Elsevier Science Publishers B.V. (North-Holland)

Multiqueue Systems with Finite Capacity and Nonexhaustive Cyclic Service

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The class of multiqueue systems with cyclic service has a broad spectrum of applications, e.g. in modelling approaches for switching systems, token ring local area networks, etc.. This paper presents an approximate analysis method for this class of models, whereby the realistic assumption of finite queue capacity is taken into account. The analysis is based on an imbedded Markov chain approach in conjunction with a two-moment approximation for the cycle time. The validation of the approximation is done by means of computer simulations. A number of numerical results are shown in order to illustrate the accuracy of the calculation method, over a wide range of system parameters.

1. INTRODUCTION

In performance investigations of communications systems, especially switching systems with distributed control structure or token-ring local area networks, the class of multiqueue systems with cyclic service is often employed, belonging to which diverse polling mechanisms are considered. Most of these modelling approaches consider queues with infinite capacities, where several approximation techniques for the system analysis are proposed.

In the literature, multiqueue systems served by a single server have been the subject of numerous investigations [1-14]. Various polling mechanisms like cyclic or priority order and several service disciplines, e.g. exhaustive, nonexhaustive or gating are considered. Some of these studies take into account the switchover time, i.e. the time spent by the server to switch over from one queue to the succeeding one. In most of the investigations the queues are assumed as of infinite capacity and the model is analysed by means of the imbedded Markov chain technique [1,2,3,4,5,8].

An approximative solution for symmetrically loaded systems with cyclic polling, constant switchover time and gating service was presented by Leibowitz [7]. Cooper and Murray [1,2] have considered a cyclic polling system with gating or exhaustive service and zero switchover time using an imbedded Markov chain. The case of two queues with general switchover time was treated by Eisenberg [3]. Results of Cooper and Murray are generalized by Eisenberg [4] and Hashida [5] to non-zero switchover time. An approximation technique for cyclic queues with non-exhaustive service and general switchover time has been developed by Kuehn [8]. This method has been extended in [9] for performance investigations of hierarchical polling systems with feedback. In most of investigations for a larger number of queues computational difficulties arise in the numerical evaluation of the mean delay. A simple approximation approach for the mean delay in cyclic queuing systems with exhaustive service was presented by Bux and Truong [11]. Further investigations of some specific cyclic queueing systems was performed by Manfield [12] for systems with two-way traffic and priority cyclic service and by Morris and Wang [15] for multiqueue systems with multiple cyclic servers. An exact solution for a system with two queues and nonzero switchover time was presented by Boxma [13]. A survey on polling system analysis, where various system classes are considered, was provided by Takagi and Kleinrock [14].

In real systems all buffers are of finite capacity. In the case of modelling such systems under nonsymmetrical load conditions or overload, in which blocking of incoming messages can occur, the finite capacity of some overloaded queues must be taken into account. For the investigation of such finite capacity systems, established analysis methods for infinite multiqueue systems, which usually operate with generating functions or in Laplace domain, do not lead to closed expressions for system characteristics of interest.

This paper presents an approximate analysis method for multiqueue systems with finite capacity and nonexhaustive cyclic service, whereby a numerical algorithm is developed. The accuracy of the method will be illustrated by means of numerical results, which have been obtained for symmetrically as well as for nonsymmetrically loaded systems.

2. MODEL DESCRIPTION

The basic model of a multiqueue system with finite capacity is depicted in Fig. 1, which consists of a number g of queues. The queues are served nonexhaustively in a cyclic order by a single server with a generally distributed service time. The service time distribution function can be individually chosen for each queue. Also individual arrival rates and sizes of the buffer capacity are considered. After the service of a queue the server will move to the succeeding queue. This switchover time, which models all overheads spent and procedures performed by the server to move to and to scan the succeeding queue, is assumed to have a queue-dependent general distribution function. At the scanning epoch, i.e. at the end of the corresponding switchover time, the server will process one message in the queue, if there is at least one message waiting for service. If the queue is empty, the observed interscan period consists of just the switchover time. The arrival processes are assumed to be Poissonian with queue-individual rates λ_i, according to which the system can be modelled to be symmetrically or nonsymmetrically loaded.

