

DELAY SYSTEMS WITH LIMITED ACCESSIBILITY

by

Martin H. Thierer

Dipl. Ing., scientific assistant
Institute for Switching and Data Technics
Technical University Stuttgart
Federal Republic of Germany

Summary

The study deals with the traffic problems of delay systems with limited accessibility. The probabilities of state, the probability of delay and the mean waiting time for Erlang's ideal gradings are derived.

The solution may be named "Interconnection Delay Formula (IDF)". It is similar to the wellknown Interconnection Formula of Erlang (EIF) for limited-access trunkgroups in loss systems having ideal Erlang gradings.

It will be shown that the IDF yields good approximate values for usual non-ideal gradings too.

Introduction

In loss systems the most important quantity for characterizing traffic problems is the probability of loss.

In delay systems the corresponding quantity is the probability of delay. For the description of the traffic problems in delay systems a second important quantity is necessary: the mean waiting time.

One gets these quantities by the calculation of the probabilities of state in the system.

Full-access trunkgroups in loss systems and in delay systems can be calculated exactly. A.K. Erlang has developed the solution of both problems /1/.

For economical reasons limited-access trunkgroups are often used. Generally one cannot calculate these arrangements exactly because of the large number of unknowns.

For loss systems A.K. Erlang has found an exact solution, valid for so-called "ideal Erlang gradings". This solution is known as Erlang's Interconnection Formula (EIF).

The ideal gradings are mostly not realizable because of the large number of necessary incoming grading groups. In case

of non-ideal gradings the EIF-loss often underrates slightly the real values of loss.

A first approximation for delay systems with limited accessibility has been published by E. Gambe on the 3rd ITC 1961 /2/.

In this study an Interconnection Delay Formula (IDF) is developed for delay systems using Erlang's ideal gradings. It permits calculating the probabilities of state, the probability of delay and the mean waiting time.

It will be shown that the results of IDF yield good approximations for usual non-ideal gradings in delay systems.

The System

A single-stage connecting array with n outgoing trunks is considered. The traffic is offered to g incoming groups. A call offered to an incoming group has access to k out of n outgoing trunks. The number k is called accessibility. The trunkgroup of n outgoing trunks is a limited-access trunk-group.

Erlang's ideal grading consists of

$$g = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

incoming grading groups. $\binom{n}{k}$ is the number of ways of selecting k trunks out of n . Every incoming grading group has access to another combination of k trunks chosen from a total of n trunks.

In fig.1 an example of an ideal Erlang grading is shown.



Fig.1 Ideal Erlang grading

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A call offered to an incoming grading group is queuing in front of this group, if all k trunks of this incoming group are busy. The maximum number of queuing calls may be infinite.

If one occupation terminates on a trunk, which is hunted by the waiting calls of one or more queues, one waiting call out of only one of those queues can be served. In the following theory it is not necessary to distinguish which waiting call in which of those queues has been served. The state probabilities, the probability of delay and the mean waiting time are independent of the queuing discipline in Erlang's ideal grading.

The state probabilities of the outgoing trunks are also independent of the order of hunting in Erlang's ideal grading.

The Traffic

The offered traffic is assumed to be Poissonian having an average number of calls c_A offered per unit time.

The traffic offered may be evenly distributed among all incoming groups.

The probability of a call occurring during the time dt is

$$c_A \cdot dt \quad (1)$$

The holding time is assumed to have a negative exponential distribution with the mean h .

The probability that an occupation will terminate during dt holds

$$\frac{1}{h} \cdot dt$$

If there are x simultaneous occupations the probability that any one of them will terminate during dt will be

$$\frac{x}{h} \cdot dt \quad (2)$$

The traffic offered is defined by

$$A = c_A \cdot h \quad (3)$$

In the delay system the traffic offered is equal to the traffic carried, because no call is rejected and no call refuses to wait.

Therefore, the assumption of statistical equilibrium is only satisfied, if the traffic offered does not exceed the number of outgoing trunks n .

$$A \leq n$$

The Transition Probabilities

The state of a grading in delay systems may be characterized both by the number of busy trunks x and the sum z of calls waiting simultaneously in front of all incoming groups.

The probability of the state $\{x, z\}$ is denoted by $p(x, z)$.

If a system is in the state of statistical equilibrium the sum of the transition probabilities from within a state $\{x, z\}$ into all neighbour states is equal to the sum of the transition probabilities from these neighbour states into the state $\{x, z\}$. A neighbour state changes the number of simultaneous occupations by one only.

The state $\{x, z\}$ has 4 neighbour states, if it is no boundary state.

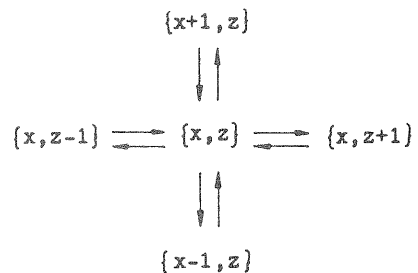


Fig.2 The neighbour states of $\{x, z\}$

The transition probabilities to and from this 4 neighbour states will be calculated now.

If the number of busy trunks x is greater or equal to the accessibility k , a call offered to an incoming group will not always find an idle trunk. In this case, the call waits in a queue in front of the incoming grading group.

In Erlang's ideal grading a call will find no idle trunk in exactly

$\binom{x}{k}$ incoming groups out of total $\binom{n}{k}$, if there exist x busy trunks among n .

