

Performance Analysis of the FDDI Media Access Control Protocol

Michael Tangemann, Klaus Sauer

Institute of Communications Switching and Data Technics
University of Stuttgart
Federal Republic of Germany
Telephone: +49 711 121-2475
Telefax: +49 711 121-2477
E-Mail: Tangemann@DS0IND5.Bitnet

Abstract

The Fibre Distributed Data Interface (FDDI) is a Token Ring Protocol to provide communication over fibre optic rings with a transmission rate of 100 Mbps. The protocol is an ANSI standard and supports a synchronous traffic class which offers guaranteed response time and guaranteed bandwidth as well as an asynchronous traffic class where two token modes and up to eight different priority levels can be provided. The access to the medium for these traffic classes is controlled by the so-called Timed Token Protocol.

In this paper, we focus on the performance behaviour of the asynchronous traffic class without priorities. An iterative algorithm to calculate the first two moments of the cycle time will be presented. Furthermore, the major system characteristics in terms of mean waiting times and queue lengths will be derived from this analysis. Our model considers queues with limited buffer size, and the loss probability of data packets can also be calculated. The results which are obtained by the approximate analysis will be discussed, and the accuracy will be validated by detailed computer simulations.

Keywords: HSLAN, FDDI, Timed Token Protocol, Iterative Analysis

1 Introduction

FDDI is a protocol designed for a 100 Mbps token passing ring using a fibre optic medium. The FDDI standard has been developed by the American National Standards Institute X3T9 Committee. It supports two traffic classes, both handle packet switched traffic [3]. The so-called synchronous traffic class allows data transmission with a pre-allocated bandwidth. The transmission of frames in the asynchronous traffic class can be based on two token modes. Using a so-called non-restricted token, up to eight priority levels can be distinguished. A special restricted token mode in the asynchronous traffic class allows dialogue oriented connections between some selected stations.

The access to the medium for these traffic classes is controlled by the so-called Timed Token Protocol. During ring initialization all stations negotiate a target token rotation time (TTRT). Each station is assigned a percentage of the TTRT for its synchronous packet transmissions. The residual bandwidth is available for asynchronous traffic. The transmission of asynchronous packets is controlled by the token rotation timer (TRT), which measures the time between successive token arrivals at a station. If the token is in time, i.e. the TRT value is less than the negotiated TTRT when the token arrives, then the value of the TRT is copied into the token holding timer (THT), which starts counting upward. Asynchronous

frames may now be transmitted until the THT reaches the TTRT level. Then the transmission of the last packet is completed before the token is passed to the next station. If the token is late, i.e. the TRT has exceeded the TTRT, only synchronous traffic may be served. If the token is in time, the TRT is reset upon token arrival, otherwise the TRT is reset after reaching the TTRT level [3].

The FDDI protocol has been developed from the ideas of Grow [4] and Ulm [13], and the formal description of the protocol can be found in the standard proposal [3]. Johnson [5, 7] has made various investigations on the robustness and the reliability of the FDDI protocol. Some basic properties of the Timed Token Protocol have been proved by Sevcik and Johnson [9, 6]. A procedure to estimate the throughput of each asynchronous traffic class has been presented by Dykeman and Bux [1, 2], and an estimation of the cycle time and the station throughput has been derived by Pang and Tobagi [8], who consider the deterministic behaviour of the Timed Token Protocol under heavy load.

The analysis presented in this paper is based on a modified polling system with timer-controlled gated service and non-zero switch-over times. The iterative algorithm uses an imbedded Markov chain approach in conjunction with a cycle time analysis. It is an extension of the method used by Tran-Gia and Raith [12] and provides the system characteristics over the whole range of the offered load for arbitrary packet length distributions [11]. The accuracy of the presented analysis is validated by detailed computer simulations.

The queueing model which is the basis for our analysis of the FDDI media access control part is given in the following section. In Section 3 the analysis is presented in detail, and some results obtained by the analysis are discussed and compared with simulation results in Section 4.

2 Modelling

The queueing model we have derived from the FDDI MAC protocol is depicted in Figure 1. We only consider one non-restricted asynchronous priority level. Our FDDI system consists of N stations and is interpreted as a polling system. Every station is modelled by a single queue i . The number of packets that can be buffered in a queue is limited by the value m_i . Packets arrive at station i according to a Poisson process with rate λ_i . The server of the polling model represents the transmission channel, which is allocated cyclically to the stations according to the FDDI MAC protocol. The random variables of the service time of a packet T_{H_i} and the switchover time T_{U_i} from station i to station $i + 1$ have general distributions. They can be chosen individually for each station and are characterized by their first two moments.

Due to the complexity of the FDDI MAC protocol none of the well-known service disciplines limited-k, gated or exhaustive [10] are appropriate for our model. Basically, the queues are served exhaustively. However, this exhaustive service can be interrupted by the token holding timer (THT). Furthermore, the timer value of the THT is not fixed but depends on the duration of the last token rotation. We call this "timer controlled exhaustive service". For reasons of numerical tractability we have adopted a slightly different service discipline for our model, which we call "timer controlled gated service". With this service discipline, packets arriving during service of a station cannot be served in this cycle but have to wait until the next token arrival. Obviously, this assumption yields some inaccuracies under heavy load, but it works quite well for light and medium load. Furthermore, if the number of stations N is large, the station times will be small compared to the cycle time and therefore the error of this approximation will not be significant.