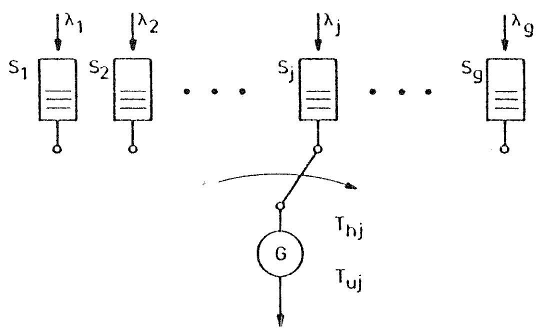

Fig. 1
The basic queueing model

The following symbols and random variables (r.v.) are used in this paper :

- g number of queues in the system
- λ_j arrival rate of messages offered to queue j
- T_{hj} r.v. for the service time of messages in queue j
- T_{uj} r.v. for the switchover time corresponding to queue j
- S_j capacity of queue j

The following notations are used, e.g. for a r.v. T

- $F(t)$ probability distribution function
- $f(t)$ probability density function
- $\Phi(s)$ Laplace-Stieltjes-Transform of F(t)
- $\overline{T}$ mean value of T
- $c[T]$ coefficient of variation of T.

3. ANALYSIS

In this section, a numerical algorithm for an approximate analysis of finite capacity multiqueue systems will be derived. Basically the analysis draws upon approaches presented in [6,8], using the technique of the imbedded Markov chain. However some modifications must be provided in order to take into account the blocking effect and the finiteness of queue capacities.

3.1 Markov Chain State Probabilities

A particular queue j is considered in the following, which is observed at scanning instants. Let t_n be the time of the n-th scanning epoch and let $X^{(n)}(0^-)$ be the number of messages in this queue at time t_n^-, just prior to the n-th scanning epoch, we define the Markov chain state probabilities

$$P^{(n)}_{k,j} = \Pr\{ X^{(n)}(0^-) = k \} , \quad k = 0,1,\dots,S_j \tag{3.1}$$

and the steady state probabilities of the Markov chain are defined from

$$P_{k,j} = \Pr\{ X(0^-) = k \} , \quad k = 0,1,\dots,S_j \tag{3.2}$$

For ease of reading, the subscript j indicating the observed queue will be suppressed, e.g., the notation P_k will be used instead of $P_{k,j}$. In order to calculate the transition probabilities of the Markov chain

$$P_{jk} = \Pr\{ X^{(n+1)}(0^-) = k \mid X^{(n)}(0^-) = j \} \tag{3.3}$$

we observe the system state $X^{(n)}(t)$ of the queue at time t_n+t. Considering the pure birth process in the queue between two consecutive scanning epochs, i.e. during a scanning cycle, the state probabilities at time t_n+t denoted by

$$P^{(n)}_k(t) = \Pr\{ X^{(n)}(t) = k \} , \quad k = 0,1,\dots,S_j \tag{3.4}$$

can be obtained as follows

$$P^{(n)}_k(t) = P^{(n)}_0 a_k(t) + \sum_{i=0}^{k} P^{(n)}_{i+1}\, a_{k-i}(t) , \quad k = 0,1,\dots,S_j-1$$

$$P^{(n)}_{S_j}(t) = P^{(n)}_0 \sum_{i=S_j}^{\infty} a_i(t) + \sum_{i=0}^{S_j-1} P^{(n)}_{i+1} \sum_{m=S_j-i}^{\infty} a_m(t) \tag{3.5}$$

where

$$a_m(t) = \frac{(\lambda_j t)^m}{m!} e^{-\lambda_j t} \tag{3.6}$$

Using the consideration of conditional cycle time [8], where the following random variables (r.v.) are defined

- T_C r.v. for the cycle time with respect to the observed queue j
- T'_C r.v. for a cycle, conditioning on an empty queue at the previous scanning instant (i.e., without service of queue j during the cycle)
- T''_C r.v. for a cycle, conditioning on an non-empty queue at the previous scanning instant (i.e., with service of queue j during the cycle),

the state equations which implicitly contain the transition probabilities can be written as

$$p_k^{(n+1)} = p_0^{(n)} \int_0^\infty a_k(t)\, f_{C'}(t)dt + \sum_{i=1}^{k+1} p_i^{(n)} \int_0^\infty a_{k-i+1}(t)\, f_{C''}(t)dt \qquad k=0,1,\ldots,S_j-1$$

$$p_{S_j}^{(n+1)} = p_0^{(n)} \sum_{i=S_j}^{\infty} \int_0^\infty a_i(t)\, f_{C'}(t)dt + \sum_{i=1}^{S_j} p_i^{(n)} \sum_{m=S_j-i+1}^{\infty} \int_0^\infty a_m(t)\, f_{C''}(t)dt. \qquad (3.7)$$

