried out with respect to

- variable overhead
- load control mechanisme in the CPU
- external traffic
- data traffic.

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