In the state $\{x\}$ the conditional probability $\sigma(x)$ that all k trunks in an incoming grading group will be busy is given by

$$\sigma(x) = \frac{\binom{x}{k}}{\binom{n}{k}} \quad (4)$$

It may be defined $\sigma(x)=0$ $x=0, 1, \dots, k-1$

The probability $\sigma(x)$ is independent of the number of waiting calls z .

State {x,z+1}

The absolute probability of a transition from the state {x,z} to {x,z+1} during dt is

$$p(x,z) \cdot \sigma(x) \cdot c_A \cdot dt \quad \begin{matrix} x=0,1,\dots,n \\ z=0,1,\dots \end{matrix} \quad (5)$$

If the number of busy trunks x is less than the accessibility k, an incoming call always will find a free trunk. It is impossible that a call has to wait.

$$p(x,z) = 0 \quad \begin{matrix} x=0,1,\dots,k-1 \\ z=1,2,\dots \end{matrix} \quad (6)$$

State {x+1,z}

The state {x,z} will change into the state {x+1,z}, if an offered call can find a free trunk. The transition probability is given by the following expression:

$$p(x,z) \cdot [1 - \sigma(x)] \cdot c_A \cdot dt \quad \begin{matrix} x=0,1,\dots,n-1 \\ z=0,1,\dots \end{matrix} \quad (7)$$

The termination probability of an occupation during dt was found to be

$$\frac{x}{h} \cdot dt \quad x=1,2,\dots,n \quad \text{cf. (2)}$$

After termination of a busy trunk the state {x,z} changes into {x-1,z} or into {x,z-1}.

State {x-1,z}

The transition probability from {x,z} to {x-1,z} is

$$p(x,z) \cdot \varrho(x,z) \cdot \frac{x}{h} \cdot dt \quad \begin{matrix} x=1,2,\dots,n \\ z=0,1,\dots \end{matrix} \quad (8)$$

$\varrho(x,z)$ is defined as the conditional probability that after termination of a busy trunk the state {x,z} changes into the state {x-1,z}.

The following special values of $\varrho(x,z)$ hold:

a) $\varrho(k,z) = 0 \quad z = 1,2,\dots \quad (9)$

The state {k-1,z} for z=1,2,... can not exist cf. (6); therefore a transition from {k,z} to {k-1,z} is impossible.

b) $\varrho(x,0) = 1 \quad x = 1,2,\dots,n \quad (10)$

By termination on a busy trunk a state {x,0} changes always to a state {x-1,0}.

It will be seen later, that the other probabilities $\varrho(x,z)$ are not necessary for the calculation of the state probabilities p(x,z).

State {x,z-1}

The conditional probability of transition from {x,z} to {x,z-1} is

$$1 - \varrho(x,z)$$

Hence, finally, the transition probability:

$$p(x,z) \cdot [1 - \varrho(x,z)] \cdot \frac{x}{h} \cdot dt \quad \begin{matrix} x=0,1,\dots,n \\ z=1,2,\dots \end{matrix} \quad (11)$$

The transitions of the states can be demonstrated in a state diagram. The coefficients of the state probabilities multiplied by h and omitting dt are added.

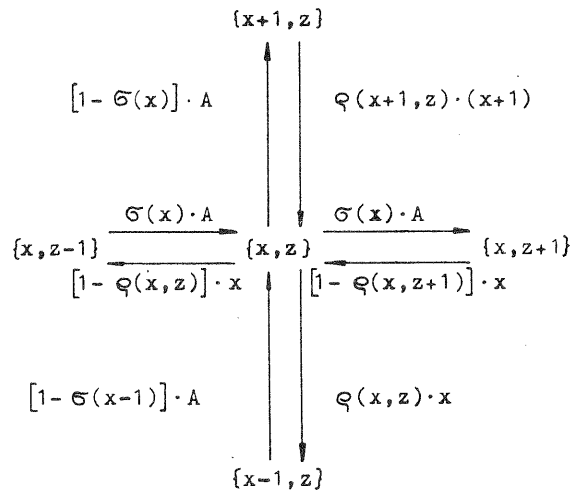


Fig.2 State diagram

The Equations of State

The sum of transition probabilities from within the state {x,z} is with eq.(5,7,8,11)

$$p(x,z) \left\{ \begin{matrix} [1 - \sigma(x)] \cdot c_A \cdot dt \\ + \sigma(x) \cdot c_A \cdot dt \\ + \varrho(x,z) \cdot \frac{x}{h} \cdot dt \\ + [1 - \varrho(x,z)] \cdot \frac{x}{h} \cdot dt \end{matrix} \right\} = p(x,z) \cdot \left\{ c_A + \frac{x}{h} \right\} \cdot dt \quad (12)$$

The sum of the transition probabilities into the state {x,z} is equal (12) because of the assumption of statistical equilibrium:

$$p(x,z) \cdot \left\{ c_A + \frac{x}{h} \right\} \cdot dt =$$

$$p(x,z-1) \cdot \sigma(x) \cdot c_A \cdot dt$$

$$+ p(x-1,z) \cdot [1 - \sigma(x-1)] \cdot c_A \cdot dt$$

$$+ p(x+1,z) \cdot \varrho(x+1,z) \cdot (x+1) \cdot dt \cdot \frac{1}{h}$$

$$+ p(x,z+1) \cdot [1 - \varrho(x,z+1)] \cdot x \cdot dt \cdot \frac{1}{h}$$

The solution of this linear equation system is not known because of the infinite number of unknowns. To find a solution it was necessary to introduce an additional assumption.

