ON COMBINED DELAY AND LOSS SYSTEMS WITH NONPRE-EMPTIVE PRIORITY SERVICE

by

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Summary

The number of queue places is considered as being infinite for delay systems. It is finite for combined delay and loss systems which were investigated by H. Störmer /18/, and by J.R.W. Smith and J.L. Smith /17/. On account of their different importance and urgency calls are arranged in priority classes. Nonpre-emptive priority service and delay systems with infinite number of queue places were studied by A. Cobham /4/, /5/, S.A. Dressin and E. Reich /8/, H. Kesten and J.T. Runnenburg /13/, H. Störmer /19/, and other authors /7/, /11/. (See also /6/, /16/, /20/, /22/.)

This paper extends nonpre-emptive priority service to combined delay and loss systems. With a finite queue line a call may wait or may be lost. It is considered to be intolerable that a call of low priority is waiting in the queue line while an important call of high priority should be rejected, because the queue line is totally occupied. Nevertheless a call of high priority enters the queue line and begins waiting. A waiting call of lower priority is pushed away and is lost. The specifications of the system and traffic considered are:

1. combined delay and loss system with s queue places including the special case of delay system $s \to \infty$;
2. full-access trunkgroup of $n \geq 1$ trunks;
3. nonpre-emptive priority for calls which have occupied a trunk; for waiting calls priority pushing away in case of a combined delay and loss system;
4. independent holding times with one negative exponential distribution for all calls;
5. K priority classes with distinct Poisson inputs;
6. no defections;
7. lost calls cleared.

The stationary probability $p(\varrho_k, \leq k)$ - ϱ_k calls of priority class k or of higher priority are waiting in front of the totally occupied trunkgroup - is derived. From this the probabilities are deduced that an arriving call of priority class k

1. occupies a free trunk at once, or
2. is rejected at once, or
3. begins waiting and will be pushed away out of the queue line later, or
4. begins waiting and will occupy a trunk becoming idle.

The formulae are deduced for the mean waiting times

1. referred to all calls,
2. referred to delayed calls,
3. referred to calls which wait successfully, and
4. referred to calls which wait in vain.

(The calls belong to the priority class k.) The delay probability distribution of waiting times can be deduced from a system of differential equations given in this paper. The results determine the quality of service of the system considered.

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1. Introduction

1.1 Combined delay and loss system

Calls arrive at a connecting system. n trunks are leading to the aim desired by the calling subscriber. If there is an idle trunk at the arrival instant of a call, the call occupies this trunk at once. The trunkgroup of n trunks is fully available. If the trunkgroup is totally occupied, an arriving call is stopped and has to wait. The number of queue places is limited. Therefore the connecting system considered is a "combined delay and loss system". Sometimes it is termed "delay system with finite queue line". A call arrives during a period when the trunkgroup is totally occupied. If the call finds a free queue place, it begins waiting. No priorities are assumed. If all s queue places are occupied by calls waiting, a call arriving is dismissed and is lost. /18/, /17/, /21/, /20/, /16/, /3/

1.2 Quality of service

If a connecting system works as a loss system, the quality of service is given by the probability of loss. The quality of service of a delay system is determined by the probability of delay and by the probability of exceeding a waiting time. It is given by the probability distribution of waiting times referred to delayed calls. The mean value of this distribution is the mean waiting time of delayed calls. /3/ The analysis of a combined delay and loss system requires to know the probabilities

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of loss, of delay, and of exceeding a waiting time.

1.3 Priority

Demands which are connected from inlets to outlets of an exchange are not of the same importance. There are different types of messages. For example some messages may call for a physician or may report a catastrophe. They are most urgent. Others are less pressing, because they are private communications and conversations. It is desirable that the first class of messages should take precedence of the other. There are also different types of telegrams in the general telegraph exchange service or in special telecommunication networks. For common telegrams a much longer waiting time may be tolerated than for express telegrams. On account of their different importances and urgencies the calls may be arranged in priority classes. These classes are usually numbered $k=1,2,...K$ in order of decreasing urgency. Higher priorities correspond to smaller indices k of priority classes. The system considered here has K priority classes. /4/, /11/, /5/, /7/, /8/, /13/, /19/, see also /20/, /6/, /16/, /22/.

Calls of higher priority take precedence when waiting. The operation of the delay and loss system considered is specified by the following rules:

1. Calls having occupied a trunk are never interrupted (nonpre-emptive priority).
2. Calls of higher priority queue up in front of calls of lower priority.
3. The order of service is "first come - first served" or strict queueing within each priority class.

1.4 Holding times

All calls belonging to any priority class have the same probability distribution of holding times (respectively service times). The holding times are mutually independent random variables, each with negative exponential distribution. The mean holding time is unity.

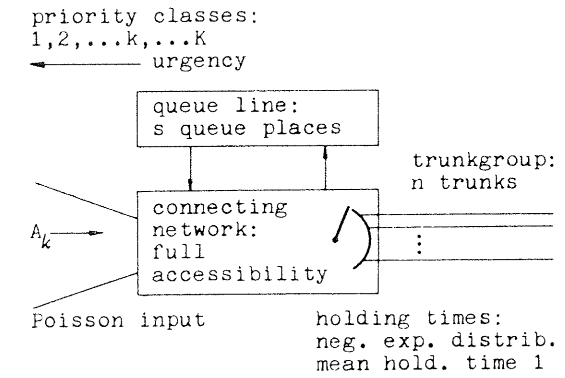

Fig.1. Combined delay and loss system with priorities

1.5 Traffic offered

The average number of calls of priority class k arriving per mean holding time is A_k, i.e. the traffic offered of the priority class k. For each priority class a distinct Poisson input is assumed. The calls arrive "individually and collectively at random". The pooled traffic input of calls of arbitrary priority classes is a Poisson input, too. The total traffic offered is A.

2. Probabilities of state

2.1 Calls of arbitrary priority classes

At first priority classification is ignored. The state of the system at any instant is given by the total number x of calls which are present in the system. In the number x both, the calls occupying a trunk and the calls waiting on a queue place are contained. (See fig.2.) Provided that the process is stationary the probabilities of state are independent of time. Let be p_x the stationary or equilibrium probability that the system is in state x. The probabilities are defined by formulae given by H. Störmer /18/, and by J.R.W. Smith and J.L. Smith /17/.