Defining the arrival probabilities, i.e. the probabilities for m arrivals during a conditional cycle of type T_C' or T_C''

$$b_m' = \int_0^\infty a_m(t)\, f_{C'}(t)\, dt$$
$$b_m'' = \int_0^\infty a_m(t)\, f_{C''}(t)\, dt \qquad (3.8)$$

we obtain from (3.7) the following set of Markov chain state equations

$$p_k^{(n+1)} = p_0^{(n)} b_k' + \sum_{i=1}^{k+1} p_i^{(n)} b_{k-i+1}'' \quad , k=0,1,\ldots,S_j-1$$
$$p_{S_j}^{(n+1)} = p_0^{(n)} \sum_{i=S_j}^{\infty} b_i' + \sum_{i=1}^{S_j} p_i^{(n)} \sum_{m=S_j-i+1}^{\infty} b_m'' \qquad (3.9)$$

Eqn. (3.9) will be used for the numerical calculation of the steady state probabilities $\{P_k\}$. It remains to determine the arrival probabilities $\{b_m'\}$ and $\{b_m''\}$ in (3.8), which is the subject of the next subsection.

3.2 Conditional Cycle Time Approximation

Define $T_{E,j}$ to be the random variable for the time interval between the scanning epochs of queue j and $(j+1)$, i.e., the segment of the cycle time corresponding to queue j, with the Laplace-Stieltjes-Transform (LST)

$$\Phi_{E,j}(s) = \Phi_{uj}(s)\cdot((1-P_{0,j})\,\Phi_{hj}(s) + P_{0,j}). \qquad (3.10)$$

Under the assumption of independency between $T_{E,j}$, $j=1,2,\ldots,g$, the LST of the conditional cycle times can be given as follows

$$\Phi_{C',j}(s) = \Phi_{uj}(s)\cdot \prod_{\substack{k=1\\k\neq j}}^{g} \Phi_{E,k}(s) \qquad (3.11a)$$

$$\Phi_{C'',j}(s) = \Phi_{uj}(s)\cdot\Phi_{hj}(s)\cdot \prod_{\substack{k=1\\k\neq j}}^{g} \Phi_{E,k}(s) \qquad (3.11b)$$

Eqns. (3.10) and (3.11a,b) yield the first two moments of the conditional cycle times, thus

$$\overline{T}_{C,j}' = \overline{T}_{uj} + \sum_{\substack{k=1\\k\neq j}}^{g} \overline{T}_{E,k} \quad ,VAR[T_{C,j}'] = VAR[T_{uj}] + \sum_{\substack{k=1\\k\neq j}}^{g} VAR[T_{E,k}]$$
$$\overline{T}_{C,j}'' = \overline{T}_{uj} + \overline{T}_{hj} + \sum_{\substack{k=1\\k\neq j}}^{g} \overline{T}_{E,k} \quad ,VAR[T_{C,j}''] = VAR[T_{uj}] + VAR[T_{hj}] + \sum_{\substack{k=1\\k\neq j}}^{g} VAR[T_{E,k}] \qquad (3.12)$$

where

$$\overline{T}_{E,j} = \overline{T}_{uj} + (1-P_{0,j})\,\overline{T}_{hj}$$
$$VAR[T_{E,j}] = VAR[T_{uj}] + (1-P_{0,j})VAR[T_{hj}] + P_{0,j}(1-P_{0,j})\,\overline{T}_{hj}^2 . \qquad (3.13)$$

In general eqns. (3.11a,b) which determine the conditional cycle times can be used in conjunction with the state equations (3.9) for the numerical calculation of the Markov chain state probabilities.

The main idea in the calculation method presented in this paper is to develop an alternating calculation algorithm to obtain values for the Markov chain state probabilities $\{P_k\}$ and the cycle times $\{T_C', T_C''\}$, which fulfil the eqns. (3.9) and (3.11a,b). Since the expression of the conditional cycle times (3.11a,b) are given in the Laplace-Stieltjes domain, a Laplace inversion procedure should have been utilized during each iteration cycle, in order to enable the calculation of eqns. (3.7) or (3.8). However, for reasons of computing efforts, the two-moment approximation technique proposed in [16] is used in this paper. This will briefly be described in the following.

According to the method presented in [16], a given random variable T, which is described by the mean $\overline{T}$ and the coefficient of variation c, is approximated by means of the following substitute probability distribution function F(t) :

i. $0 \leqslant c \leqslant 1$:

$$F(t) = \begin{cases} 0 & 0 \leqslant t \leqslant t_1 \\ 1 - e^{-(t-t_1)/t_2} & t \geqslant t_1 \end{cases} \quad \text{where } t_1=\overline{T}.(1-c) \text{ and } t_2=\overline{T}.c \qquad (3.14a)$$

ii. $c \geqslant 1$

$$F(t) = 1 - p\,e^{-t/t_1} - (1-p)\,e^{-t/t_2}, \quad \text{where } t_{1,2} = \overline{T}\left(1 \pm \sqrt{\frac{c^2-1}{c^2+1}}\right)^{-1} \qquad (3.14b)$$
$$\text{and } p = \overline{T}/2t_1 \ , \ p\,t_1 = (1-p)t_2$$