Assumption:

The transition probabilities between two neighbour states are always equal.

- a) Transitions $\{x-1, z\} \rightleftharpoons \{x, z\}$ are equally probable

$$p(x, z) \cdot Q(x, z) \cdot \frac{x}{h} \cdot dt = p(x-1, z) \cdot [1 - \sigma(x-1)] \cdot c_A \cdot dt \quad (13)$$

- b) Transitions $\{x, z-1\} \rightleftharpoons \{x, z\}$ are equally probable

$$p(x, z) \cdot [1 - Q(x, z)] \cdot \frac{x}{h} \cdot dt = p(x, z-1) \cdot \sigma(x) \cdot c_A \cdot dt \quad (14)$$

By addition of eq.(13) and (14) and multiplication by h one gets the following equation system:

$$p(x, z) \cdot x = p(x-1, z) \cdot [1 - \sigma(x-1)] \cdot A + p(x, z-1) \cdot \sigma(x) \cdot A \quad (15)$$

$x=1, 2, \dots, n$
 $z=1, 2, \dots$

The equation of state of the boundary state $\{x, 0\}$ is obtained appropriately:

$$p(x, 0) \cdot x = p(x-1, 0) \cdot [1 - \sigma(x-1)] \cdot A \quad (16)$$

$x=1, 2, \dots, n$

The equation system (15), (16), together with the normalizing condition

$$\sum_{x=0}^n \sum_{z=0}^{\infty} p(x, z) = 1$$

yields a recurrence formula for the state probabilities $p(x)$ in the outgoing trunkgroup.

The Probability of State $\{x\}$

Be $p(x)$ the probability of the state $\{x\}$ which means that x trunks of the outgoing trunkgroup are busy.

One gets the probability $p(x)$ by the following summation:

$$p(x) = \sum_{z=0}^{\infty} p(x, z) \quad x=0, 1, \dots, n \quad (17)$$

Eq.(15) summed up from $z = 1$:

$$x \cdot \sum_{z=1}^{\infty} p(x, z) = [1 - \sigma(x-1)] \cdot A \cdot \sum_{z=1}^{\infty} p(x-1, z) + \sigma(x) \cdot A \cdot \sum_{z=1}^{\infty} p(x, z-1)$$

and together with eq.(16):

$$x \cdot p(x, 0) = [1 - \sigma(x-1)] \cdot A \cdot p(x-1, 0)$$

one obtains :

$$x \cdot p(x) = [1 - \sigma(x-1)] \cdot A \cdot p(x-1) + \sigma(x) \cdot A \cdot p(x)$$

or

$$[x - \sigma(x) \cdot A] \cdot p(x) = [1 - \sigma(x-1)] \cdot A \cdot p(x-1) \quad x=1, 2, \dots, n \quad (18)$$

The probabilities of the states $\{0, z\}$ are

$$p(0, z) = 0 \quad z=1, 2, \dots \text{ cf. (6)}$$

therefore with (17)

$$p(0) = p(0, 0) \quad (19)$$

Successive addition of eq.(18) yields

$$p(x) = p(0) \cdot A^x \cdot \frac{\prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x [i - \sigma(i) \cdot A]} \quad (20)$$

With the condition that the sum of all $p(x)$ is unity one obtains:

$$\frac{1}{p(0)} = \sum_{x=0}^n A^x \cdot \frac{\prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x [i - \sigma(i) \cdot A]} \quad (21)$$

The probabilities $p(x)$ has been checked by artificial traffic trials on a digital computer of the German Research Society.

Fig.3 shows the probabilities $p(x)$ calculated by eq.(20,21). The results of artificial traffic are plotted by the dotted line.

The tested grading is an ideal Erlang grading with $n=9$ trunks, accessibility $k=6$ and $g = \binom{9}{6} = 84$ incoming groups.

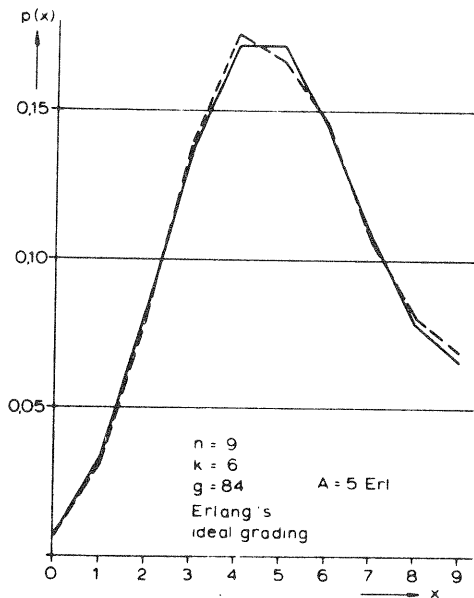


Fig.3 The probabilities $p(x)$ of the states $\{x\}$

The Probability of Delay

If a call arrives at the delay system, it will find x trunks busy with the probability $p(x)$. If x trunks are busy, the call will have no access to a free trunk with the conditional probability $\sigma(x)$. The absolute probability of delay in the state $\{x\}$ therefore is $p(x) \cdot \sigma(x)$.

