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Performance Analysis of Load-Balancing Semidynamic Scheduling Mechanisms in Distributed Systems

by Phuoc Tran-Gia* and Erwin Rathgeb**

Semidynamic scheduling mechanisms used in load-sharing, routing or job-scheduling problems form a class of strategies to optimize delay characteristics in distributed systems. An essential question arising in this class of problems is: how to distribute the system load among a number of heterogeneous service units in order to optimize the overall delay performance of the system. In this paper a performance investigation for semidynamic scheduling strategies using discrete-time analysis methods is presented. The interarrival process and the service process are assumed to be general. The arising performance model is decomposed into submodels in an exact way. The analysis of these submodels includes the discrete-time investigation of the general class of $G/G/1$ models with general service and cyclic input processes. To compare semidynamic scheduling strategies with random scheduling schemes, a model example is taken, for which numerical results are provided showing the influences of a range of system parameters, e.g., the types of input and service processes, the traffic intensity, etc., on the system performance.

Leistungsanalyse von semidynamischen Lastteilungsstrategien in verteilten Systemen

Semidynamische Lastteilungsstrategien finden häufig Anwendung in Systemen mit verteilter Steuerung. Diese Klasse von Mechanismen hat die Optimierung von Warte- und Durchlaufzeiten in derartigen Systemen zum Ziel. Beispiele sind Lastteilungs-, Wegesuch- oder Anforderungsverwaltungs-Mechanismen in Rechner- und Kommunikationssystemen. Dieser Beitrag stellt ein neues Verfahren zur Leistungsbewertung der semidynamischen Lastteilungsstrategien vor, das auf Methoden der zeitdiskreten Verkehrstheorie basiert. Sowohl die Ankunftsprozesse als auch die Bedienzeiten können dabei allgemein verteilt sein. Für die Analyse wird das gewonnene Gesamtmodell in Teilmodelle exakt dekomponiert. Das Spektrum der entstehenden Teilmodelle beinhaltet u. a. eine allgemeine Klasse von $G/G/1$- Systemen mit allgemein verteilten Bedienzeiten und zyklischen Ankunftsprozessen, für die ein neuer Analyse-Algorithmus entwickelt wird. Anhand eines Beispiels wird anschießend ein Leistungsvergleich von semidynamischen und zufallgesteuerten Lastteilungsstrategien durchgeführt. Weitere Ergebnisse zeigen den Einfluß der Systemparameter, wie z. B. Typ des Ankunfts- und Bedienprozesses, Verkehrsintensität etc. auf die Systemleistung, unter Anwendung unterschiedlicher Lastteilungsstrategien.

1. Introduction

In distributed systems the messaging delays are strongly influenced by the load-sharing mechanism, routing strategy or scheduling scheme implemented. In a common class of distributed processing systems a decentralized architecture is employed, where a number of heterogeneous processing units of different speeds and service characteristics share service of an incoming job stream. This architecture can be found, e. g., in file server systems, distributed databases, switching processors in stored program controlled (SPC) systems, transmission groups in computer networks, etc. In such systems, the incoming traffic has to be distrubuted among a number of distributed controlled, heterogeneous processing units (cf. Fig. 1). The connectivity is provided via an interconnection network. The load distribution is done according to a predefined scheduling strategy, taking into account the load conditions and the properties of the dedicated servers. Two problems have to be considered: 1) Define the amount of traffic to be routed to each of the processing units acting as servers in the system and 2) determine the scheduling mechanism to distribute these traffic streams in such a way, that the burstiness of inputs offered to the processing units is reduced and the delays can be minimized.

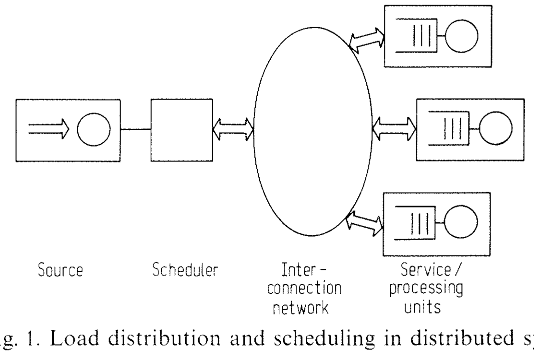

Fig. 1. Load distribution and scheduling in distributed systems.

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The aim of the scheduling-strategy design is an optimization of the overall delays for customers or jobs. A scheduler can be characterized by means of the following characteristics:

1) *Load Distribution Scheme:* Defines the amount of load which is directed to each processing unit. Examples are:

- *Load balancing*: Load is distributed in such a way, so that the heterogeneous servers will have the same utilization factor.
- *Load-driven or dynamic:* Load is distributed depending on the actual system load, in order to balance the actual load in all processing units (e. g., an incoming job will join the queue with lowest actual load level).