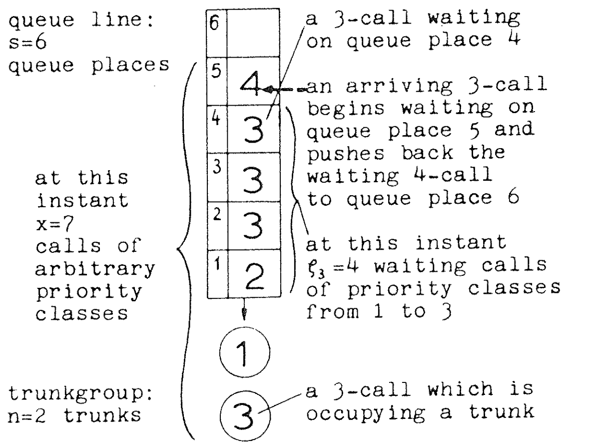

Fig.2. An example for a momentary random state

2.2 Calls of priority classes from 1 to k

The influence of priorities will be investigated henceforth. In the sequel calls of priority class k will be termed k-calls /13/. A k-call arriving inserts in the queue in front of waiting calls of lower priority. The new k-call finds ζ_k waiting calls of the priority class k or of higher priorities.

The state of the system is determined by the random number ζ_k of calls of priority classes from 1 up to k which have been waiting in front of the totally occupied trunkgroup. Let be $p(\zeta_k \leq k)$ the stationary probability that both all n trunks are busy and ζ_k calls of the priority classes from 1 up to k waiting.

The number of waiting calls ζ_k is diminished by one when any of the n occupations ends. The call waiting at the head of the queue occupies the trunk which has become idle. The probability density of ending is n, because the mean holding time is the unit of time.

The number of waiting calls ζ_k is raised by one, when a call of one of the priority classes from 1 up to k arrives. The call probability density is /19/

$$A(\leq k) = A_1 + A_2 + \dots + A_k \qquad (1)$$

The state that both, the trunkgroup is totally occupied, and no call of priority classes from 1 to k is waiting, has the probability $p(0,\leq k)$. The next call arriving must wait. If it belongs to one of the priority classes from 1 to k, ζ_k is raised to the value 1.

The queue line with s queue places is common to all the calls of arbitrary priority classes. Even if all s queue places should be occupied, the priority rule is in force: Calls of higher priority queue up in front of calls of lower priority. If all s calls waiting are of the same priority as the call arriving or even of higher priority than it, the call arriving is rejected. If, on the other hand, at least one call of lower priority is waiting, the arriving call begins waiting. It inserts in the queue behind the calls of higher and of the same priority which have been waiting. All calls of lower priority are pushed back by one place in the queue. The call which has been waiting on queue place s finds no queue place left. Although it had been waiting up to now, it is pushed away and lost.

2.3 Lost calls cleared and no defections

Lost calls have either been rejected or have been pushed away. Both kinds of lost calls may be disregarded. They leave the system and have no effect on the traffic input: lost calls cleared. There are no retrials. /16/

Each call on any queue place continues waiting till it is allowed to occupy a trunk or till it is pushed away by an arriving call of higher priority. No call refuses to wait; waiting is not conditional: no defections /16/, /14/.

2.4 State diagram and equations of state

The state ζ_k=s means that all s queue places are occupied by waiting calls of the priority class k or of a higher priority. Whether an arriving call of priority classes from 1 to k will either be rejected or will begin waiting by pushing away a call which has been waiting on queue place s till now: in both cases the state ζ_k=s does not change. It is left only by the end of any of the n occupations. The ending rate is n and is valid for all other states from ζ_k=0 up to ζ_k=s, too.

A k-call may arrive during a period when the trunkgroup is totally occupied. The question whether the k-call will either be rejected or will begin waiting, depends merely on the number of waiting calls of priority classes from 1 to k. The transitions from state ζ_k to the neighbouring state ζ_k+1 do not depend on the number of waiting calls of priority classes from k+1 to K. The set of possible states extends from ζ_k=0 to ζ_k=s.

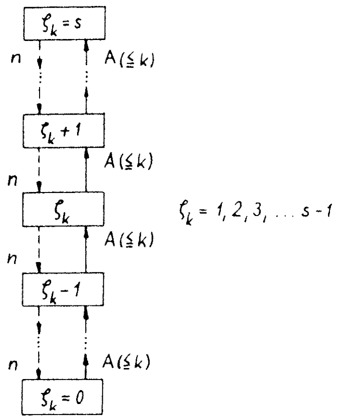

Fig.3. State diagram

By reference to the state diagram it can be seen that the equations of statistical equilibrium for the delay and loss system considered reduce to

$$A(\leq k)\cdot p(\zeta_k-1,\leq k) = n\cdot p(\zeta_k,\leq k) \qquad (2)$$

for ζ_k=1,2,...s and k=1,2,...K

At any state all n trunks are busy. Otherwise stated, at least n calls of arbitrary priority classes are present in the system.

$$\sum_{\zeta_k=0}^{s} p(\zeta_k,\leq k) = \sum_{x=n}^{n+s} p_x = E \qquad (3)$$

To repeat the specifications of the connecting system and traffic considered:

1. combined delay and loss system with s queue places, and delay system with an infinite number of queue places;
2. full-access trunkgroup of $n\geq 1$ trunks;
3. nonpre-emptive priority for calls which have occupied a trunk; for waiting calls priority pushing away in case of delay and loss system;
4. independent holding times for all calls, negative exponential distribution with mean value 1;
5. K priority classes with distinct Poisson inputs, A_k being the traffic offered of priority class k;
6. no defections;
7. lost calls cleared, no retrials.

2.5 Probability of blocking

The probability E is the time congestion which is the probability for "the condition where the further establishment of a connection is stopped (blocked), because the submitted occupation (call) cannot be extended at once" /3/. Write E for the probability of blocking or time congestion following the notation of A.K. Erlang and the papers of various Swedish teletraffic theorists.

Write α for the specific traffic offered, i.e. the total traffic offered per trunk:

$$\alpha = \frac{A}{n} = \frac{A_1 + A_2 + \ldots + A_k + \ldots + A_K}{n} \quad (4)$$

A.K. Erlang gave the probability of blocking for a loss system /2/:

$$E = E_{1,n}(A) = \frac{\frac{A^n}{n!}}{\sum_{x=0}^{n} \frac{A^x}{x!}} \quad (5)$$

H. Störmer /18/, and J.R.W. Smith and J.L. Smith /17/ derived the probability of blocking for a combined delay and loss system:

$$E = \frac{\frac{1-\alpha^{s+1}}{1-\alpha}}{\frac{1}{E_{1,n}(A)} + \alpha \cdot \frac{1-\alpha^s}{1-\alpha}} \quad \text{for } s=0,1,2,\ldots \text{ finite} \quad (6)$$

For the special case of total traffic offered A being equal to the number of trunks n:

$$E = \frac{s+1}{\frac{1}{E_{1,n}(n)} + s} \quad \text{for } \alpha=1 \text{ and } s=0,1,2,\ldots \text{ finite} \quad (7)$$

A.K. Erlang deduced the probability of blocking for a delay system /2/:

$$E = E_{2,n}(A) = \frac{\frac{1}{1-\alpha}}{\frac{1}{E_{1,n}(A)} + \frac{\alpha}{1-\alpha}} \quad \text{for } \alpha<1 \text{ and } s \to \infty \quad (8)$$

If the traffic offered A equals or exceeds the number of trunks n, the trunkgroup is always totally occupied:

$$E = 1 \quad \text{for } \alpha \geq 1 \text{ and } s \to \infty \quad (9)$$

2.6 Probabilities of state for combined delay and loss systems

The s equations of state (2) together with the normalization condition (3) determine the solution. Write $\alpha(\leq k)$ for the traffic offered of priority classes from 1 to k per trunk:

$$\alpha(\leq k) = \frac{A(\leq k)}{n} = \frac{A_1 + A_2 + \ldots + A_k}{n} \quad (10)$$

For brevity write

$$[\alpha(\leq k)]^j = \alpha(\leq k)^j \quad (11)$$

for the powers of $\alpha(\leq k)$.