Using the substitute distribution functions in (3.14a,b), the conditional cycle time T_C' and T_C'', which are derived in (3.11a,b) and their means and variances in (3.12), will be approximately described. With the approximate description the arrival probabilities $\{b_m'\}$ and $\{b_m''\}$ can be calculated according to the following expressions, where $\{b_m\}$ are given for $\{b_m'\}$ and $\{b_m''\}$

i. $0 \leqslant c \leqslant 1$

$$b_m = \int_0^\infty a_m(t)\, f_C(t)\, dt$$
$$= \int_{t=t_1}^{\infty} \frac{(\lambda t)^m}{m!} e^{-\lambda t} \frac{1}{t_2} e^{-(t-t_1)/t_2} dt$$
$$= \frac{(\lambda t_2)^m}{(1+\lambda t_2)^{m+1}} e^{-\lambda t_1} \sum_{k=0}^{m} \left(\frac{t_1}{t_2}\right)^k (\lambda t_2+1)^k / k! \qquad (3.15a)$$

whereby the following relationship [17] is utilized

$$\int_0^x t^m e^{-at} dt = \frac{m!}{a^{m+1}} \left(1 - \sum_{k=0}^{m} \frac{(ax)^k}{k!} e^{-ax}\right).$$

ii. $c \geq 1$

$$b_m = p \frac{(\lambda t_1)^m}{(1+\lambda t_1)^{m+1}} + (1-p) \frac{(\lambda t_2)^m}{(1+\lambda t_2)^{m+1}} \tag{3.15b}$$

The arrival probabilities according to (3.15a,b) are subsequently used in (3.9) for the calculation of the Markov chain state probabilities.

3.3 Calculation Algorithm for Markov Chain State Probabilities

Using the expressions derived in subsections 3.1 and 3.2 for the Markov chain state probabilities and the conditional cycle times, a numerical algorithm is developed . The main elements of the algorithm are :

- alternate iteration of the Markov chain state probabilities and the cycle time
- during an iteration cycle the state probabilities of all queues are determined in a cyclic manner ; the calculation for each queue is done according to eqn.(3.9)
- during an iteration cycle, depending on the actual state probabilities the conditional cycle times are updated; these values will be used in the next iteration cycle
- to evade numerical Laplace transform inversion and evaluation of integrals (e.g.eqn.(3.8)), a two-moment approximation according to [16] is utilized in conjunction with a substitute process description.

The convergence criteria for the iteration are defined from

$$\Sigma\Delta_j = \sum_{j=1}^{g} \Delta_j < \varepsilon \quad \text{with} \quad \Delta_j = \sum_{k=1}^{S_j} kP_{k,j}^{(n)} - \sum_{k=1}^{S_j} kP_{k,j}^{(n-1)} \tag{3.16}$$

More details of the algorithm are given in Appendix A1.

3.4 Arbitrary Time State Probabilities

In order to calculate system characteristics, e.g. blocking probability for messages or mean waiting time in the queue, it is useful to obtain first the arbitrary time state probabilities (cf.[17]). Again, the subscript j indicating the observed queue will be suppressed in this subsection.

Define $\{P_k^*, k=0, 1, \ldots, S_j\}$ to be the arbitrary time state probabilities, i.e. the distribution of the number of messages in the considered queue j at an arbitrary observation instant. According to the two types of conditional cycle times we define π' and π'' to be the probability of an outside observer to see a cycle of type T_C' or T_C'' respectively. It can be seen clearly that

$$\pi' = P_0 \bar{T}_C' / \bar{T}_C \quad \text{where} \quad \bar{T}_C = P_0 \bar{T}_C' + (1 - P_0) \bar{T}_C''$$

and

$$\pi'' = 1 - \pi' \qquad \ldots(3.17)$$

The time interval from the last scanning epoch until the observation point is the backward recurrence time with the probability density function (pdf)

$$f_{C'}^r(t) = (1 - F_{C'}(t)) / \bar{T}_C' \tag{3.18a}$$

and

$$f_{C''}^r(t) = (1 - F_{C''}(t)) / \bar{T}_C'' \tag{3.18b}$$

The arrival probabilities during the backward recurrence time $T_C'^r$ and $T_C''^r$ can be given as

$$b_m'^* = \int_0^\infty a_m(t) f_{C'}^r(t)\, dt \qquad b_m''^* = \int_0^\infty a_m(t) f_{C''}^r(t)\, dt \tag{3.19}$$