2) *Scheduling Scheme:* Determines the order to distribute the load according to the applied load-distribution scheme. Examples are:

- *Random scheduling:* The amount of load which is dedicated to the i-th processing unit will be formulated as a branching probability p_i, according to which the incoming traffic will be randomly split (Bernoulli branching).
- *Semidynamic scheduling:* The load distribution according to a deterministic scheduling cycle. The number of scheduling positions of a queue is proportional to the amount of traffic distributed to this queue.

The different load-sharing strategies require different levels of information about the system state. From an implementation viewpoint, it is obvious that the class of semidynamic scheduling schemes requires a smaller amount of processing overhead compared with other scheduling schemes. In the case of load-driven dynamic scheduling strategies, the scheduler has to known the actual load levels of all processing units; this often requires extensive on-line signalling overheads. To route a job according to the random scheduling, processing overhead of a random number generator implementing the branching probabilities is required. To dimension semidynamic scheduling strategies, only the processing speeds of the servers are needed as off-line input information during the system initialization phase.

An overview, including an extensive classification of scheduling strategies has been given in Wang and Morris [17]. Buzen and Chen [5] presented an algorithm to define the optimal load-distribution scheme assuming the random-scheduling strategy, Poisson input processes and general service times. In Yum [16] semidynamic scheduling strategies have been presented and investigated in the context of routing problems in computer communication networks, whereby Markovian input and service processes are taken into account. Analysis and performance comparisons concerning the random and semidynamic scheduling schemes have been given in Agrawala and Tripathi [3], [4] and Ephremides et al. [7].

The performance analysis of models for scheduling strategies, especially for semidynamic schemes leads to a number of submodels, for which a closed-form solution or a numerical algorithm in continuous-time domain and related transforms are not available. A typical example of this is the class of $G/G/1$ systems with non-renewal cyclic input processes. In most of the performance studies mentioned above, Markovian assumptions for arrival or service processes are considered, where solutions in transform domain can be obtained (cf. [4]).

In this paper, general assumptions are made for arrival and service processes, by means of which effects appearing in real systems like the influence of heterogeneous servers and non-Poisson arrival processes on the performance of the scheduling strategies can be investigated. The analysis employs methods developed for discrete-time queueing systems [1], [8], [15].

2. Models of Scheduling Strategies

In this section the queueing model and the according scheduling strategies will be defined. The basic queueing model is shown in Fig. 2.

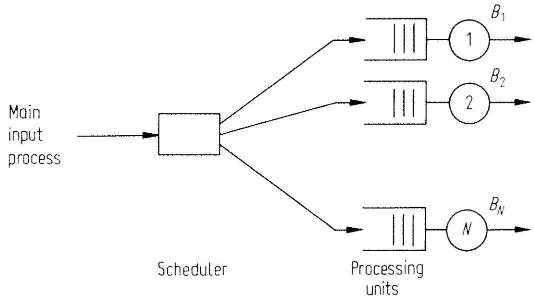

Fig. 2. The basic model.

2.1. Load Distribution and Scheduling Model

As illustrated in Fig. 2 the queueing system consists of a number N of single server queueing stations representing the processing units. The offered traffic of each queue results from the load distribution performed by the scheduler. As mentioned above, the main subject of interest is the quantitative load distribution among N heterogeneous servers of different capacities and service-time distributions.

The main arrival process is general with an arbitrary discrete-time interarrival distribution. The service processes are general discrete-time processes which can be individually chosen for each processing unit as well. The following random variables (r. v.) are used:

A is the r.v. for the interarrival time of the main arrival process,
B_i is the r.v. for the service time of server i

The queues are infinite and the service discipline is first-in, first-out (FIFO). Hence, the waiting time of a job only depends on the amount of unfinished work in the system (queue and server) seen upon arrival. The following notations for functions belonging to a discrete-time random variable X will be used:

$$x(k) = \Pr\{X = k\}, \; -\infty < k < +\infty: \quad \text{distribution of } X, \tag{1a}$$

$$X(k) = \sum_{i=-\infty}^{k} x(i), \; -\infty < k < +\infty: \quad \text{distribution function of } X, \tag{1b}$$

$$x_{ZT}(z) = \sum_{k=-\infty}^{\infty} x(k) z^{-k}: \quad Z\text{-transform of } x(k) \tag{1c}$$

EX, c_X are the mean and the coefficient of variation of X, respectively.

For the sums of independent random variables of the same type, the r.v. and the according distribution obtained by convolution are denoted as:

$$X^{(j)} = \sum_{i=1}^{j} X, \tag{2a}$$

$$x^{(j)}(k) = x(k)^{[j*]}. \tag{2b}$$

where the symbol $x(k)^{[j*]}$ is used for the j-fold convolution of the distribution $x(k)$ with itself.