Thus the probability of state that at an arbitrary instant ζ_k calls of priority classes from 1 up to k are waiting in front of n occupied trunks turns out to be

$$p(\zeta_k, \leq k) = \alpha(\leq k)^{\zeta_k} \cdot \frac{1-\alpha(\leq k)}{1-\alpha(\leq k)^{s+1}} \cdot E \quad \text{for } \zeta_k = 0,1,\ldots s \quad (12)$$

Both in loss systems and in combined delay and loss systems the stochastic traffic process reaches stationarity for any value of traffic offered. The reason is: there are lost calls because of the finite number of queue places.

If the traffic offered $A(\leq k)$ equals the number of trunks n, all $s+1$ states are equally probable.

$$p(\zeta_k, \leq k) = \frac{E}{s+1} \quad \text{for } \alpha(\leq k)=1 \text{ and } \zeta_k=0,1,\ldots s \quad (13)$$

For the lowest priority (i.e. priority class k=K) eq. (12) and (13) change to equations which are valid for the combined delay and loss system without priorities /18/, /17/.

2.7 Probabilities of state for delay systems

For delay systems the set of equations of state (2) contains an infinite number of equations since the number of queue places tends to infinity. If the traffic offered $A(\leq k)$ of priority classes from 1 to k is inferior to the number of trunks n, the solution is

$$p(\zeta_k, \leq k) = \alpha(\leq k)^{\zeta_k} \cdot [1-\alpha(\leq k)] \cdot E \quad (14)$$
$$\text{for } \alpha(\leq k)<1 \text{ and } \zeta_k=0,1,\ldots \text{ and } s \to \infty$$

This formula was derived for the case n=1 by S.A. Dressin and E. Reich /8/, /16/. Eq. (8) or (9) yield the probability of blocking E for n=1. Without the factor E the probabilities $p(\zeta_k, \leq k)$ belong to a geometric distribution. /10/

For calls of priority classes from 1 up to k* the stochastic traffic process reaches

$$\text{stationarity, if } A(\leq k^*) < n, \text{ but } A(\leq k^*+1) \geq n \quad \text{for } s \to \infty, \quad (15)$$

/4/, /5/, /13/

though the total traffic process of all calls of arbitrary priority classes does not reach stationarity. The probabilities are constant and independent of time only for the priority classes from 1 to k*. The number of waiting calls grows, on the average, continuously. The increase of queue length results, on the average, only from less urgent calls of priority classes from k*+1 up to K. The mean numbers of calls which are present in the trunkgroup or in the queue line are independent of time for the priority classes from 1 to k*. All calls of priority classes from 1 to k* will occupy any trunk after a finite waiting time. During remaining gaps of time some of the (k*+1)-calls will occupy some trunks. The probability that a call will succeed in occupying a trunk is finite for priority class k*+1. It tends to zero for priority classes from k*+2 to K. This holds during a period of "saturation" /13/, /1/, with $A(\leq k^*+1) \geq n$.

Starting with the probability of state according to eq. (12) we shall determine the probabilities of delay and of loss in section 3. In section 4 waiting times will be dealt with.

3. Probabilities of delay and of loss

3.1 Account of traffic intensities

A call either occupies a trunk or is lost. We make up the accounts of the mean number of calls only of priority class k which are termed k-calls shortly (see fig. 4). The mean number of k-calls which arrive during a unit of time (during a mean holding time) equals the traffic offered A_k of priority class k. The partition of these calls according to their lot - they become either an occupation or a lost call - gives the account of traffic intensities.

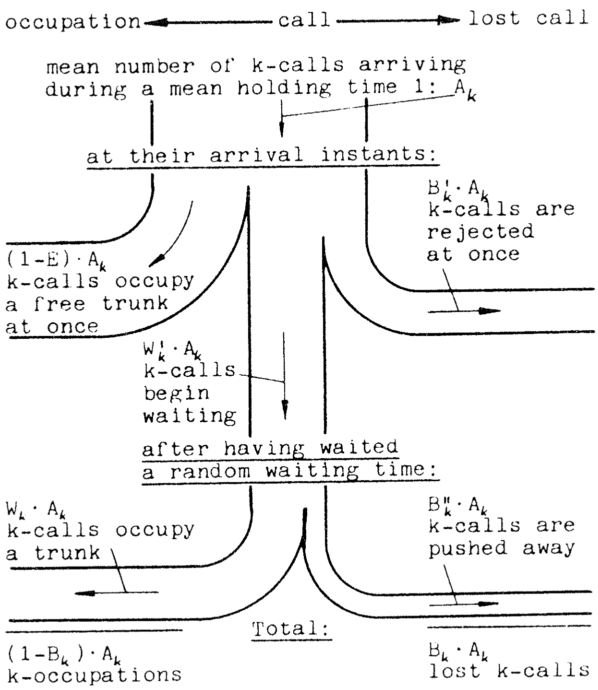

Fig.4. The account of the mean number of k-calls during a mean holding time 1; the account of traffic intensities

3.2 Calls at their arrival instants

Arriving k-calls, which succeed in finding an idle trunk, occupy it at once. The trunkgroup is not blocked, if at least one idle trunk is left: probability 1-E. No one but the probability of blocking E is independent of the priority class considered; E is equal for all calls.

Some of the k-calls which arrive during a period of blocking or time congestion are rejected at once. They are rejected, if all s queue places are occupied by calls of the priority classes from 1 to k. Figure 5 shows a realization of a random instantaneous state. All s=6 queue places are occupied by 1-calls, 2-calls, or 3-calls. 3-calls or 4-calls arriving are rejected immediately, because this instantaneous state belongs to the state $\xi_3=6$ and at the same time to the state $\xi_4=6$.