Considering both types of conditional cycle times and combining the above results the arbitrary time state probabilities can be written as follows

$$P_k^* = \pi' b_k'^* + \pi'' \sum_{i=1}^{k+1} \frac{P_i}{1-P_0} b_{k-i+1}''^* \quad , k=0,1,\ldots,S_j-1$$
$$P_{S_j}^* = \pi' \sum_{i=S_j}^{\infty} b_i'^* + \pi'' \sum_{i=1}^{S_j} \frac{P_i}{1-P_0} \sum_{m=S_j-i+1}^{\infty} b_m''^* \tag{3.20}$$

Finally we obtain from (3.17) and (3.20)

$$P_k^* = \frac{\bar{T}_C'}{\bar{T}_C} P_0 b_k'^* + \frac{\bar{T}_C''}{\bar{T}_C} \sum_{i=1}^{k+1} P_i b_{k-i+1}''^* \quad , k=0,1,\ldots,S_j-1$$
$$P_{S_j}^* = \frac{\bar{T}_C'}{\bar{T}_C} P_0 \sum_{i=S_j}^{\infty} b_i'^* + \frac{\bar{T}_C''}{\bar{T}_C} \sum_{i=1}^{S_j} P_i \sum_{m=S_j-i+1}^{\infty} b_m''^* \tag{3.21}$$

Analogous to the approximate calculation of the arrival probabilities in eqns. (3.15a,b) using the substitute distribution function in eqns (3.14a,b) [16] the arrival probabilities during the backward recurrence conditional cycle times given by eqn. (3.19) can be explicitly written as follows:

i. $0 \leq c \leq 1$

$$b_m^* = \frac{1}{\lambda(t_1+t_2)} \left(1 - \sum_{k=0}^{m} \frac{(\lambda t_1)^k}{k!} e^{-\lambda t_1} \right) + \frac{t_2}{t_1 + t_2} b_m \tag{3.22a}$$

where b_m is given in eqn.(3.15a)

ii. $c \geq 1$

$$b_m^* = \frac{1}{2} \frac{(\lambda t_1)^m}{(1+\lambda t_1)^{m+1}} + \frac{1}{2} \frac{(\lambda t_2)^m}{(1+\lambda t_2)^{m+1}} \tag{3.22b}$$

3.5 System Characteristics

With the arbitrary time state probabilities and taking into account the Poisson arrival process offered to the observed queue j, the blocking probability for messages in queue j can be determined :

$$B_j = P_{S_j}^* \tag{3.23}$$

The mean delay in queue j , referred to transmitted messages, is found from Little's law as

$$\bar{T}_{wj} = \frac{L_j}{\lambda_j(1-B_j)} \quad , \quad L_j = \sum_{i=1}^{S_j} i\, P_i^* \quad , \tag{3.24}$$

L_j is the mean length of queue j. It should be noted here that the well-known formula for the mean cycle time [8] can be modified for the case of finite queue capacity

$$\bar{T}_C = \left(\sum_{j=1}^{g} \bar{T}_{uj} \right) / \left(1 - \sum_{j=1}^{g} \lambda_j \bar{T}_{hj}(1-B_j) \right) \tag{3.25}$$

4. APPROXIMATION ACCURACY

In the following, numerically obtained results will be presented and discussed for the two cases of symmetrically and nonsymmetrically loaded multiqueue systems, in order to illustrate the accuracy of the derived algorithm. For both systems the time variables are standardized by $\bar{T}_{hj} = 1$, $j=1,2,\ldots,g$, and the switchover time is chosen to be constant. In order to validate the approximation, computer simulations are provided. The simulation results will be depicted with their 95 percent confidence intervals.

4.1 Multiqueues with Symmetrical Load Conditions

In this subsection, two symmetric systems with $g = 8$ and $g = 32$ are taken into account. Figs. 2 and 3 depict the mean and the coefficient of variation of the cycle time as function of the offered traffic intensity

$$\rho_0 = \sum_{j=1}^{g} \lambda_j \bar{T}_{hj} \tag{4.1}$$

for different values of the number g of queues and the service time coefficient of variation. Fig. 2 exhibits the effect that servers with higher variance lead to shorter mean cycle times in the range of traffic intensity (0.5,0.75). This effect can be explained considering the higher blocking probability by large $c[T_{hj}]$.