In the following, attention is devoted to the load-balancing scheme used as load-distribution principle. Accordingly, we distribute the amount of incoming traffic to the servers in such a way that the servers in the system have the same utilization factor (i.e., the normalized traffic intensity)

$$\varrho_i = \frac{EB_i}{EA_i} = \varrho, \qquad i = 1, \ldots, N. \tag{3}$$

With the service time factors k_i defined by

$$k_i = \frac{EB_i}{EB_1}, \tag{4}$$

the mean interarrival time at queue i can be given as follows

$$EA_i = k_i \, EA_1 . \tag{5}$$

Considering the conservation of flows in the system we arrive at

$$\frac{1}{EA} \sum_{i=1}^{N} \frac{1}{EA_i}. \tag{6}$$

Thus, we obtain for the mean interarrival time of the input process offered to queue i

$$\frac{EA_i}{EA} = k_i \sum_{j=1}^{N} \frac{1}{k_j}. \tag{7}$$

2.2. *Semidynamic vs Random Scheduling*

The characterization of input processes at individual queues depends on the applied scheduling scheme, as specified in the following:

1) *Random Scheduling Scheme (RS):* According to this strategy the traffic offered to a queue i results from a Bernoulli branching of the main input process with the routing probability given by

$$p_i = \frac{EA}{EA_i}, \tag{8}$$

the input process at each queue is again a renewal process. The analysis of these decomposed processes in discrete-time domain will be dealt with in Section 3.1.

2) *Semidynamic Scheduling Scheme (SD):* The jobs are distributed in a cyclic manner. In general, the input process can be described by means of a non-renewal cyclic input process (or alternating renewal process [6]). The analysis, therefore, requires an algorithm to analyze general single server queues with cyclic inputs. This new method will be described in Section 3.3.

2.3. *Model Example*

1) *Model Parameters:* For the numerical computations in Section 4, a system with three servers ($N = 3$) is considered. The service time factors are chosen as follows [cf. (4)]:

$$k_1 = 1, \quad k_2 = \frac{EB_2}{EB_1} = \frac{3}{2}, \quad k_3 = \frac{EB_3}{EB_1} = 3. \tag{9}$$

From (7), (8) and (9) the routing probabilities p_i are obtained

$$p_1 = \tfrac{1}{2}, \quad p_2 = \tfrac{1}{3}, \quad p_3 = \tfrac{1}{6}. \tag{10}$$

For the semidynamic scheme, in order to fulfill the load-distribution scheme given in (8) the minimal cycle length is 6. During each arrival cycle the servers 1, 2 and 3 will receive 3, 2 and 1 jobs, respectively. Clearly, ten alternatives exist to design such an arrival cycle using different groupings for the appearances of the servers.

According to this cycle length, two semidynamic scheduling schemes will be defined and investigated for comparison purposes:

- Semidynamic Scheduling Scheme 1 (SD 1) defined by the sequence 1 1 1 2 2 3.
  In this scheme the appearances of the servers are grouped together in blocks.
- Semidynamic Scheduling Scheme 2 (SD 2) defined by the sequence 1 2 1 2 1 3.
  In this scheme the appearances are distributed as regularly as possible over the arrival cycle.

It is obvious that the choice of the scheduling cycle strongly affects the variation of the input processes offered to the servers, where grouping of appearances of a server leads to a more bursty arrival process offered to it. As a consequence, the grouping effect will lead to longer waiting time in the processing unit, as we shall see in the results presented in Section 4.

2) *Description of Submodels:*

a) Random scheduling: As mentioned in Section 2.2, the processes resulting from a random branching of a renewal process still have the renewal property. Therefore, the waiting-time distributions for the RS scheme can be evaluated using standard methods for discrete-time $G/G/1$ systems with renewal inputs as described in Section 3.2.

b) Semidynamic SD1: According to the SD1 scheme the jobs are distributed as depicted in Fig. 3,

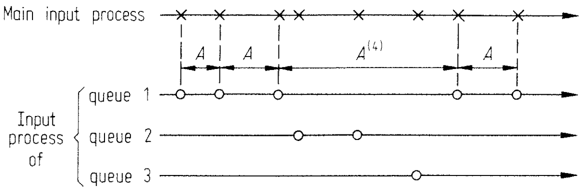

Fig. 3. Input process characteristics according to the semidynamic scheduling SD1.

where the interarrival processes at individual processing units are illustrated.

As a result, we observe at processing unit 1 a cyclic input with an arrival cycle consisting of three segments. The interarrival interval is of length A for the first and second segment and of length $A^{(4)}$ for the third segment. These three segments yield to an arrival process which is obviously non-renewal and has to be analyzed using the methods described in Section 3.3. Similarly, the cyclic input process at processing unit 2 has an arrival cycle consisting of two segments of length A and $A^{(5)}$, respectively.