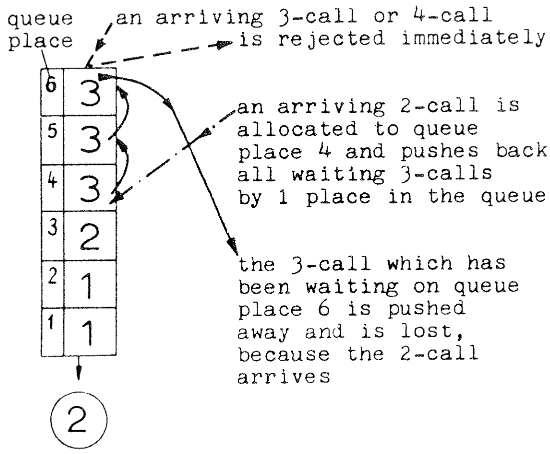

Fig.5. An example for an instantaneous state. It is a possibility for the following states: $\xi_1=2$; $\xi_2=3$; $\xi_3=6=s$; $\xi_4=6=s$.

The probability of the state $\xi_k=s$ - all s queue places are occupied by calls of priority classes from 1 to k at any instant - equals the probability that a k-call arriving is rejected at once and is lost. This probability is termed <u>probability of loss by</u> immediate <u>rejection</u> B'_k.

$$B'_k = p(s,\leq k) = \alpha(\leq k)^s \cdot \frac{1-\alpha(\leq k)}{1-\alpha(\leq k)^{s+1}} \cdot E \qquad (16)$$

Let us look on k-calls only. On the average, $(1-E)\cdot A_k$ calls occupy a free trunk at once, and $B'_k \cdot A_k$ calls are rejected immediately at their arrival instants. The remaining $W'_k \cdot A_k$ calls begin waiting. Hence the <u>probability of delay</u> W'_k:

$$W'_k = \sum_{\xi_k=0}^{s-1} p(\xi_k,\leq k) = E-B'_k = \frac{1-\alpha(\leq k)^s}{1-\alpha(\leq k)^{s+1}} \cdot E \qquad (17)$$

The primes note the probabilities for the lot of calls at their arrival instants.

3.3 Calls having waited

If a k-call is delayed, it begins waiting; there are two lots possible. On the average, $W_k \cdot A_k$ calls occupy a trunk at the very instant, when an occupation ends. The remaining $B''_k \cdot A_k$ calls are pushed away out of the queue line. $W_k \cdot A_k$ calls are waiting successfully and $B''_k \cdot A_k$ calls are waiting in vain. (See fig.4.)

Calls of higher priority (priority classes from 1 to k-1) insert in the queue according to the priority rule, i.e. in front of all waiting calls of lower priority. They push back all waiting calls of lower priority by one place in the queue. Thus they may cause the pushing away of a call which has been waiting on queue place s .

Let us return to the example of an instantaneous state (see fig.5). A 2-call arriving is allocated to queue place 4. All calls, which have waited on queue places behind the place allocated, are pushed back by one place. The 3-call, which has been waiting on queue place 6, is pushed away out of the queue line and is lost. In order to register the calls pushed away it is insufficient to make up an account of nothing but the k-calls. We must also consider possible events and calls of higher priorities which may cause the pushing away of k-calls.

We use the formal presentation of all possible events as domains within a rectangle: the sample space /10/ (see fig.6). The state $\zeta_{k-1}=s$ - all s queue places are occupied by calls of priority classes from 1 to k-1 - corresponds to the area within the inner circle. Therefore calls of priority classes from k-1 to K are rejected immediately. If the state $\zeta_k=s$ prevails, arriving calls of priority classes from k to K are rejected immediately. If $\zeta_{k-1}=s$ occurs, $\zeta_k=s$ occurs, too; if s calls of priority classes from 1 to k-1 wait, s calls of priority classes from 1 to k wait, too. The area of the circle $\zeta_{k-1}=s$ is contained in the area of the circle $\zeta_k=s$; the event $\zeta_{k-1}=s$ implies $\zeta_k=s$.

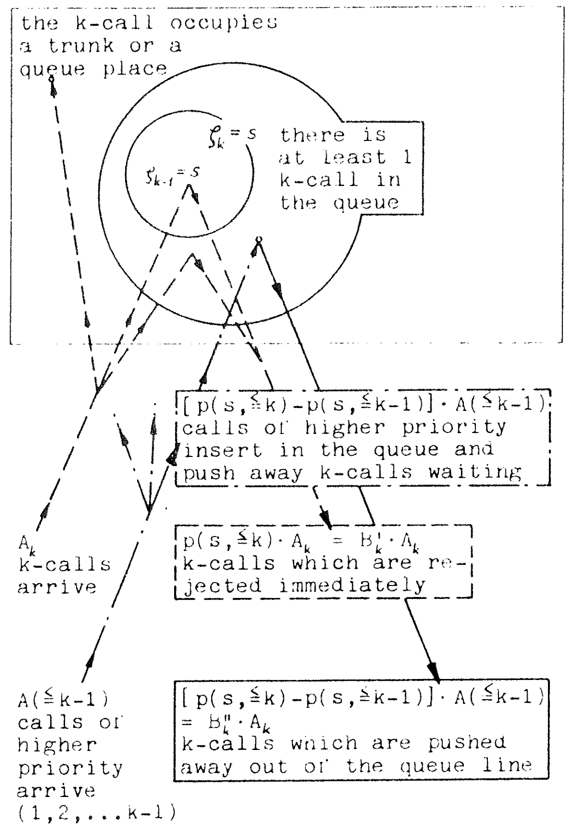

Fig.6. Sample space with domains representing possible events. Account of the mean number of k-calls and of calls of higher priority during a mean holding time 1; account of traffic intensities.

The ring-shaped domain between the two circles represents the events that there is at least one k-call among s waiting calls of priority classes from 1 to k, but no waiting call of priority classes from k+1 to K. The probability for these events is $p(s,\le k)-p(s,\le k-1)$. A call arriving of any of the priority classes from 1 up to k-1 enters the queue and pushes away one waiting k-call.

During a mean holding time the mean number of k-calls which are rejected at once is

$$B_k' \cdot A_k = p(s,\le k)\cdot A_k$$

During a mean holding time the mean number of k-calls which are pushed away out of the queue line turns out to be

$$B_k'' \cdot A_k = [p(s,\le k)-p(s,\le k-1)]\cdot A(\le k-1) = (B_k' - B_{k-1}')\cdot A(\le k-1) \qquad (18)$$

B_k'' is termed the <u>probability of loss by pushing away</u>.

Introduce the traffic offered of priority class k per trunk:

$$\alpha_k = \frac{A_k}{n} \qquad (19)$$

The <u>probability of total loss</u> B_k is the probability that a k-call is lost either by rejection or by pushing away.

$$B_k = B_k' + B_k'' = \frac{B_k'\cdot\alpha(\le k) - B_{k-1}'\cdot\alpha(\le k-1)}{\alpha_k} \qquad (20)$$

The <u>probability of waiting successfully</u> W_k results from the probability of delay W_k' and the probability of loss by pushing away B_k'' (cf. fig.4):

$$W_k = W_k' - B_k'' = E - B_k \qquad (21)$$

Now we have succeeded in determining all probabilities for all possible ways which are listed in the account of k-calls (see fig.4). The probabilities of not being stopped 1-E, of waiting successfully W_k, and of total loss B_k determine the final lot of a k-call.

