THEORETICAL ASPECTS OF FINITE SOURCE INPUT PROCESS SIMULATION

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ABSTRACT

By applying the event-by-event simulation method for a finite number of sources, each source must be treated individually. For large systems this method is very computer time and storage consuming. To meet this problem an approximate method for simulating the input process is investigated with respect to the accuracy.

The inter-arrival times of calls from an idle source are assumed to be negative exponentially distributed. The suggested simulation method is however applicable to all input processes generated by a finite number of sources.

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1. INTRODUCTION

The mathematical investigation of telecommunication systems often leads to large systems of linear equations or differential equations for which the subsequent analytical or numerical treatment mostly is beyond human or computorial power. Furthermore the structures of the systems are of such a complexity that an explicit mathematical solution or numerical evaluation is impossible. In all these cases, the simulation of the systems on a computer is a useful tool for the determination of the traffic characteristics of the system. An excellent survey on the principles of simulation is given in /1/.

In many applications, where the time-dependent behaviour of the system is investigated, the event-by-event simulation method has to be used. For the specification of the input process, the number of traffic sources is of importance. In respect to the number of sources there are two different source-models: Infinite number and finite number of sources. In the case of infinite number of sources the input process is described independently of the busy sources and in the case of finite number of sources the intensity of the input process depends on the number of busy sources.

The following considerations deal only with the simulation of the input process caused by a finite number of sources. The inter-arrival times of calls from an idle source are assumed to be negative exponentially distributed. This assumption often meets reality quite well /2/.

2. SIMULATION METHODS

2.1 INDIVIDUAL SOURCE SIMULATION

The input process of calls is assumed to be originated by a finite number of sources (subscribers). Therefore the sources must be treated individually, i.e. each source must be simulated and for each idle source a call must be generated and organized in a time table. This method is called INDIVIDUAL SOURCE SIMULATION (ISS).

The time passed when a source becomes idle until the next call of this source arrives, i.e. the inter-arrival time t, can be calculated by the formula

$$t = \frac{-\ln v}{\alpha}, \qquad (1)$$

where α is the mean value of the arrival rate of an idle source and v is an equally-distributed random variable ($0 \le v \le 1$).

By simulating each source individually for large systems (great number of sources) a lot of information about inter-arrival times must be stored, i.e. extensive storage capacity is necessary. Furthermore, the filing of future events (calls) in the time table is very computer time consuming.

2.2 COMMON SOURCE SIMULATION

The disadvantage of the ISS method can be overcome, if an approximate method for simulating the input process is applied. This method is called COMMON SOURCE SIMULATION (CSS).

By this simplified method, all individual q sources are replaced by one "common source". The arrival rate of this common source is $(q-x)\cdot\alpha$, where q is the total number of sources and x is the number of busy individual sources. If a call arrives, the inter-arrival time for the next call can be determined by the formula

$$t = \frac{-\ln v}{(q-x)\cdot\alpha} \qquad (2)$$

where v is again an equally-distributed random variable ($0 \leq v \leq 1$) and α is the mean value of the arrival rate of an idle source.

This CSS method reduces the necessary storage capacity and the computer time for simulation, because only one call must be planned in the time-table. Each following call is generated when the previous one is executed and cleared from the time-table.

The disadvantage of this method is, that the input process is reproduced only approximately, because the sources becoming idle during the inter-arrival time of the succeeding call have not been considered for the determination of this inter-arrival time according to equ.(2).

3. COMPARISON OF THE TWO SIMULATION METHODS

The input process to the investigated telecommunication system can be characterized by the distribution function of the inter-arrival times of the calls arriving at the system. These distribution functions of the inter-arrival times resulting from one of the two methods ISS or CSS respectively cannot be compared directly with each other because the (exact) ISS input process depends on the distribution function of the holding times.

In order to investigate the accuracy of the CSS method a simple model of a trunk-group arrangement for the simulation is regarded.