A special case of a cyclic input can be observed for processing unit 3, where only one segment of length $A^{(6)}$ occurs. This process still is a renewal process and can be obtained from the main arrival process by means of convolutions. Because of this characteristic this class of processes will be referred to as *convolved* inputs. Their renewal property makes convolved inputs amenable to the more efficient analysis with standard methods operating in frequency domain for discrete-time $GI/G/1$ systems.

c) Semidynamic SD2: The processing units 1 ($A^{(2)}$) and 3 ($A^{(6)}$) in the SD2 scheme fall into the class of submodels with convolved inputs, whereby for processing unit 3 the input process is the same as in the case of the SD1 scheme. Thus, the processing unit 2 has to be analyzed using the algorithms for $G/G/1$ queues with cyclic inputs due to its two-segments cyclic arrival process.

2.4. System Characteristics

The performance measures of the whole model are obtained from the characteristics calculated in the submodel analysis. Denote W to be the waiting time for an arbitrary job entering the whole system and W_i the waiting time of jobs routed to the i-th processing unit, the overall waiting-time distribution can be determined by a weighted summation

$$w(k) = \sum_{i=1}^{N} p_i \, w_i(k). \tag{11}$$

Out of the waiting time W_i and the service time B_i the distribution of the sojourn time F of an arbitrary job of the main arrival process is given by:

$$f(k) = \sum_{i=1}^{N} p_i [w_i(k) * b_i(k)]. \tag{12}$$

3. Discrete-Time Analysis of Submodels

In this section, analysis methods for the submodels arising out of the scheduling models described above will be presented. As we have discussed in the previous section, the variation of arising submodels can be reduced to two basic types:

- Standard $GI/G/1$ systems (with renewal inputs) and
- $G/G/1$ systems with non-renewal cyclic inputs, which have to be investigated to form the compound solution of the whole system.

The solution for the standard $GI/G/1$ systems is well known in the literature and will be outlined in Section 3.2. For the analysis of systems with cyclic inputs a new iterative algorithm is developed and will be presented in Section 3.3.

An overview of the submodels and analysis methods applied is given in Fig. 4. Starting with a load-

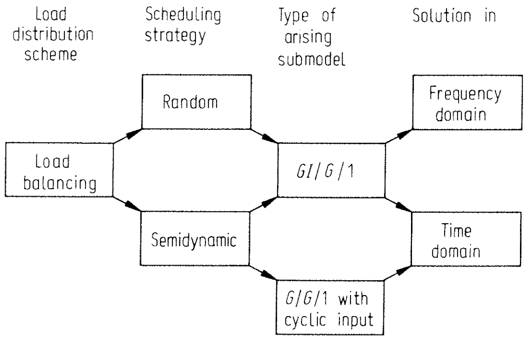

Fig. 4. Overview of submodels and analysis methods.

distribution scheme, i.e. the load-balancing scheme in our case, the two scheduling strategies, Random Scheduling and Semidynamic Scheduling, are analyzed. The standard $GI/G/1$ type of submodels can be found in the analysis of both scheduling schemes. The analysis of the waiting-time distribution for these systems can be done in the time domain as well as in the frequency domain. The submodel type with cyclic inputs appears in the analysis of the Semidynamic Scheduling schemes. Its solution operates in the time domain, as will be described in Section 3.3.

3.1. Random Branching of Discrete-Time Renewal Processes

According to the random-scheduling scheme (RS) the main arrival process is randomly routed to the i-th processing unit with the probability p_i. The input process of queue i is a renewal process described by the r.v. A_i with the distribution given as follows (cf. [11]):

$$a_{i,\mathrm{ZT}}(z) = \frac{p_i \, a_{\mathrm{ZT}}(z)}{1-(1-p_i)\, a_{\mathrm{ZT}}(z)}. \tag{13}$$

3.2. Discrete-Time Analysis of GI/G/1-Queues

For the discrete time as well as for the continuous time $GI/G/1$ system several approaches to compute the waiting-time distribution $w(k)$ have been proposed [1]–[10], most of which are based on the Lindley Integral Equation [9], [12]. Assuming the distributions for interarrival and service times to be of finite length according to

$$a(k) = \Pr\{A = k\}, \quad k = 0, 1, \ldots, n_A - 1, \quad n_A < \infty, \quad (14\text{a})$$

$$b(k) = \Pr\{B = k\}, \quad k = 0, 1, \ldots, n_B - 1, \; n_B < \infty, \quad (14\text{b})$$

in the discrete-time domain an equivalent form of this equation is given for stationary conditions by

$$w(k) = \pi(w(k) * c(k)), \qquad (15)$$

where

$$c(k) = a(-k) * b(k),$$

and the discrete π-operator is defined by [1], [9], [15]

$$\pi(x(k)) = \begin{cases} x(k) & \text{for} \quad k > 0 \\ \sum_{i=-\infty}^{0} x(i) & \text{for} \quad k = 0. \end{cases} \qquad (16)$$