3.4 Diagrams for the probabilities of blocking, of delay, and of loss

Let us look back. As occupations never are interrupted for nonpre-emptive priority service, the probability of blocking E is valid for all priority classes. (cf. eq. (5) to (9), see fig.7) The curves show the influence of the size of the queue line. The extremes are on the one hand the delay system ($s\to\infty$), on the other hand the loss system ($s=0$). The probability of blocking E can be factored out of most equations.

The quantity B_k'/E, meaning the probability of loss by rejection divided by the probability of blocking (cf. eq. (16)), is plotted in figure 8. It does not depend on the number of trunks n, if the traffic offered of priority classes from 1 to k per trunk $\alpha(\le k)$ is used. Thus $\alpha(\le k)$ proves to be useful. Starting with the value E drawn from diagram 7 for a given number of trunks, diagram 8 is used two times: for priority classes from 1 to k, and for priority classes from 1 to k-1. When we use these three values, the formulae (18), (20), and (21) can easily be evaluated.

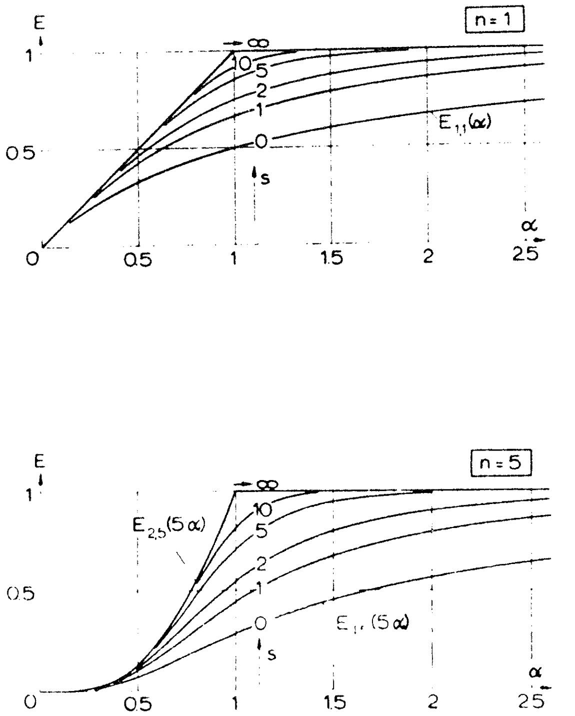

Fig.7. The probability of blocking E depends on the total traffic offered per trunk α, on the number of queue places s, and on the number of trunks n.

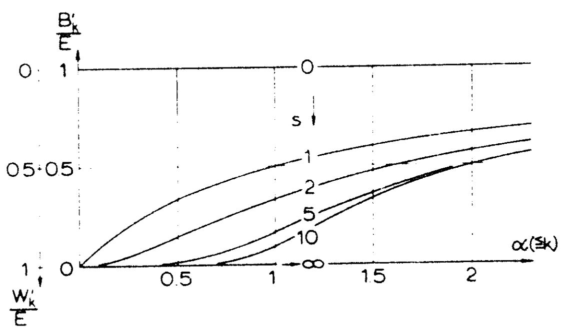

Fig.8. The probabilities of loss by rejection B'_k and of delay W'_k divided by the probability of blocking E depend on the traffic offered of priority classes from 1 to k per trunk $\alpha(\leq k)$, and on the number of queue places s.

4. Mean waiting times

4.1 Waiting traffic

The mean number of calls of priority classes from 1 to k which are waiting in the queue line equals the waiting traffic $\Omega(\leq k)$. When we use eq. (12), this expectation value turns out to be

$$\Omega(\leq k) = \sum_{\zeta_k=0}^{s} \zeta_k \cdot p(\zeta_k, \leq k)$$
$$= \left[\frac{\alpha(\leq k)}{1-\alpha(\leq k)} - \frac{(s+1)\cdot\alpha(\leq k)^{s+1}}{1-\alpha(\leq k)^{s+1}}\right] \cdot E \qquad (22)$$

For delay systems the second term within the brackets vanishes, if $\alpha(\leq k)<1$. The mean number of calls waiting increases, if the number of queue places increases.

The mean number of waiting calls of priority class k seperately is the waiting traffic Ω_k.

$$\Omega_k = \Omega(\leq k) - \Omega(\leq k-1) \quad \text{for } k=1,2,\ldots K \qquad (23)$$

(Calls of highest priority belong to priority class k=1: $\Omega_1=\Omega(\leq 1)$; $\Omega(\leq 0)=0$.)

4.2 Mean waiting time referred to all calls

The mean holding time is used as unit of time. The mean waiting time τ^*_{wk} is referred to all k-calls. If the traffic process is stationary for priority class k, the waiting traffic Ω_k is equal to the average number A_k of k-calls arriving per unit of time multiplied by the mean waiting time τ^*_{wk} referred to all k-calls (cf. /3/).

$$\Omega_k = A_k \cdot \tau^*_{wk} \qquad (24)$$

Use eq. (10) and (19), and the abbreviation

$$\alpha(\leq k-1) = \alpha(<k) \qquad (25)$$

and insert eq. (23) and (22) into eq. (24). The mean waiting time referred to all k-calls is found to be

$$\tau^*_{wk} = \frac{E}{n}\left\{\frac{1}{[1-\alpha(\leq k)]\cdot[1-\alpha(<k)]} - \frac{(s+1)\left[\alpha(\leq k)^{s+1}-\alpha(<k)^{s+1}\right]}{\alpha_k\cdot[1-\alpha(\leq k)^{s+1}]\cdot[1-\alpha(<k)^{s+1}]}\right\} \qquad (26)$$

The waiting traffic $\Omega(\leq k)$ is equal to the average number $A(\leq k)$ of calls of priority classes from 1 to k per unit of time multiplied by the mean waiting time $\tau^*_w(\leq k)$ referred to all calls of priority classes from 1 to k.

$$\Omega(\leq k) = A(\leq k)\cdot\tau^*_w(\leq k) \qquad (27)$$

Hence with eq. (10) and (22)

$$\tau^*_w(\leq k) = \frac{E}{n}\left[\frac{1}{1-\alpha(\leq k)} - \frac{(s+1)\cdot\alpha(\leq k)^{s}}{1-\alpha(\leq k)^{s+1}}\right] \qquad (28)$$

Inserting eq. (24) and (27) into eq. (23), we get with (10) and (19) (cf. /19/)

$$\tau^*_{wk} = \frac{\tau^*_w(\leq k)\cdot\alpha(\leq k) - \tau^*_w(\leq k-1)\cdot\alpha(\leq k-1)}{\alpha_k} \qquad (29)$$

Eq. (29) together with eq. (28) is equivalent to eq. (26). The structure of eq. (29) for the mean waiting time τ^*_{wk} referred to all k-calls agrees with the structure of eq. (20) for the probability of total loss B'_k. In order to evaluate τ^*_{wk} we use $\tau^*_w(\leq k)$.