The total probability of delay is the sum

$$W = \sum_{x=k}^n p(x) \cdot \sigma(x) \quad (22)$$

The probability of delay is defined, too, as the average number of delayed calls c_W divided by the average number of offered calls c_A :

$$W = \frac{c_W}{c_A} \quad (23)$$

The probability of delay (22) takes the following form with eq.(20,21):

$$W = \sum_{x=k}^n \frac{A^x \cdot \prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x [i - \sigma(i) \cdot A]} \cdot \sigma(x) \cdot \frac{\sum_{x=0}^n A^x \cdot \prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x [i - \sigma(i) \cdot A]} \quad (24)$$

This formula of the delay probability is very similar to Erlang's Interconnection Formula (EIF), which yields exactly the probability of loss for Erlang's ideal gradings:

$$B_{EIF} = \frac{\sum_{x=k}^n \frac{A^x \cdot \prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x i} \cdot \sigma(x)}{\sum_{x=0}^n \frac{A^x \cdot \prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x i}} \quad (25)$$

Two limiting values of the delay probability (24) can be derived:

a) Comparing eq.(24) with eq.(25) one may verify:

$$A \rightarrow 0 : W \rightarrow B$$

b) If the traffic offered A is equal n , the outgoing trunks are always busy and every incoming call has to wait.

$$A = n : W = 1$$

Artificial traffic trials were performed investigating the above mentioned ideal grading with $n=9, k=6, g=84$.

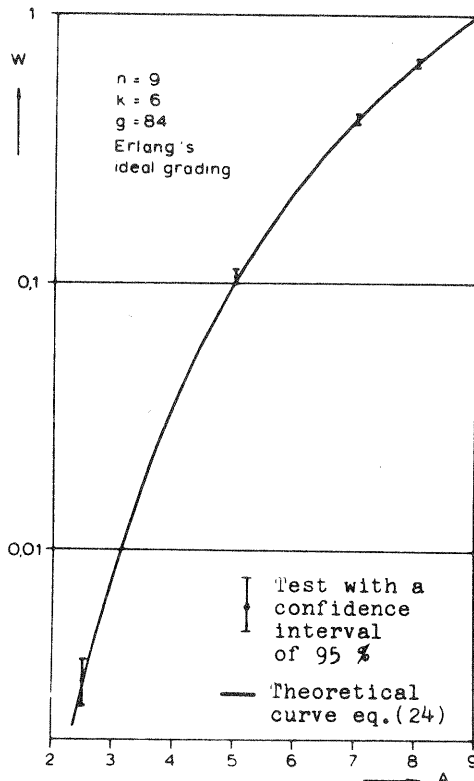


Fig.4 The probability of delay

Equation (24) for the probability of delay includes, as a special case, Erlang's formula for full-access trunkgroups, because for $k=n$ one obtains

$$\sigma(x) = \begin{cases} 0 & \text{for } x=0,1,\dots,n-1 \\ 1 & \text{for } x=n \end{cases} \quad \text{cf. eq. (4)}$$

and with eq. (24)

$$W = \frac{n \cdot \frac{A^n}{n!}}{\sum_{x=0}^n \frac{A^x}{x!}} = E_{2,n}(A)$$

$$n - A + A \cdot \frac{\frac{A^n}{n!}}{\sum_{x=0}^n \frac{A^x}{x!}}$$

according to Erlang's wellknown formula of the probability of delay in full-access trunkgroups.

The Probability "at least one call is waiting"

The probability "at least one call is waiting" E_s is equal to the sum of the state probabilities $p(x)$ for $x=k, k+1, \dots, n$ reduced by the sum of the state probabilities $p(x,0)$.

$$E_s = \sum_{x=k}^n p(x) - \sum_{x=k}^n p(x,0)$$

E_s can be interpreted as the fraction of time any queue exists, too.

By successive addition of the recurrence formula (16) the probability of the state $\{x,0\}$ is found to be

$$p(x,0) = A^x \cdot \frac{\prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x i} \cdot p(0,0)$$

$$p(0,0) = p(0) \quad \text{cf. eq. (19)}$$

$$p(0) \text{ is known} \quad \text{cf. eq. (21)}$$

With eq. (20) the probability E_s takes the following form:

$$E_s = p(0) \cdot \sum_{x=k}^n A^x \left\{ \frac{\prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x [i - \sigma(i) \cdot A]} - \frac{\prod_{i=0}^{x-1} [1 - \sigma(i)]}{\prod_{i=1}^x i} \right\}$$

In fig.5 the results of simulation with artificial traffic and the calculated curve are shown.