The derivation of (15) can be found in [15]. This equation can be solved by iteration in the time domain (probability domain) or directly, without iteration in the frequency domain [1]. The latter method has been found to be more efficient for computation, especially when the distributions $a(k)$ and $b(k)$ are relatively long ($> 2^{10}$ elements). To get (15) into a suitable form for solutions in the frequency domain we introduce the discrete probability distribution function $W(k)$ and express (15) as

$$W(k) = \begin{cases} 0 & \text{for} \quad k < 0 \\ c(k) * W(k) & \text{for} \quad k \geqq 0. \end{cases} \qquad (17)$$

Defining a sequence $W^-(k)$ similar to Kleinrock [9] as

$$W^-(k) = \begin{cases} c(k) * W(k) & \text{for} \quad k < 0 \\ 0 & \text{for} \quad k \geqq 0 \end{cases} \qquad (18)$$

and using the Z-transform to move into the frequency domain we get

$$\frac{W_{ZT}^-(z)}{W_{ZT}(z)} = c_{ZT}(z) - 1 \qquad (19)$$

or, replacing the probability distribution function by the probability distribution

$$\frac{W_{ZT}^-(z)}{w_{ZT}(z)} = \frac{c_{ZT}(z) - 1}{1 - z^{-1}} = S_{ZT}(z). \qquad (20)$$

For finite length sequences $a(k)$ and $b(k)$, $c(k)$ is also a finite length sequence. Furthermore, the function $c_{ZT}(z)$ can be shown to have a single zero for $z = 1$. Taking these properties into account $S_{ZT}(z)$ has to be a finite polynomial in $1/z$ and for that reason has no poles. It can also be shown that the term $W_{ZT}^-(z)$ is a polynomial without poles for a finite length $c(k)$. Applying the theorem of Eneström and Kakeya [1] to $W^-(-k)$ we find, that all zeros of $W_{ZT}^-(z)$ are located outside the unit circle. The function $w_{ZT}(z)$ is the Z-transform of a probability distribution and converges for $z = 1$. From this we can conclude that $w_{ZT}(z)$ has only poles inside the unit circle and so all zeros of $1/w_{ZT}(z)$ have to be located inside the unit circle as well. Since $S_{ZT}(z)$ and $W_{ZT}^-(z)$ have no poles and the latter function only has zeros outside the unit circle it is obvious that $1/w_{ZT}(z)$ can have no poles. To obtain $w_{ZT}(z)$ from (20) it is thus necessary to find the zeros of $S_{ZT}(z)$ and to separate them with respect to their location to the unit circle.

Two principles have been proposed to accomplish this separation numerically:

- The polynomial factorization algorithm as proposed by Konheim [10]. In this algorithm the zeros of the characteristic function have to be explicitly determined, which may be inefficient for interarrival and service-time distributions with a large number of elements. Furthermore, the results are given in the frequency domain only and further computations are required to get them into the probability domain.
- The complex cepstrum algorithm as presented by Ackroyd [1]. This algorithm takes advantage of the properties of the complex cepstrum [13] and all operations involved, e.g., convolutions and correlations can be computed using highly efficient Fast Fourier Transform algorithms. This algorithm delivers the waiting-time distribution of the $GI/G/1$ queueing system. For the computation of the random-scheduling scheme and investigation of the parts of the semidynamic schemes with convolved inputs, the algorithm as proposed by Ackroyd [1] has been used, in combination with a decomposition procedure in transform domain implementing the relationship in (13).

3.3. Analysis of G/G/1-Systems with Cyclic Inputs

In the literature, solutions of queueing systems with cyclic inputs can be found in continuous-time domain under simplifying assumptions, mainly for systems with Poisson input [4]. To investigate the system with more general assumptions, e.g., heterogeneous service process and general cyclic input, and to overcome numerical barriers of transform techniques, an analysis approach operating in discrete-time domain has been developed and will be presented. It should be noted here that this class of problems can be dealt with using results and algorithms employing Levinson's method for systems with cyclo-stationary behavior [2]. In this subsection the alternative applying iterative convolutions will be described.

1) Algorithm in Discrete-Time Domain: We consider a single server with arbitrary distributed service times in discrete-time domain. The input process is cyclic and consists of a number n of interarrival intervals or segments. With respect to the processing unit i these intervals will be denoted by means of the r.v. $A_{ij}, j = 1, \ldots, n$. Within an arrival cycle an interval A_{ij} is assumed to be started with the job j which experiences the waiting time W_{ij}. For the first-in, first-out service discipline, W_{ij} is the amount of unfinished work seen by job j upon arrival (cf. Fig. 5). The service time of job j is denoted by the r.v. $B_{ij}, j = 1, \ldots, n$. Considering the process development of unfinished work during an arrival cycle, as depicted in Fig. 5 the following equation system can be obtained under sta-