$\tau_w^*(\leq k)/E$ and $\tau_w^*(\leq k-1)/E$ are drawn from fig. 9, and E is drawn from diagram 7 for the number of trunks n.

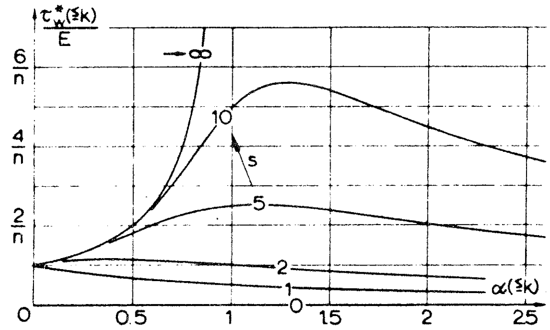

Fig.9. The mean waiting time $\tau_w^*(\leq k)$ referred to all calls of priority classes from 1 to k divided by the probability of blocking E depends on the traffic offered of priority classes from 1 to k per trunk $\alpha(\leq k)$, and on the number of queue places s.

4.3 Delayed calls of highest priority

For calls of the highest priority: k=1, $\alpha(\leq 1)=\alpha_1$, and $\alpha(<1)=0$. Eq. (26) yields the mean waiting time τ_{w1}^* referred to all 1-calls. With the probability $W_1=W_1'$ according to eq. (17) a 1-call is delayed and begins waiting. 1-calls never are pushed away. The mean waiting time τ_{w1} referred to delayed and waiting 1-calls is

$$\tau_{w1}=\frac{\tau_{w1}^*}{W_1}=\frac{1}{n}\cdot\left(\frac{1}{1-\alpha_1}-\frac{s\cdot\alpha_1^s}{1-\alpha_1^s}\right)\quad \text{for } s=1,2,\dots \text{ finite} \qquad (30)$$

For $s\to\infty$ (delay systems) the second term within the brackets vanishes, if $\alpha_1<1$. If α_1 approaches 1, the mean waiting time τ_{w1} tends to infinity. Only for $\alpha_1<1$ the traffic process is stationary (cf. section 2.7).

In delay and loss systems with one single queue place each 1-call waiting has to wait till one of the n occupations ends. (See fig.10.)

$$\tau_{w1}=\frac{1}{n} \qquad \text{for } s=1 \qquad (31)$$

If for delay and loss systems the traffic offered per trunk $\alpha_1\to\infty$, each 1-call that must wait is allocated to queue place s. It must wait till s occupations have finished, and at the very instant it is to occupy a trunk. (See fig.10.)

$$\tau_{w1}\to\frac{s}{n} \qquad \text{for } \alpha_1\to\infty \text{ and } s=1,2,\dots \text{ finite} \qquad (32)$$

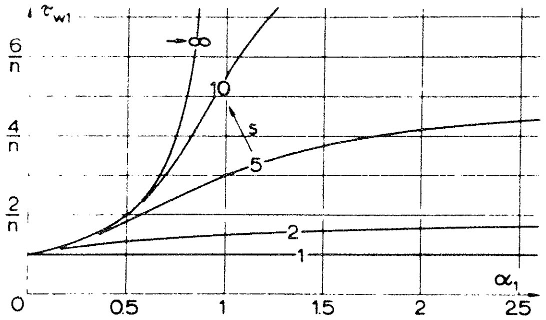

Fig.10. The mean waiting time τ_{w1} referred to the 1-calls waiting depends on the traffic offered of priority class 1 per trunk α_1, and on the number of queue places s.

4.4 Delayed calls in delay systems

With infinite number of queue places calls waiting are never pushed away. The waiting traffic Ω_k is equal to the average number $W_k\cdot A_k$ of delayed k-calls occuring per unit of time multiplied by the mean waiting time τ_{wk} referred to waiting k-calls.

$$\Omega_k = W_k\cdot A_k\cdot\tau_{wk} \qquad \text{for } s\to\infty$$

With eq. (23), (22), and with $W_k=E$ for $s\to\infty$

$$\tau_{wk}=\frac{1}{n}\cdot\frac{1}{[1-\alpha(\leq k)]\cdot[1-\alpha(<k)]} \qquad \text{for } s\to\infty \text{ and } \alpha(\leq k)<1 \qquad (33)$$

(cf. /4/, /13/, /19/)

The mean waiting time τ_w^* is changed to τ_{wk}^* by priority classification. In delay systems the probability of delay $W_k=E$ does not depend on the priority class considered.

$$\frac{\tau_{wk}^*}{\tau_w^*}=\frac{\tau_{wk}}{\tau_w}=\frac{1-\alpha}{[1-\alpha(\leq k)]\cdot[1-\alpha(<k)]} \qquad \text{for } s\to\infty \text{ and } \alpha<1 \qquad (34)$$

The formula (34) for the relative prolongation of the mean waiting times by introducing priorities is valid for

the delay system with Poisson input, negative exponential distribution of holding times, and $n\geq 1$ fully accessible trunks.

It was deduced quite differently by A. Cobham /4/. The formula (34) holds even for

a delay system with Poisson input, general probability distribution of independent holding times, and one single trunk n=1 /4/, /13/, /19/, /22/.

4.5 Delayed calls in delay and loss systems

For combined delay and loss systems the waiting traffic contains all waiting calls; we leave out of account whether they will occupy a trunk, or will be pushed out of the queue line after their waiting times. Starting from the mean waiting time τ_{wk}^*

referred to all k-calls the mean waiting time τ'_{wk} referred to delayed k-calls (i.e. k-calls that begin waiting) is

$$\tau'_{wk} = \frac{\tau^*_{wk}}{W'_k} \qquad (35)$$

As well as τ^*_{wk} (cf. eq. (26) or (29)) the mean waiting time τ'_{wk} of delayed k-calls depends on both $\alpha(\leq k)$ and $\alpha(<k)$, too.

So far the mean waiting time τ^*_{wk} referred to all k-calls cannot be divided in the mean waiting time τ_{wk} referred to k-calls which are waiting successfully (probability W_k cf. eq. (21)), and to the mean waiting time τ''_{wk} referred to k-calls which wait in vain (probability B''_k cf. eq. (18)).

$$\tau^*_{wk} = W_k \cdot \tau_{wk} + B''_k \cdot \tau''_{wk} \qquad (36)$$

In order to determine the mean waiting times τ_{wk} and τ''_{wk} we must take into consideration the queue place on which a call is waiting.