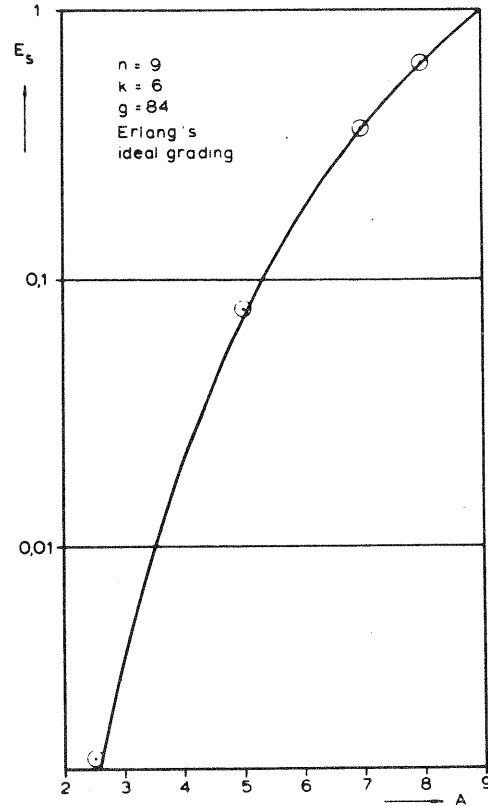


Fig.5 Probability "at least one call is waiting" E_s

For the special case $k=n$ Erlang's formula for E_s is obtained:

$$E_s = \frac{A \cdot \frac{A^n}{n!}}{\sum_{x=0}^n \frac{A^x}{x!}} \cdot \frac{1}{n - A + A \cdot \frac{\frac{A^n}{n!}}{\sum_{x=0}^n \frac{A^x}{x!}}}$$

The Waiting Traffic

The waiting traffic Ω is defined as the average number of calls waiting simultaneously.

Hence, Ω is the mathematical expectation of z .

$$\Omega = \sum_{x=k}^n \sum_{z=1}^{\infty} z \cdot p(x, z)$$

At first the following sum may be calculated:

$$\omega(x) = \sum_{z=1}^{\infty} z \cdot p(x, z) \quad (26)$$

For this reason the eq.(15) is multiplied by z .

$$x \cdot p(x, z) \cdot z = [1 - \sigma(x-1)] \cdot A \cdot p(x-1, z) \cdot z + \sigma(x) \cdot A \cdot p(x, z-1) \cdot z$$

or

$$x \cdot p(x, z) \cdot z = [1 - \sigma(x-1)] \cdot A \cdot p(x-1, z) \cdot z + \sigma(x) \cdot A \cdot p(x, z-1) \cdot (z-1) + \sigma(x) \cdot A \cdot p(x, z-1)$$

Now this linear equation system is summed up from $z = 1$:

$$x \cdot \sum_{z=1}^{\infty} p(x, z) \cdot z = [1 - \sigma(x-1)] \cdot A \cdot \sum_{z=1}^{\infty} p(x-1, z) \cdot z + \sigma(x) \cdot A \cdot \sum_{z=1}^{\infty} p(x, z-1) \cdot (z-1) + \sigma(x) \cdot A \cdot \sum_{z=1}^{\infty} p(x, z-1)$$

with the abbreviation (26):

$$x \cdot \omega(x) = [1 - \sigma(x-1)] \cdot A \cdot \omega(x-1) + \sigma(x) \cdot A \cdot \omega(x) + \sigma(x) \cdot A \cdot p(x)$$

or

$$[x - \sigma(x) \cdot A] \cdot \omega(x) = [1 - \sigma(x-1)] \cdot A \cdot \omega(x-1) + \sigma(x) \cdot A \cdot p(x)$$

This recurrence relation is valid for $x=k, k+1, \dots, n$

It holds $\omega(k-1) = 0$, because $p(k-1, z) = 0$ for $z=1, 2, \dots$ cf. (6)

After some transformations the following relation can be found :

$$\omega(x) = p(x) \cdot \sum_{i=k}^x \frac{\sigma(i) \cdot A}{i - \sigma(i) \cdot A} \quad x=k, k+1, \dots, n$$

$p(x)$ is known, so one can calculate the waiting traffic

$$\Omega = \sum_{x=k}^n \left\{ p(x) \cdot \sum_{i=k}^x \frac{\sigma(i) \cdot A}{i - \sigma(i) \cdot A} \right\}$$

The Mean Waiting Time

The waiting traffic is equal to the average number of delayed calls occurring per unit time multiplied by the mean waiting time :

$$\Omega = c_W \cdot t_W$$

or

$$t_W = \frac{\Omega}{c_A} \cdot \frac{c_A}{c_W}$$

The quotient $\frac{c_W}{c_A}$ is the delay probability cf. eq.(23).

The mean waiting time divided by the mean holding time may be denoted by τ_W .

$$\tau_W = \frac{t_W}{h} = \frac{\Omega}{W \cdot c_A \cdot h}$$

and with eq.(3):

$$\tau_W = \frac{\Omega}{W \cdot A}$$

The mean waiting time τ_W can now be calculated:

$$\tau_W = \frac{\sum_{x=k}^n \left\{ p(x) \cdot \sum_{i=k}^x \frac{\sigma(i) \cdot A}{i - \sigma(i) \cdot A} \right\}}{A \cdot \sum_{x=k}^n p(x) \cdot \sigma(x)} \quad (27)$$

Two limiting values of the mean waiting time can be derived from eq.(27):

- a) $A = 0$: $\tau_W = \frac{1}{k}$
- b) $A = n$: $\tau_W \rightarrow \infty$

Because all outgoing trunks are permanently busy the mean waiting time is infinite.

The special case $k=n$ yields Erlang's formula for full-access trunkgroups.

$$\tau_W = \frac{p(n)}{p(n)} \cdot \frac{1}{n-A} = \frac{1}{n-A}$$

Fig.6 shows the theoretical curve and the results of simulation.