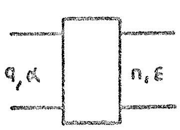


Fig.1 Simulation model

The model represents a full available trunk group with q inputs (sources) and n outlets. The mean arrival rate of calls from an idle source is α. The distribution functions of the holding times and of the inter-arrival times are assumed to be negative exponentially distributed. The mean arrival rate of an idle source is α and the mean termination rate is ε. These assumptions are valid for many applications /2,3/.

The simple model has been chosen because, for this system, the explicit solutions of the traffic characteristics are known.

For the investigation of the accuracy of the CSS method the effect of the input process is considered which is reflected in the state probabilities p(x) (x trunks are busy).

3.1 PROBABILITIES OF STATE BY INDIVIDUAL SOURCE SIMULATION (ISS)

The input process to the trunk group is represented by a Bernoulli-process. The holding times are negative exponentially distributed. The probabilities of state in statistic equilibrium are given by the Erlang's Bernoulli formula:

$$p(x) = \frac{\binom{q}{x}\left(\frac{\alpha}{\varepsilon}\right)^x}{\sum_{i=0}^{n}\binom{q}{i}\left(\frac{\alpha}{\varepsilon}\right)^i} \qquad (3)$$

3.2 PROBABILITIES OF STATE BY COMMON SOURCE SIMULATION (CSS)

The input process to the trunk group is represented by a regenerative input process because the inter-arrival times are independent random variables. The holding times are negative exponentially distributed.

The probabilities of state can be calculated by the method of the imbedded Markov chain /4/.

The set of regeneration points consists of the instants when calls arrive to the system. The state $x(t_r)$ of the system at the instant t_r depends only on the state $x(t_{r-1})$ at the instant t_{r-1}. The reason is, that the inter-arrival periods are independent and furthermore the probability that a call will terminate during the subsequent time intervall is independent of the age of the call (neg. exp. distributed holding times).

Considering the termination process between two regeneration points r-1 (the system is in the state $x(t_{r-1})=i$) and r (the system is in the state $x(t_r)=j$), the transition probability that within the time interval $t=t_r-t_{r-1}$ between the regeneration points (i-j) calls will terminate, is given by the formula:

$$p(i;j,t) = \binom{i}{j} e^{-j\varepsilon t}\,(1-e^{-\varepsilon t})^{i-j} \qquad (4)$$

$$0 \leq j \leq i$$

If a call arrives and finds i trunks busy the distribution function for the inter-arrival period of the succeeding call is given by

$$F_i(\leq t) = 1-e^{-\lambda_i t}$$
$$\lambda_i = (q-i-1)\alpha \qquad (5)$$

The formula takes into account that after the arrival of the call i+1 trunks are busy.

If a call arrives and finds i trunks busy the transition probability p(i;j) that the system will pass within the inter-arrival time of the succeeding call from state i to the state j is given by:

$$p(i;j) = \int_0^\infty \binom{i+1}{j} e^{-j\varepsilon t}(1-e^{-\varepsilon t})^{i-j+1}\,dF_i \qquad (6)$$

From equation (6) with equation (5) follows:

$$p(i;j) = \binom{i+1}{j}\lambda_i \sum_{\zeta=0}^{i+1-j} \frac{\binom{i+1-j}{\zeta}(-1)^{\zeta}}{j\varepsilon+\zeta\varepsilon+\lambda_i} \quad (7a)$$

$$i \le n-1 \quad j \le i+1 \quad (7b)$$

If i=n, the arriving call finds all trunks busy and cannot change the number of busy trunks, therefore

$$p(n;j) = p(n-1;j) \quad (8)$$

If the system is in statistic equilibrium the following equation is satisfied:

$$\sum_{i=0}^{n} p(i)\cdot p(i;j) = p(j)$$

With the conditions (7b) we get:

$$\sum_{i=j-1}^{n} p(i)\, p(i;j) = p(j) \quad (9)$$

Furthermore the normalization condition gives

$$\sum_{j=0}^{n} p(j) = 1 \quad (10)$$

Equation (9) and (10) give a linear system of equations. For this linear system of equations no explicit solution can be derived and therefore it is solved numerically by an iterative method.