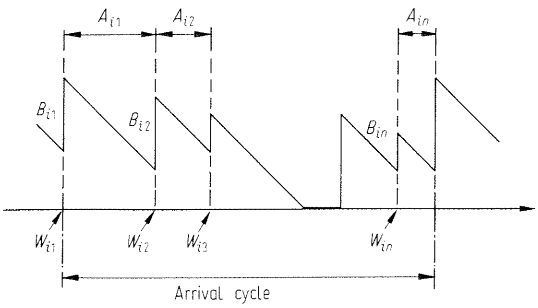

Fig. 5. A sample path of unfinished work in a single server queue with cyclic input.

tionary conditions for the random variables

$$W_{ij+1} = \max(W_{ij} + B_{ij} - A_{ij}, 0), \quad j = 1, \ldots, n-1,$$
$$W_{i1} = \max(W_{in} + B_{in} - A_{in}, 0), \qquad (21)$$

and accordingly, for the waiting-time distributions

$$w_{ij+1}(k) = \pi(w_{ij}(k) * b_{ij}(k) * a_{ij}(-k)), \quad j = 1, \ldots, n-1, \qquad (22)$$
$$w_{i1}(k) = \pi(w_{in}(k) * b_{in}(k) * a_{in}(-k)).$$

In accordance with (22), the waiting-time distributions of jobs in the next arrival cycle can be calculated successfully from those of the current arrrival cycle. Using this fact the equilibrium waiting-time distributions can be determined iteratively, as schematically depicted in Fig. 6 (cf. [15]). This iteration scheme is

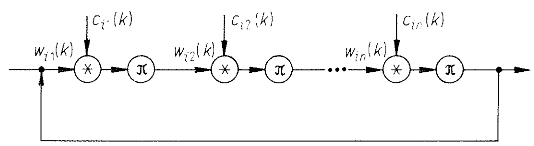

Fig. 6. Computational diagram of the algorithm in time domain.

referred to as the method of iterative convolutions. For large vector sizes of the arrival and service distributions, the discrete convolution operation can be implemented efficiently using standard algorithms, e.g., the Fast Fourier Transform (based on the Discrete Fourier Transform) [8], [13].

2) Waiting-Time Distribution: The waiting-time distributions of jobs within an observed cycle $w_j(k)$, $j = 1, \ldots, n$, as obtained by means of the iterative convolution described in the previous subsection, form the basic requirements for the calculation of further system characteristics.

The waiting-time distribution of an arbitrary job arriving at the processing unit i is determined as:

$$w_i(k) = \frac{1}{n} \sum_{j=1}^{n} w_{ij}(k). \qquad (23)$$

The overall waiting time and sojourn distributions seen from an arbitrary job entering the system are calculated from these waiting-time distributions in accordance with (11) and (12).

4. Numerical Examples

To provide a quantitative comparison of the scheduling strategies discussed above, the model example as presented in Section 2.3 will be investigated. Again, the three scheduling mechanisms considered will be referred to as

- RS: Random Scheduling.
- SD1: Semidynamic Scheduling according to the sequence 1 1 1 2 2 3.
- SD2: Semidynamic Scheduling according to the sequence 1 2 1 2 1 3.

To investigate the influences of the random processes systematically we consider the random variables having distributions given by the first two moments. With the exception of the deterministic case, we use for both, the arrival and the service processes, the negative binomial distribution for this purpose:

$$x(k) = \binom{y+k-1}{k} p^y (1-p)^k, \quad 0 < p < 1, \quad y \text{ real}. \qquad (24)$$

The mean and the coefficient of variation are given by:

$$EX = \frac{y(1-p)}{p}, \quad c_X^2 = \frac{1}{y(1-p)} \qquad (25a)$$

or

$$p = \frac{1}{EX\, c_X^2}, \quad y = \frac{EX}{EX\, c_X^2 - 1}, \quad \text{where } EX\, c_X^2 > 1. \qquad (25b)$$

The time measures will be given in a normalized form with the discrete time unit $\Delta t = 1$. For the mean-waiting time EW_i of jobs at individual queues, Fig. 7 shows a comparison of the worst performing scheme (Random Scheduling) and the scheme with the best performance (SD2). The server coefficients of variation have been set to $c_B = 0.5$, which are equivalent to continuous-time Erlangian distribution of 4-th order

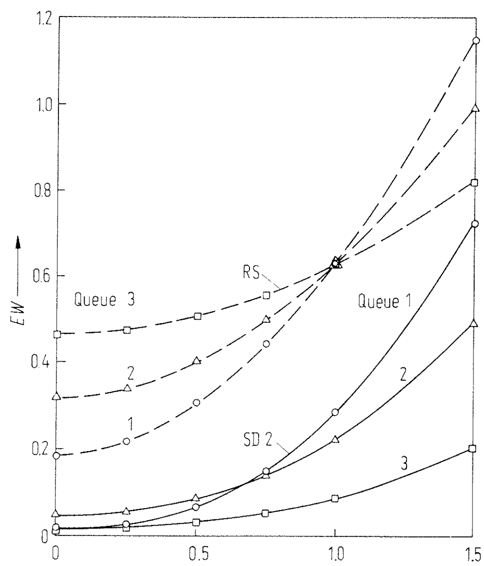

Fig. 7. Influence of scheduling strategies on queue individual waiting times.