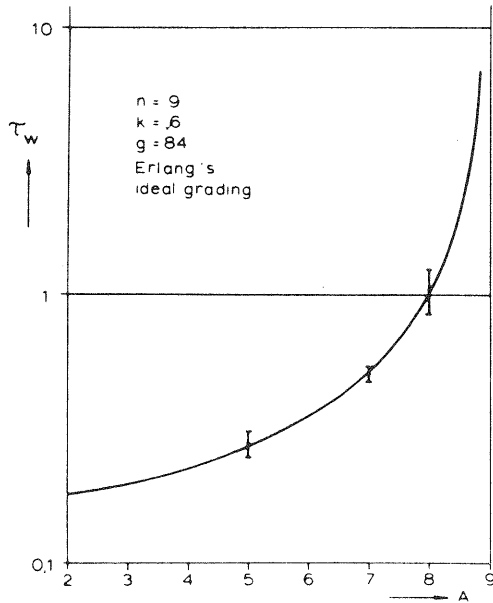


Fig.6 The mean waiting time

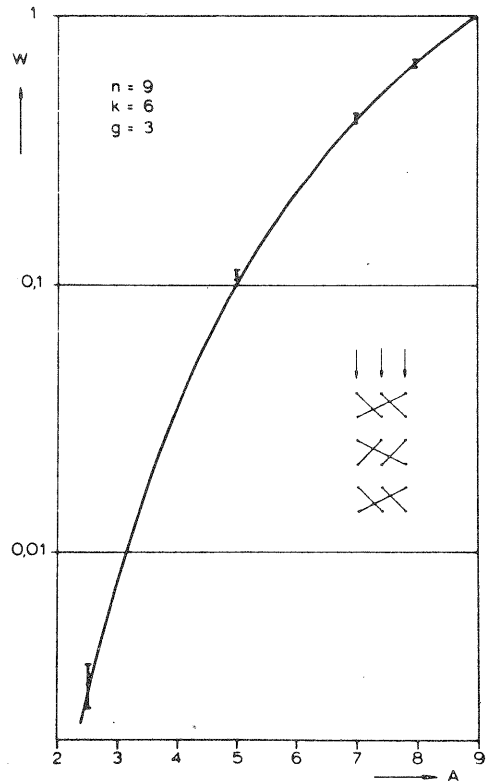


Fig.8 The probability of delay

Non-ideal Gradings

Besides Erlang's ideal grading ($n=9, k=6, g=84$) some other gradings have been tested. Especially gradings developed for loss systems were investigated by simulation in delay systems.

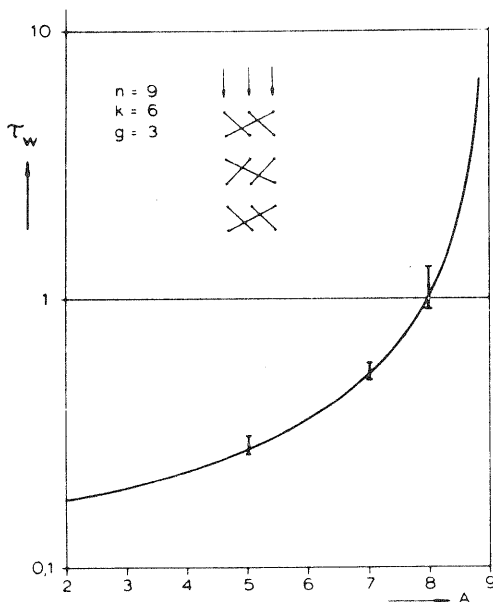


Fig.7 The mean waiting time

A grading with the same number of trunks $n=9$ and the same accessibility $k=6$ as the above shown Erlang's ideal grading has been tested. This grading has only $g=3$ incoming grading groups, while the corresponding ideal grading has $g=84$ incoming groups.

Fig.7 and Fig.8

The test results of the delay probability w and the mean waiting time T_w do not change significantly although the number of waiting queues is reduced from 84 to 3.

Therefore it has been investigated whether the derived solution represents a good approximation too, for usual but non-ideal gradings in waiting systems. The results of simulation of 3 gradings standardized by the German Post Office are compared with the derived solution.

The theoretical curves, calculated with eq.(24) resp. (27), and the results of the tests as well as the confidence intervals of 95 % are shown for the probability of delay w and the mean waiting time T_w in the following diagrams.