3.3 COMPARISON

The probabilities of state which result from the two simulation methods for a system ($q=40$, $n=10$, $\alpha/\varepsilon=0.10$) are shown in figure 2.

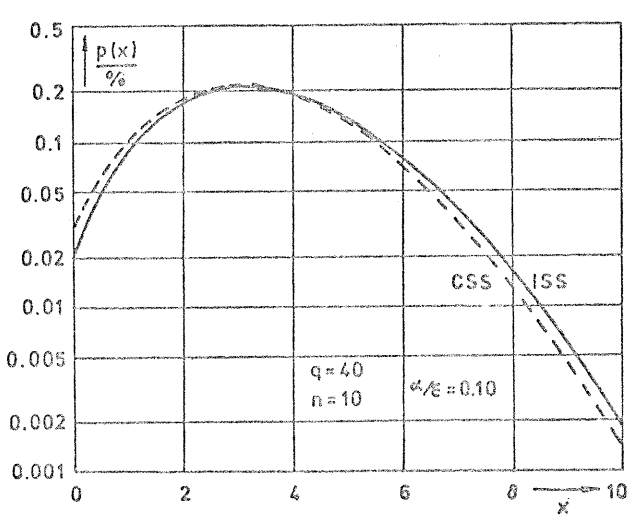


Fig.2 Probabilities of state p(x)

The diagram shows that the values of the probabilities of state x resulting from the ISS method are for low values of x smaller than the values resulting from the CSS method.

For high values of x the values resulting from the ISS method are greater than the values resulting from the CSS method.

The reason for this effect is that by using the CSS method the arrival rate during the inter-arrival time between consecutive calls is constant whereas by the ISS method the arrival rate is changing when a busy source becomes idle. In the case of the CSS method the sources which become idle during the inter-arrival time of the succeeding call have not been taken into account when generating this succeeding call. Therefore the global arrival rate of all sources of the CSS method is smaller than that of the ISS method.

The difference between the two curves is according to the above mentioned reasons systematic and has the same tendency for all systems.

The inaccuracy of the CSS method takes a severe effect on the system for the state x=n (all n trunks are busy). This state is relevant for the congestion of the system.

The relative deviations r(n) of the probability of state p(n) between the exact (ISS) simulation and the approximate (CSS) simulation for a group with n=10 lines is shown in fig. 3. Parameter in the figure is the ratio of the number of subscribers q to the number of trunks n. Many numerical evaluations have shown that r(n) depends mainly on q/n and not essentially on the absolute value of n.

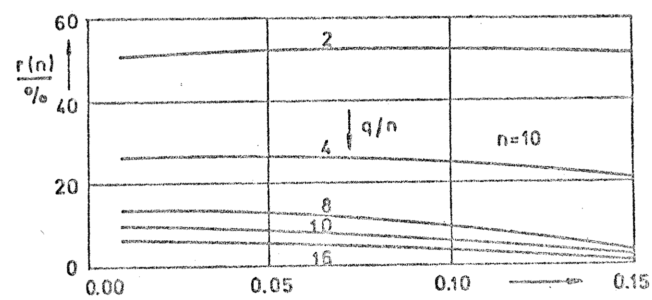


Fig.3 Relative deviation of the probability of state p(n).

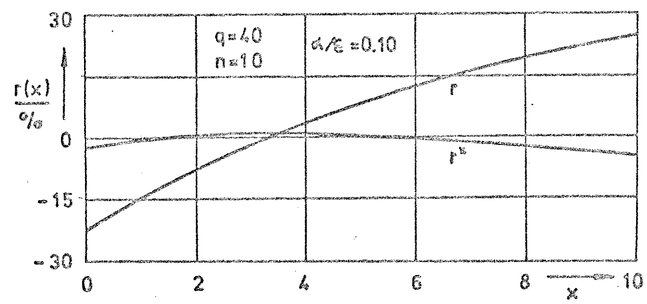
If an inaccuracy of ≤ 10% in the probability of state p(n) is admitted the CSS method can be used for systems with a ratio of q/n ≥ 10.