4.6 Conditional probability distribution of the waiting times for given queue place

We are looking for the probability that the waiting time of a k-call exceeds the time τ on condition that this k-call waits on queue place j at the instant $\tau=0$. Because of the Markov property of the traffic process (/10/, /14/) it is identical to the conditional probability that the waiting time of an arriving k-call exceeds the time τ on condition that this k-call is allocated to queue place j. On condition that the k-call occupies the queue place j, various waiting times must be distinguished:

a) referred to all k-calls that begin waiting,
b) on additional condition of waiting successfully,
c) on additional condition of waiting in vain.

a) All calls that begin waiting

The distribution function $W'_k(>\tau/j)$ expresses the probability that during a time τ a k-call that occupies the queue place j at the beginning of the interval τ has kept waiting. It waits at least time τ, till it is allowed to occupy a trunk immediately after an occupation has ended, or till it is pushed away out of the queue line by an arriving call of higher priority.

At the instant $\tau=0$ a k-call is waiting on queue place j. During the subsequent short interval $d\tau$ one of the following possibilities may occur:

1. A new call of higher priority arrives and pushes back the call considered to queue place j+1.
2. No call of higher priority arrives, and no occupation ends. The k-call considered stays on queue place j.
3. One of the n occupation ends. The call considered advances to queue place j-1.

After this the k-call considered is assumed to wait at least time τ. There are 2 neighbouring queue places, if j=2,3,...s-1. The input of calls and the terminations of occupations are mutually independent. When $d\tau$ is so small that multiple arrivals, terminations, and arrivals-terminations have probabilities of the order of $(d\tau)^2$ they can be ignored. Hence (cf. fig.11)

$$\begin{aligned} W'_k(>\tau+d\tau/j) &= A(<k)\cdot d\tau\cdot W'_k(>\tau/j+1) \\ &+ [1-A(<k)\cdot d\tau]\cdot[1-n\cdot d\tau]\cdot W'_k(>\tau/j) \\ &+ n\cdot d\tau\cdot W'_k(>\tau/j-1) \quad \text{for } j=2,3,\ldots s-1 \end{aligned} \qquad (38a)$$

The deduction follows the method introduced by C. Palm /14/, /16/.

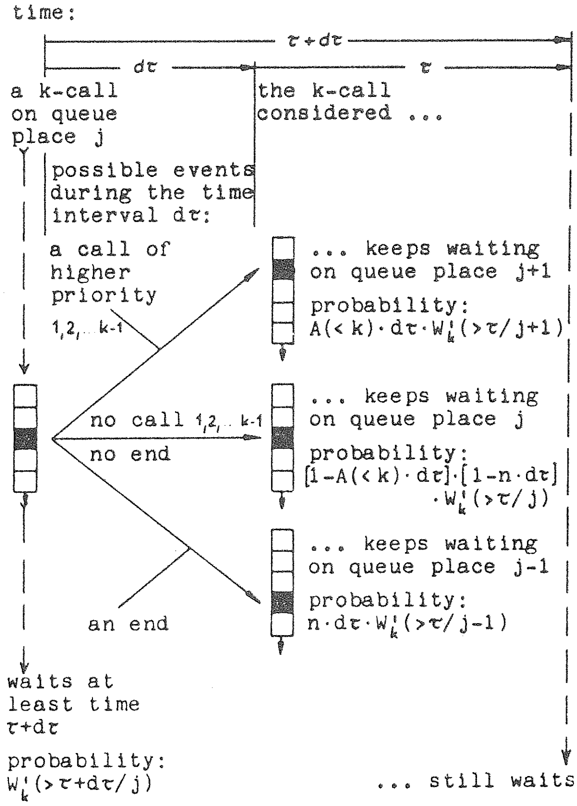

Fig.11 is designed to illustrate the deduction of the differential equation for the distribution function $W'_k(>\tau/j)$ (cf. /14/).

When the k-call considered is waiting at the head of the queue (i.e. on queue place j=1), it occupies a trunk becoming free during the time interval $d\tau$ at once. It brings its waiting time to a successful end. A k-call on queue place 1 has to wait (either successfully or in vain) at least $\tau+d\tau$, if during $d\tau$ one of the following possibilities may occur:

1. A call of higher priority arrives and pushes back the k-call to queue place 2.
2. No call of higher priority arrives, and no occupation ends. The k-call stays on queue place 1.

After this the k-call is assumed to wait at least time τ. Hence

$$\begin{aligned} W'_k(>\tau+d\tau/1) &= A(<k)\cdot d\tau\cdot W'_k(>\tau/2) \\ &+ [1-A(<k)\cdot d\tau]\cdot[1-n\cdot d\tau]\cdot W'_k(>\tau/1) \quad \text{for } j=1 \end{aligned} \qquad (37a)$$

When the k-call considered is waiting at the end of the queue (i.e. on queue place j=s), it is pushed away by a call of higher priority which has arrived during the time interval $d\tau$. A k-call on queue place s has to wait (either successfully or in vain) at least $\tau+d\tau$, if during $d\tau$ one of the following possibilities may occur:

1. No call of higher priority arrives, and no occupation ends. The k-call stays on queue place s.
2. An occupation ends. The k-call advances to queue place s-1.

After this the k-call is assumed to wait at least time τ. Hence

$$W_k'(>\tau+d\tau/s) = [1-A(<k)\cdot d\tau]\cdot[1-n\cdot d\tau]\cdot W_k'(>\tau/s) + n\cdot d\tau\cdot W_k'(>\tau/s-1) \quad \text{for } j=s \tag{39a}$$

Differential equations. Probabilities that both an arrival and an end occur during a time interval $d\tau$ are of second order in $d\tau$. When $d\tau$ tends to zero, we get a system of s linear differential equations.

$$\frac{dW_k'(>\tau/1)}{d\tau} = A(<k)\cdot W_k'(>\tau/2) - [n+A(<k)]\cdot W_k'(>\tau/1) \quad \text{for } j=1 \tag{37}$$

$$\frac{dW_k'(>\tau/j)}{d\tau} = A(<k)\cdot W_k'(>\tau/j+1) - [n+A(<k)]\cdot W_k'(>\tau/j) + n\cdot W_k'(>\tau/j-1) \quad \text{for } j=2,3,\ldots s-1 \tag{38}$$

$$\frac{dW_k'(>\tau/s)}{d\tau} = -[n+A(<k)]\cdot W_k'(>\tau/s) + n\cdot W_k'(>\tau/s-1) \quad \text{for } j=s \tag{39}$$

$$W_k'(>0/j) = 1 \quad \text{for } j=1,2,\ldots s \tag{40}$$

Starting from the system of differential equations the delay distribution function $W_k'(>\tau)$ for any delayed k-call without condition of a given queue place can be deduced on principle. To cite the formulae which have been derived for the cases s=1 and s=2 is beyond the scope of this paper.