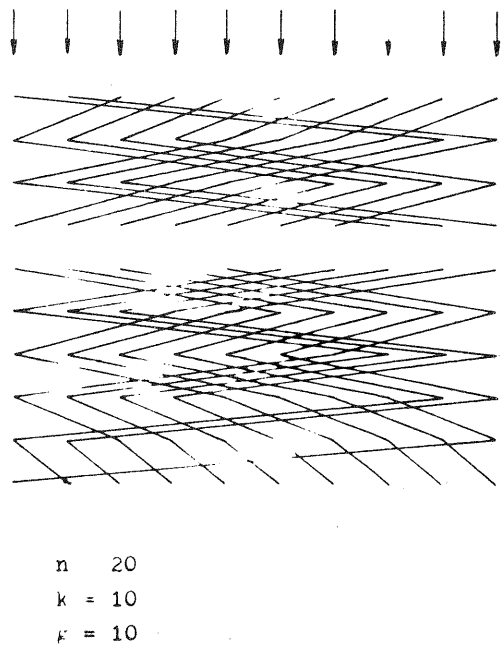


Fig.9 Grading No.3

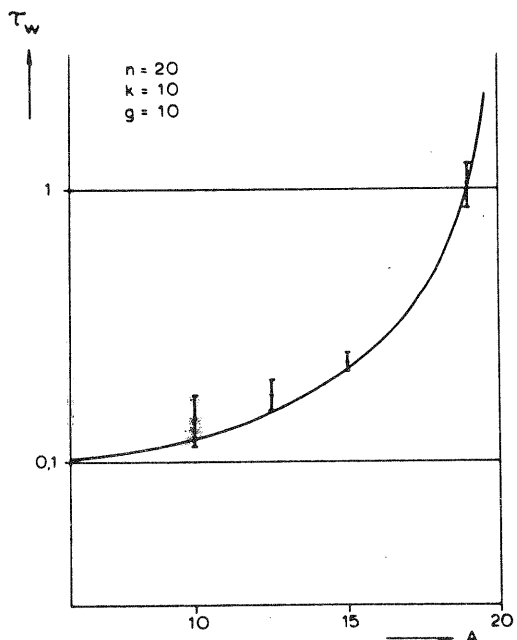


Fig.11 The mean waiting time (grading No.3)

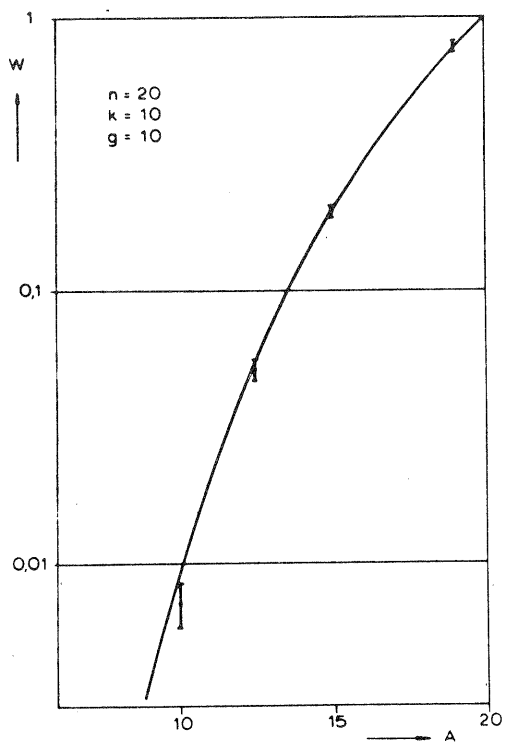


Fig.10 The probability of delay (grading No.3)

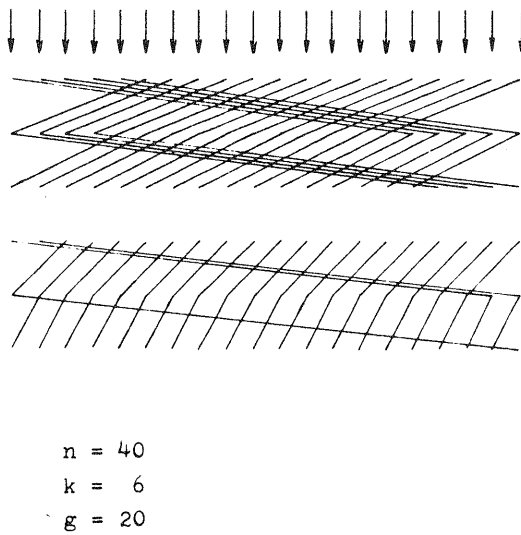


Fig.12 Grading No.4

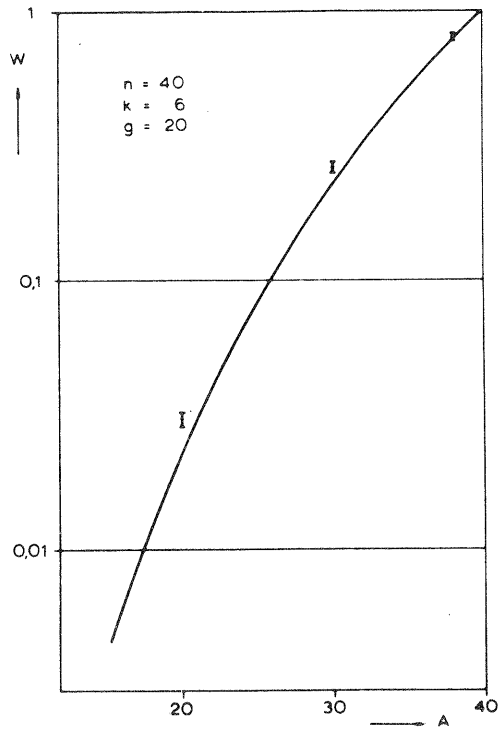
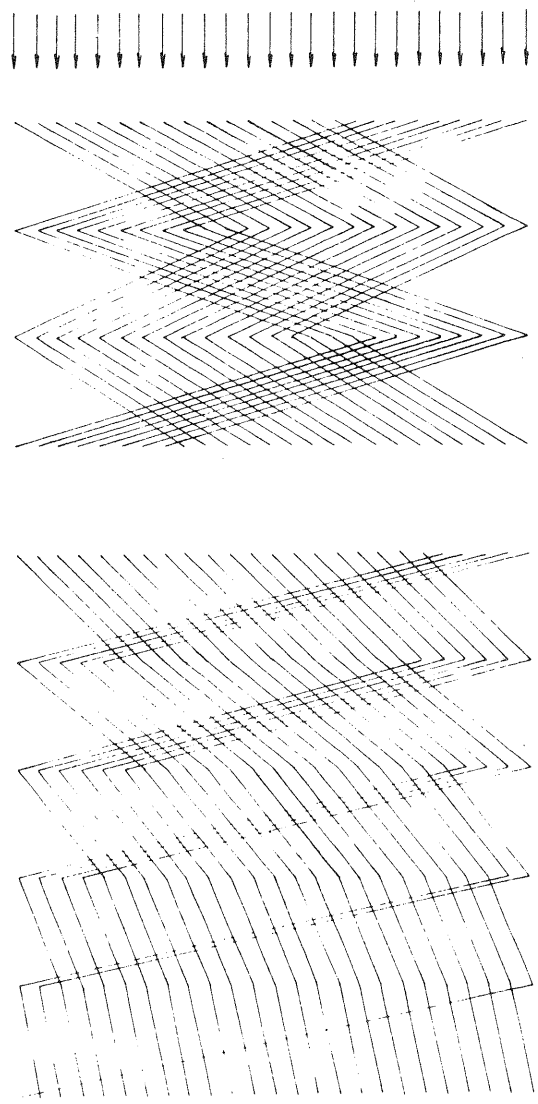