It can also be seen in Fig. 2 that the mean cycle time approaches the limiting value of cycle time (the maximal cycle), given by the sum of switchover times and service times for all queues. As expected, for the two limiting cases of the cycle time, which correspond to lower traffic level (the empty cycle) and overloads (the maximal cycle), the cycle time coefficient of variation is very small (Fig. 3). This effect is caused by the very high (or low) probability for a queue to be empty at the considered low (or high) traffic intensity. As depicted in Fig.3, the

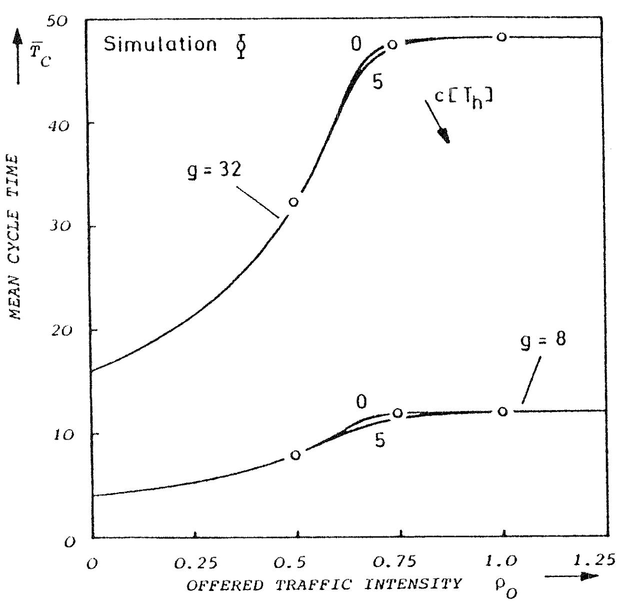

Fig. 2

Mean cycle time vs offered traffic (symmetrical load)

Parameters:
$\bar{T}_{uj} = \bar{T}_u = 0.5\ \bar{T}_h$
$S_j = 10;\ j=1,\ldots,g$
$g = 8\ /\ 32$

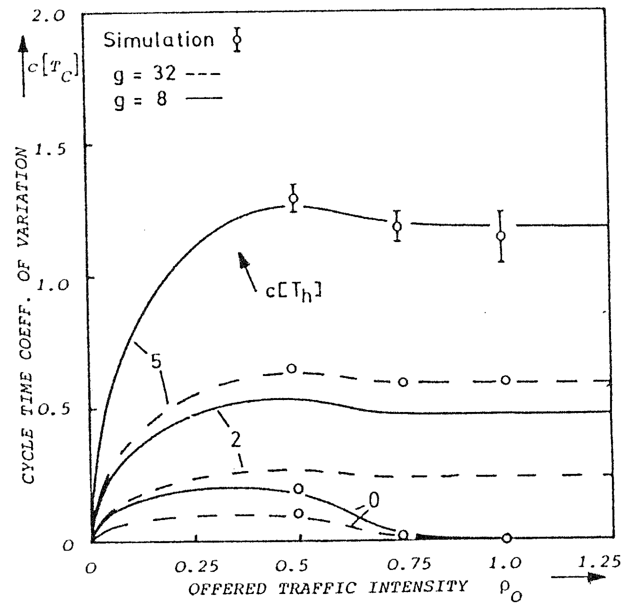

Fig. 3

Cycle time coefficient of variation vs offered traffic (symmetrical load)

Parameters:
$\bar{T}_{uj} = \bar{T}_u = 0.5\ \bar{T}_h$
$S_j = 10;\ j=1,\ldots,g$
$g = 8\ /\ 32$

cycle time coefficient of variation has a maximum, which increases by increasing service time variation or decreasing mean switchover time.

The mean waiting time and the blocking probability for messages are shown as functions of the offered traffic intensity in Figs. 4 and 5, respectively, for different server coefficients of variation. In Fig. 4 a crossover effect of the waiting time characteristics can be recognized. While the overall approximation accuracy is good concerning the waiting time, the algorithm presented is less accurate for large values of the service time coefficient of variation ($c[T_{hj}]=5$, c.f. Fig.5).

In Fig. 6, the blocking probability of messages is drawn for symmetrical systems with an extreme limitation of waiting places (S = 1), as can be found in telecommunication systems with a large number of terminating circuits in the peripheral environment.

The overall approximation accuracy for the given system parameters is good. However, it depends very strongly on the number of queues and the mean switchover time. In general, the accuracy of the algorithm increases with increasing values of switchover time (c.f.[8]). Results delivered by the presented method always show the same tendencies and phenomena as they are obtained by computer simulations.

4.2 Multiqueues with Nonsymmetrical Load Conditions

Nonsymmetrical load conditions exist in polling systems in which overload occurs in a part of the system. This phenomenon can be observed, e.g., in systems with distributed control (switching systems, LAN's, etc.), where an induction of the overload situation throughout the whole system can arise.

The example in this subsection considers a multiqueue system with $g = 3$, in which the first queue is overloaded with the traffic intensity $\rho_1=\lambda_1 T_{h1}$. Symmetrical conditions are assumed for the remainder queues.