($EB_1 = 30$, $\varrho = 0.5$). The mean-waiting times have been normalized to the mean-service times of the corresponding servers. It is obvious from Fig. 7 that for any queue the normalized mean-waiting time is higher for the RS scheme than for the SD 2 scheme over the whole range of c_A. The coefficient of variation of the interarrival process at queue i for the RS scheme is known as [11]:

$$c_{Ai}^2 = p_i\, c_A^2 + (1 - p_i). \qquad (26)$$

According to this formula $c_{A3,\mathrm{RS}}$ is the highest of the coefficients of variation $c_{Ai,\mathrm{RS}}$ for $c_A < 1$, for $c_A = 1$ there is a cross-over point with $c_{A1,\mathrm{RS}} = c_{A2,\mathrm{RS}} = c_{A3,\mathrm{RS}} = 1$. In the range of $c_A > 1$, $c_{A3,\mathrm{RS}}$ is lower than $c_{A1,\mathrm{RS}}$ and $c_{A2,\mathrm{RS}}$.

Since a higher interarrival time coefficient of variation implies a higher normalized mean-waiting time, the same behavior can be observed for the corresponding mean-waiting times.

As far as the SD 2 scheme is concerned, queue 1 and queue 3 have got convolved inputs. In this context, it shall be mentioned here, that mean value and coefficient of variation of a convolved input process are defined by

$$E[A^{(j)}] = j\,EA, \quad c_{A(j)} = \frac{c_A}{\sqrt{j}}. \qquad (27)$$

According to (27) the normalized mean-waiting time of queue 1 ($A_1 = A^{(2)}$) has to be higher compared to that of queue 3 ($A_3 = A^{(6)}$) over the full range except for $c_A = 0$, where $c_{A1,\mathrm{SD2}} = c_{A3,\mathrm{SD2}} = 0$. For the cyclic input queue 2, due to the alternating input process, $c_{A2,\mathrm{SD2}}$ is greater than 0 even for $c_A = 0$, which results in a higher normalized waiting time compared to the queues 1 and 3 at this point.

Fig. 8 shows the waiting-time coefficients of variation of the queues for the same parameters. Interpreting these results one has to take into account the small absolute value of the mean-waiting time for the SD 2 scheme and small interarrival time coefficients of variation. In these cases the whole waiting-time distribution is dominated by the probability of having disappearing waiting time, which approaches the value 1. Fig. 9 shows the complementary waiting time probability distribution functions for the case where $c_A = c_B = 0.5$. It can be seen that all distributions tend to have geometric tail characteristics.

Figs. 10 to 12 show a comparison of the mean overall waiting times for the three scheduling schemes over a variation of systems parameters. These are the service-time coefficient of variation c_B (Fig. 10), the interarrival time coefficient of variation c_A (Fig. 11) and the traffic intensity of the arrival stream (Fig. 12).

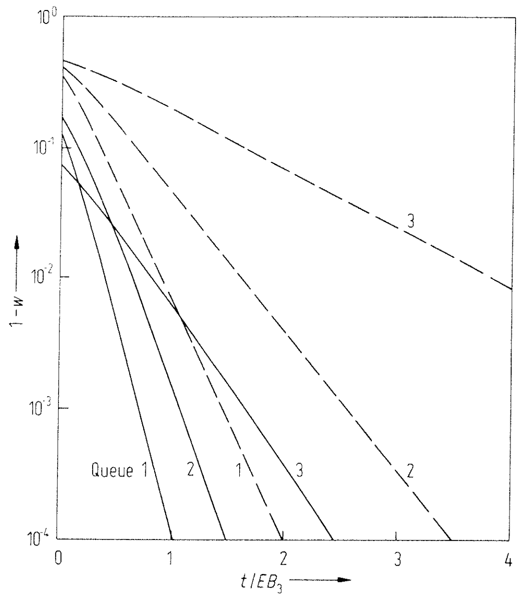

Fig. 9. Queue individual complementary waiting-time distribution functions. --- RS, —— SD 2, $c_A = 0.5$, $c_B = 0.5$.