The distribution functions of the inter-arrival times were measured for the two simulation methods by simulation. The test results showed that the curves for q/n = 10 of the two methods did only deviate within the 95 % confidence intervalls.

4. MODIFICATION OF THE COMMON SOURCE SIMULATION METHOD

The probabilities of state p(x) for low values of x resulting from the CSS method are systematically greater than the probabilities obtained by the ISS method, i.e. the mean arrival rate of all sources is greater for the ISS method than for the CSS method; (cf. section 3.3)

The mean arrival rate of an idle source for the CSS method can be increased so that the traffic carried on the trunk group, which is obtained by the two simulation methods, takes the same value. The resulting relative deviation r(x) and r*(x) resp. of the probability of state p(x) is shown in fig. 4. (r* is the relative deviation between the exact (ISS) and the approximate (CSS) simulation method in case of the CSS method with a "corrected" arrival rate which was increased by a factor 1.05 . r is the relative deviation in case of the CSS method without increased arrival rate.)



Fi[illegible] Relative deviation of the probability of state p(x)

The curves show that for an increased arrival rate the relative deviation decreases remarkably.

If, analogously to chapter 3.3, a relative deviation of $\leq 10\%$ of the probability of state p(n) is allowed, the modified CSS method can be applied for systems with a ratio of $q/n \geq 2$. Instead of the true value α of the mean arrival rate one uses an increased value $\alpha^* = \alpha \cdot k$.

The values of the factor k for multiplying the mean arrival rate α for the modified CSS method are shown for a trunk group with q/n=2 in fig.5. The factor k varies for different numbers of n.

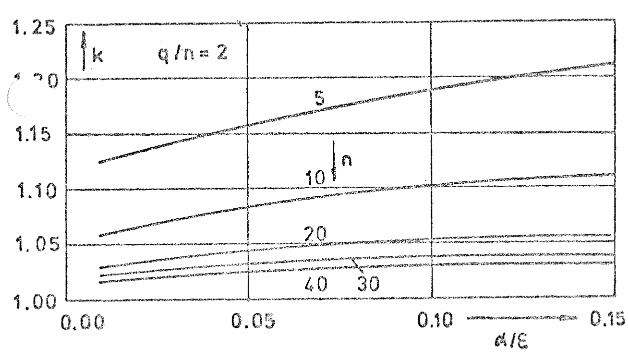


Fig.5 Multiplying factor k

In practice the multiplying factor k is not known. Therefore for the simulation with the modified CSS method a mean arrival rate α^* is assumed. In order to determine the true value α the traffic carried is measured in simulation. For this value of the traffic carried the value of α for the ISS method is determined. This is be done iteratively by means of the formula for p(x) (eq.3). The value α, determined in this way, is the true value of the mean arrival rate in simulation.

5. CONCLUSION

A method for an approximate simulation of the input process of a finite number of sources was described. It was shown that for the ratio q/n (source/trunks) ≥ 10 the COMMON SOURCE SIMULATION method can be used without significant inaccuracy. If the mean arrival rate of a source is increased for simulation, the method can be used even for values of $q/n \geq 2$.

REFERENCES

/1/ Kosten, L. : Simulation in traffic theory.
6. ITC München 1970, Congressbook, 411/1-8.

/2/ Iversen, V.B., Nygard, J. : Investigations of traffic arrival processes based upon extensive traffic measurements.
6. ITC München 1970, Congressbook, 534/1-7.

/3/ Hayward, W.S., Wilkinson, R.I. : Human factors in telephone systems and their influence on traffic theory, especially with regard to future facilities.
6. ITC München 1970, Congressbook, 431/1-10.

/4/ Syski, R. : Introduction to Congestion Theory in Telephone Systems.
Oliver and Boyd, Edinburgh and London, 1960.