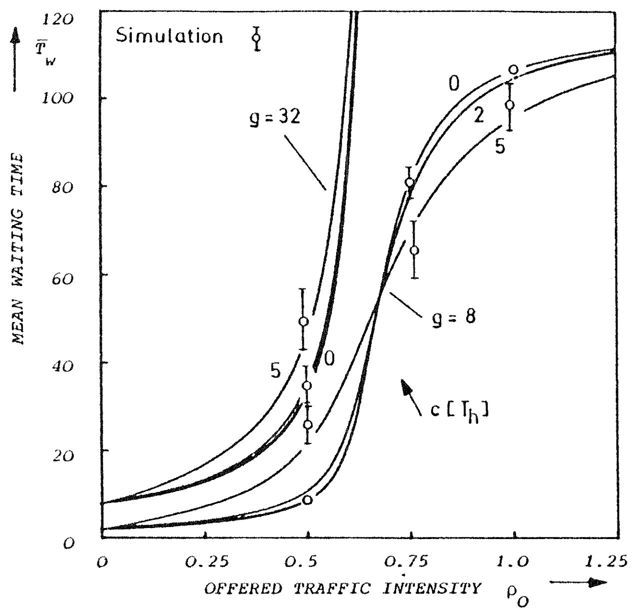

Fig. 4

Mean waiting time vs offered traffic (symmetrical load)

Parameters:
$\bar{T}_{uj} = \bar{T}_u = 0.5\ \bar{T}_h$
$S_j = 10;\ j=1,\dots,g$
$g = 8\ /\ 32$

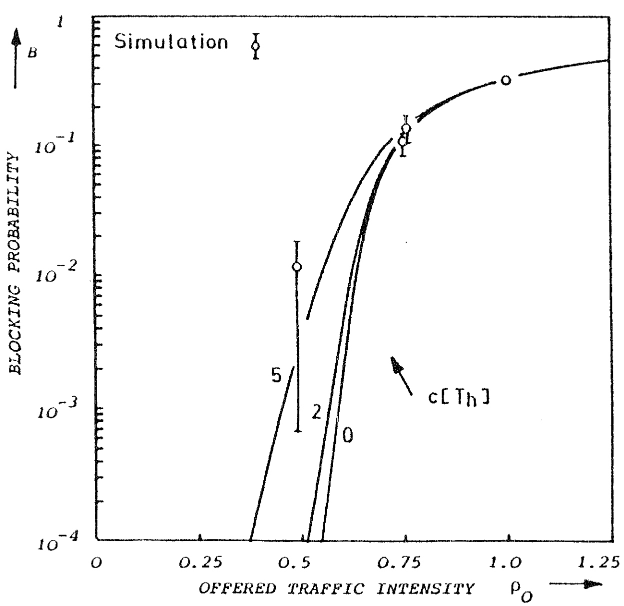

Fig. 5

Blocking probability vs offered traffic (symmetrical load)

Parameters:
$\bar{T}_{uj} = \bar{T}_u = 0.5\ \bar{T}_h$
$S_j = 10;\ j=1,\dots,g$
$g = 8$

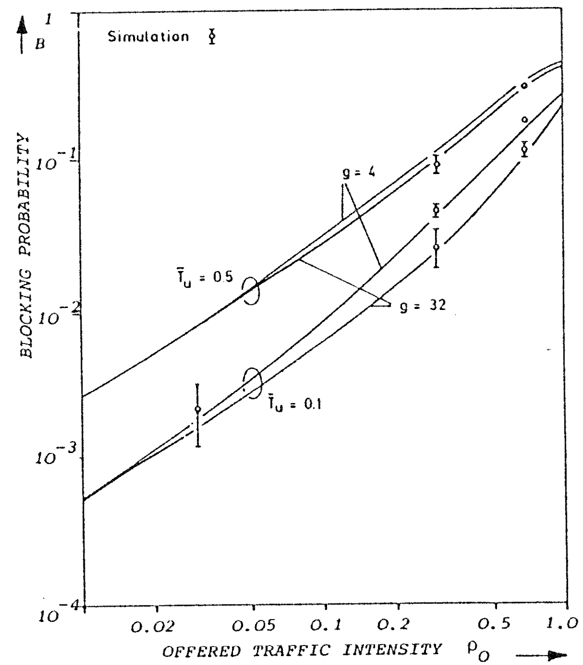

Fig. 6

Blocking probability vs offered traffic (symmetrical load)

Parameters!
$\bar{T}_{uj} = \bar{T}_u\ ;\ \bar{T}_{hj} = \bar{T}_h$
$S_j = 1\ ;\ j=1,\dots,g$
$g = 4\ /\ 32$