4.7 Mean waiting times of delayed calls for given queue place

The mean waiting time $\tau_w'(j,k)$ referred to delayed k-calls which begin waiting on queue place j is the mean value of the delay distribution function. In order to find $\tau_w'(j,k)$ we integrate $W_k'(>\tau/j)$ /14/.

$$\tau_w'(j,k) = \int_{\tau=0}^{\infty} W_k'(>\tau/j)\cdot d\tau \tag{41}$$

Because $-\frac{W_k'(>\tau/j)}{d\tau}$ is the probability density we get

$$-\int_{\tau=0}^{\infty} \frac{dW_k'(>\tau/j)}{d\tau}\cdot d\tau = 1 \tag{42}$$

By integrating the differential equations (37), (38), (39) we get a system of linear equations for the mean waiting times $\tau_w'(j,k)$ of k-calls that begin waiting on queue place j.

$$[n+A(<k)]\cdot\tau_w'(1,k) = A(<k)\cdot\tau_w'(2,k) + 1 \quad \text{for } j=1 \tag{43a}$$

$$[n+A(<k)]\cdot\tau_w'(j,k) = A(<k)\cdot\tau_w'(j+1,k) + n\cdot\tau_w'(j-1,k) + 1 \quad \text{for } j=2,3,\ldots s-1 \tag{44a}$$

$$[n+A(<k)]\cdot\tau_w'(s,k) = n\cdot\tau_w'(s-1,k) + 1 \quad \text{for } j=s \tag{45a}$$

Write

$$D_j' = [n+A(<k)]\cdot\tau_w'(j,k) \tag{46}$$

then

$$D_1' = \frac{A(<k)}{n+A(<k)}\cdot D_2' + 1 \quad \text{for } j=1 \tag{43}$$

$$D_j' = \frac{A(<k)}{n+A(<k)}\cdot D_{j+1}' + \frac{n}{n+A(<k)}\cdot D_{j-1}' + 1 \quad \text{for } j=2,3,\ldots s-1 \tag{44}$$

$$D_s' = \frac{n}{n+A(<k)}\cdot D_{s-1}' + 1 \quad \text{for } j=s \tag{45}$$

4.8 Roulette method of simulation

Let us consider the roulette method of simulation which was introduced by L. Kosten /23/, /24/. Kosten's traffic model uses call-originating and call-terminating random numbers. It is well known and is frequently used, e.g. /25/, /26/, etc. The program advances from step to step. At each step it decides whether an event may occur. We disregard the steps when no event occurs. The k-call waiting on queue place j advances to queue place j-1 with probability q.

$$q = \frac{n}{n+A(<k)} \tag{47}$$

The k-call waiting is pushed back by one queue place from j to j+1 with probability p=1-q.

$$p = \frac{A(<k)}{n+A(<k)} \tag{48}$$

The roulette method is based on Bernoulli trials. W. Feller treated problems on Bernoulli trials and used the language of betting /10/. Remember the classical ruin problem. A gambler wins a dollar with probability p, or loses a dollar with probability q=1-p. His initial capital is j, the capital of his adversary is s+1-j. The game continues until the gambler possesses no dollar (ruin), or until he possesses the combined capital s+1 (gain). The expected duration of the game until one of the two players is ruined is D_j'. The eq. (43), (44), and (45) with (47), (48) yield D_j' /10/.

The roulette method of simulation gives the probabilities of delay, and of loss. It yields the mean values, too. The mean waiting times may be determined by roulette simulation /26/. We use the correspondence between the real traffic process in the queue line and the process of a roulette simulation. A k-call waiting moves up and down in the queue line. Its queue place j corresponds to the gambler's capital.

The simplest case is a combined delay and loss system with one single queue place. A k-call begins waiting on queue place 1. Consider the process of the simulation. At the next step when an event occurs, the program ends the waiting duration of the k-call; the waiting duration of a delayed k-call is $D_1'=1$. The k-call is successful with probability q and a waiting duration of 1 step. We consider merely k-calls which wait successfully. We intend to determine the mean waiting time τ_{wk} referred to k-calls waiting successfully. The waiting duration of a k-call which waits successfully is $D_1=1$ step. Steps when no event occurs are disregarded. The conversion from the waiting duration $D_1=1$ of the simulation to the mean waiting time τ_{wk} is done by eq. (46) (cf. /10/ problem no. XIV.1).

$$\tau_{wk} = \tau_w(1,k) = \frac{D_1}{n+A(<k)} = \frac{1}{n}\cdot\frac{1}{1+\alpha(<k)} \qquad (49)$$

The k-call waiting on queue place 1 waits in vain with probability p. The simulation gives a waiting duration of 1 step. Convert $D_1''=1$ to the mean waiting time τ_{wk}'' referred to k-calls which wait in vain. With eq. (46)

$$\tau_{wk}'' = \frac{1}{n}\cdot\frac{1}{1+\alpha(<k)} = \tau_{wk} \quad \text{for } k=2,3,\ldots K \text{ and } s=1 \qquad (50)$$

For s=1 the following equivalence is valid: The mean waiting time τ_{wk}' referred to delayed k-calls (i.e. k-calls that begin waiting) equals the mean waiting time referred to k-calls which wait successfully and will occupy a trunk, and equals the mean waiting time τ_{wk}'' referred to k-calls which wait in vain and will be pushed away.

The investigation of the classical ruin problem for different values of the total capital s+1, e.g. s=2, and all possible values of the initial capital j=1,2,...s yields the expected durations of the games which are terminated by the ruin of the first gambler. Further calculations will give the formulae for the mean waiting times τ_{wk} referred to k-calls waiting successfully, and τ_{wk}'' referred to k-calls waiting in vain. In this paper we restrict ourselves to the special results $\tau_{wk}=\tau_{wk}''=\tau_{wk}'$ for s=1 cf. eq. (50), and $\tau_{wk}=\tau_{wk}'$ and $\tau_{wk}''=0$ for $s\to\infty$ (cf. section 4.4).

5. Conclusion

There are different types of calls with different tolerable waiting times. The connecting system is adopted to these requirements by priority classification. Technical systems have small or very large, but finite queue lines. Known formulae are based on the assumption of an infinite number of queue places. The formulae deduced in this paper are intended as an aid to study the influence of the finite number of queue places together with nonpre-emptive priority service. For usual assumptions on the traffic the probabilities of delay and of loss, and the mean waiting times have been deduced. Some diagrams give an idea of the influence and importance of the size of the queue line. They show the way of solution. The formulae yield exact values. To shorten the calculation, approximate values of the interesting probabilities and the mean waiting times can easily be calculated by reading out the formula-terms from the diagrams. The starting-points are given for the further calculation of the delay distribution function, and of the mean waiting times referred to calls waiting successfully, respectively referred to calls waiting in vain.

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