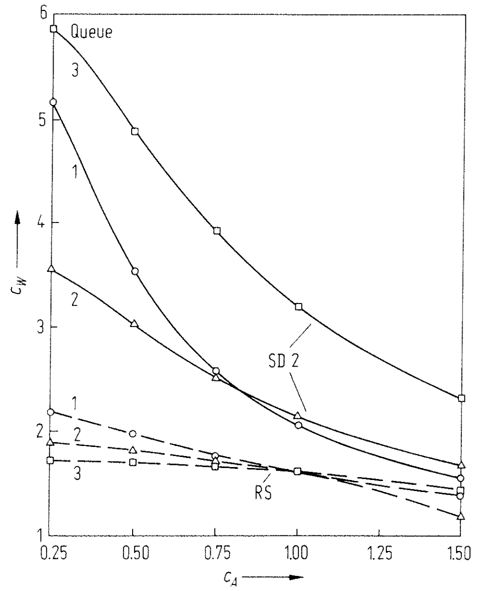

Fig. 8. Influence of scheduling strategies on the waiting-time variation.

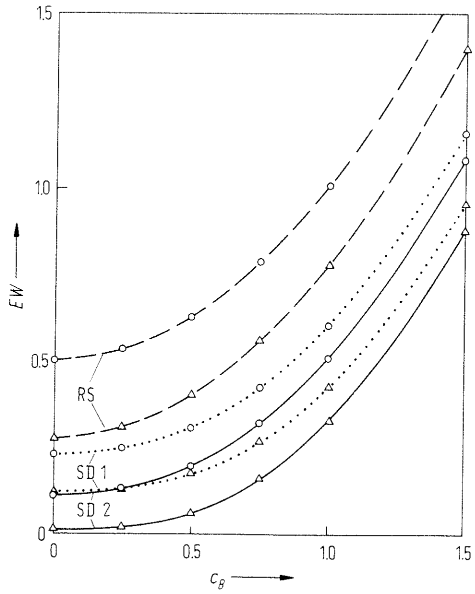

Fig. 10. Influence of scheduling strategies and types of servers on waiting times. (△) $c_A = 0.5$, (○) $c_A = 1$.

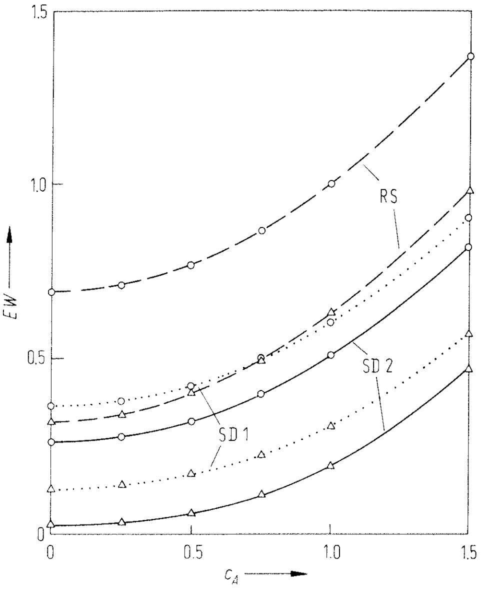

Fig. 11. Influence of scheduling strategies and types of arrival processes on waiting times. (△) $c_B = 0.5$, (○) $c_B = 1$.

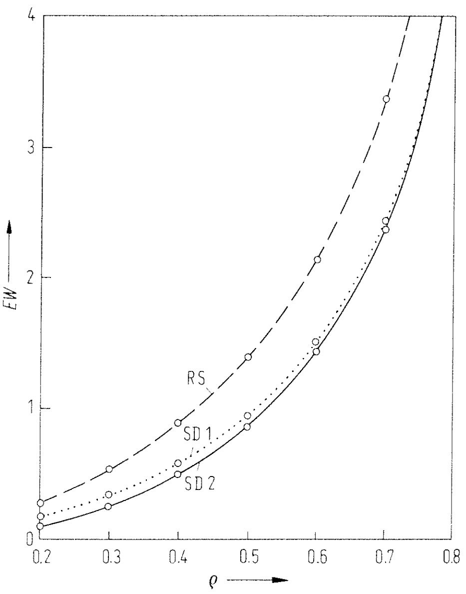

Fig. 12. Influence of scheduling strategies and server utilizations on waiting times. $c_A = 0.5$, $c_B = 1.5$.

The mean-waiting times have been normalized for these figures to the overall mean service time $EB = 45$. The figures demonstrate that in all cases the semi-dynamic schemes lead to better performance than the random scheme, independent of the system parameters.

5. Conclusion and Outlook

In this paper, we present an exact analysis for a generic class of scheduling strategies using methods operating in discrete-time. Attention is devoted to the class of semidynamic scheduling strategies which can be found in load-sharing, routing, and job-scheduling problems in distributed systems. General assumptions are made for arrival and service processes, by means of which effects appearing in real systems like the influence of heterogeneous servers and non-Poisson arrival processes on the performance of the scheduling strategies are investigated.

The model is decomposed into submodels in an exact manner. The analysis of the resulting submodels includes a new class of models with non-renewal processes, i.e., $G/G/1$ models with general service and cyclic input processes. Different variations of semi-dynamic scheduling strategies are compared with the random-scheduling scheme using a model example, for which numerical results are provided to show the influences of system parameters on the overall system performance.

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