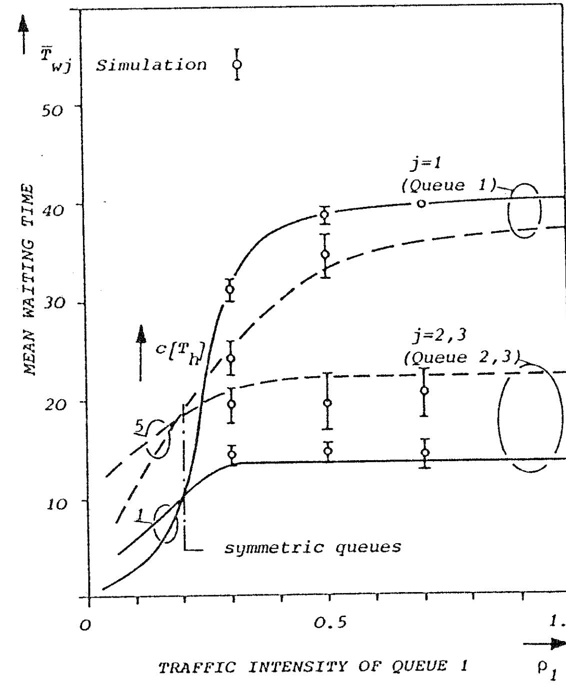

Fig. 7

Mean waiting time vs offered traffic of queue 1 (nonsymmetrical load)

Parameters:
$\bar{T}_{uj} = \bar{T}_u = 0.5\ \bar{T}_h$
$S_j = 10\ ;\ j=1,\dots,g$
$g = 3\ ;\ \rho_2 = \rho_3 = 0.2$

Fig. 7 shows the influence of the overload in queue 1 to the mean waiting times of queue 2 and 3, for different values of service time coefficient of variation. It can be seen clearly that the mean waiting time in queues 2 and 3 increases rapidly with increasing traffic intensity until a certain level (ρ_1 = 0.35). Above this level, according to the blocking effects in the system, the influence of the local overload situation in queue 1 to other queues in the system is limited. For the system parameters discussed the approximation accuracy is higher for smaller values of the service time coefficient of variation. Further studies show, as mentioned in subsection 4.1, a decrease of the accuracy for decreasing values of mean switchover time.

5. CONCLUSIONS

This paper provides an approximative analysis for finite capacity multiqueue systems with nonexhaustive cyclic service, where an effective numerical algorithm is developed. Under the realistic assumption of finite queue capacity results for mean cycle time, blocking probability, etc., for symmetrical as well as for non-symmetrical load conditions are derived. The accuracy of the presented algorithm as well as its convergence is good over a wide range of parameters. The class of multiqueue systems considered can be applied to modelling of computer and communication systems, such as token-ring local area networks or stored program controlled switching systems with distributed structure.

ACKNOWLEDGEMENT

The authors would like to express their thanks to Prof. P. J. Kuehn for reviewing the manuscript and to Mr. M.Weixler for his programming efforts.

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APPENDIX A1

The algorithm can be formulated as follows

I. Initialization of Markov chain probabilities for all queues and cycle times

II. REPEAT (Iteration cycle)

FOR ALL QUEUES DO
BEGIN

i. Calculate the mean and coefficient of variation for the approximate conditional cycle time T_C' and T_C'' according to eqn. (3.12)

ii. Calculate the parameters for the substitute conditional cycle times according to eqns. (3.14a,b), depending on the range of coefficient of variation

iii. Calculate the arrival probabilities $\{b_m'\}$ and $\{b_m''\}$ according to eqns. (3.8) and (3.15a,b)

iv. Calculate the Markov chain probabilities $\{P_{k,j}\}$ for the current iteration cycle according to (3.9)

v. Update $\bar{T}_E$ and $c[T_E]$ for the current iteration cycle according to (3.13)

vi. Calculate the current convergence indicator Δ_j according to (3.16)

END
UNTIL $\Sigma\Delta_j < \varepsilon$ (e.g., $\varepsilon = 10^{-6}$)

III. Calculate system characteristics

i. Calculate the mean and the coefficient of variation for the backward recurrence time $T_C'^{v}$ and $T_C''^{v}$ according to eqn. (3.18)

ii. Calculate $\{b_m'^{*}\}$ and $\{b_m''^{*}\}$ according to eqns. (3.18) and (3.22a,b)

iii. Calculate the arbitrary time state probabilities $\{P_k^*\}$ according to eqn. (3.21)

iv. Calculate system characteristics, such as blocking probabilities, mean waiting time etc. according to eqns. (3.23-26)