Fig.13 The probability of delay (grading No.4)



$n = 50$
 $k = 10$
 $g = 25$

Fig.15 Grading No.5

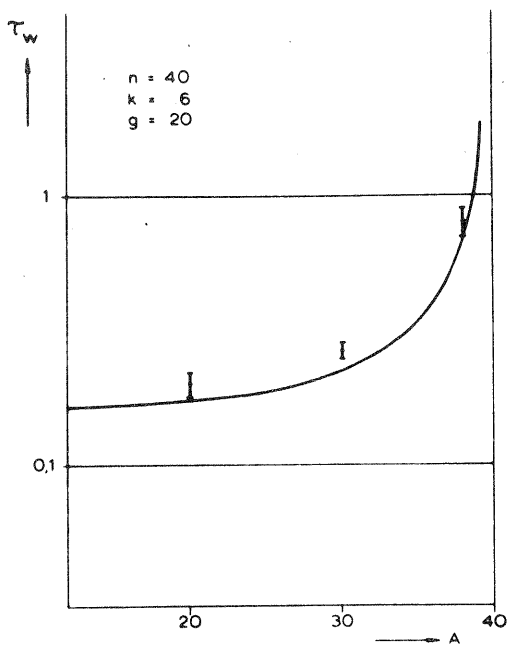


Fig.14 The mean waiting time (grading No.4)

The results of simulation show that the Interconnection Delay Formula (IDF) developed above for ideal Erlang gradings, yields also good approximate values for usual gradings.

Besides the probability of delay and the mean waiting time other interesting quantities describing the traffic problems in delay systems may be calculated too, because the state probabilities of the system have been derived.

References

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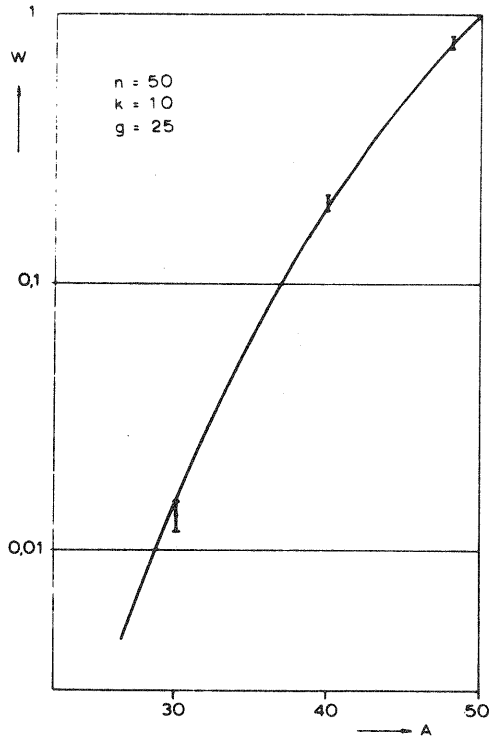


Fig.16 The probability of delay (grading No.5)

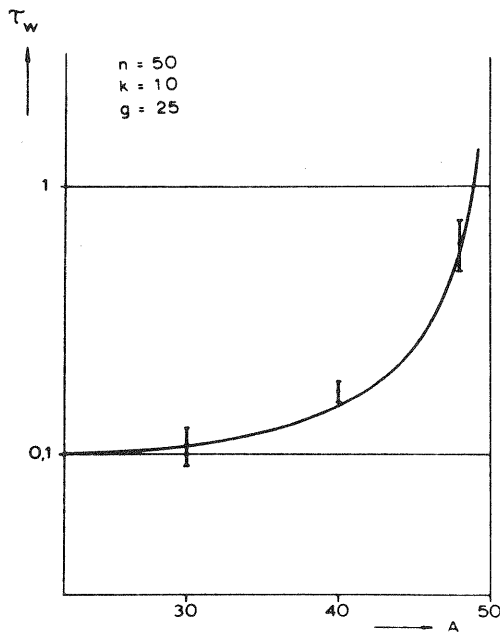


Fig.17 The mean waiting time (grading No.5)