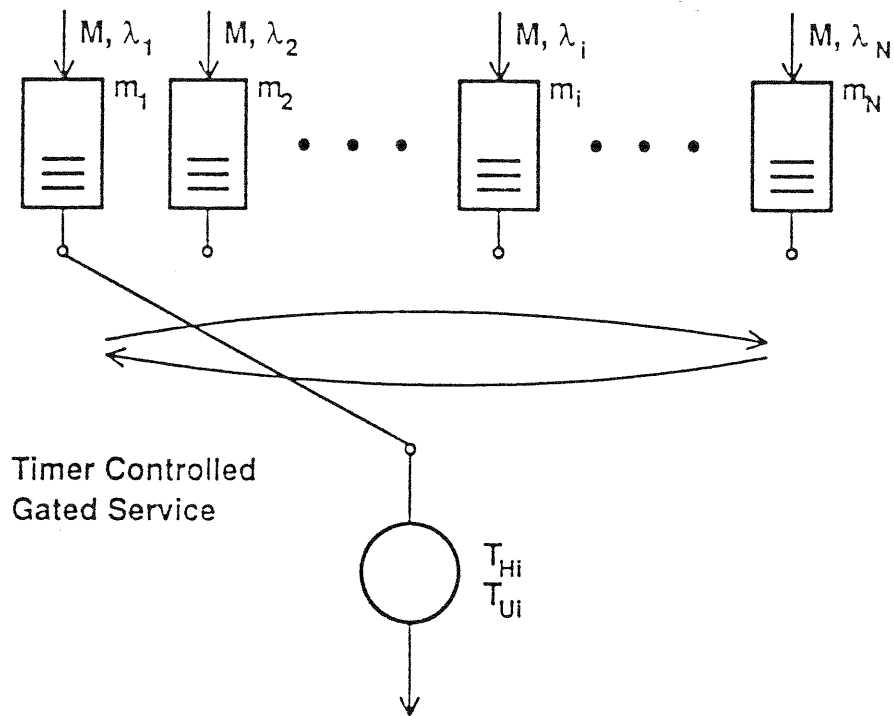


Figure 1: Queuing Model of the FDDI System

3 Analysis

The analysis consists of three parts. After a summary of the notation, first the cycle time will be analysed by evaluating the station times T_{E_i} , which represent the amount of time contributed by station i to the cycle time. This approach will be improved in Section 3.3, where additionally the dependence between successive station times is taken into account. In Section 3.4 finally the queuing analysis of the individual stations is performed. Here, the steady state distributions of the queue lengths at arbitrary times are obtained, from which loss probabilities and mean waiting times of the packets can be derived.

3.1 Notation

In our analysis, we use the following notation:

$f_x(t)$	distribution density function of the random variable T_x
$F_x(t)$	distribution function of the random variable T_x
$\Phi_x(s)$	Laplace-Stieltjes transform of $F_x(t)$
Q_{x_i}	random variable of the number of Poisson arrivals with rate λ_i during the interval T_{x_i}
$G_Y(z)$	generating function of the random variable Y
N	total number of stations
m_i	number of waiting places in queue i
λ_i	packet arrival rate at queue i
T_{H_i}	packet service time at station i
T_{U_i}	switchover time from station i to $i+1$
$A_i^{(n)}$	number of packets waiting in queue i at the n -th token arrival

$B_i^{(n)}$	maximum number of packets to be served at station i after the n -th token arrival
$C_i^{(n)}$	number of packets to be served at station i after the n -th token arrival
$D_i^{(n)}$	number of packets that cannot be served at station i after the n -th token arrival
$T_{TRT_i}^{(n)}$	token rotation time measured by station i after the n -th token arrival
T_{TTRT}	operative target token rotation time (constant value)
$T_{THT_i,j}^{(n)}$	token holding time of station i after the n -th token arrival and j packet service times
$T_{E_i}^{(n)}$	station time of station i after the n -th token arrival
$T_{C_i}^{(n)}$	cycle time measured by station i after the n -th token arrival
$T_{V_i,l}$	conditional type- l vacation time of station i
$p(j)$	state probability of a queue at fixed points of time
$p^*(j)$	state probability of a queue at arbitrary points of time
p_{L_i}	loss probability of packets at queue i
L_i	length of queue i at arbitrary points of time
T_{W_i}	waiting time of packets in queue i

3.2 Cycle Time Analysis

The analysis is based on an imbedded Markov chain approach, where the regeneration points are represented by the token arrival times at the stations. The system state of station i at the n -th token arrival is given by the token rotation time $T_{TRT_i}^{(n)}$, measured by station i and the number of packets $A_i^{(n)}$ waiting at station i with the corresponding probabilities

$$a_i^{(n)}(j) = P\{A_i^{(n)} = j\} \quad (1)$$

The maximum number of packets $B_i^{(n)}$ that can be served at station i after the n -th token arrival depends on the token rotation time measured by station i . The value of the token rotation timer is copied into the token holding timer, which is then started counting upward. It expires upon reaching the level T_{TTRT} where the token has to be passed to the next station after finishing the current packet transmission. This behaviour can be modelled by composing the random variable $T_{THT_i,j}^{(n)}$ of the measured token rotation time $T_{TRT_i}^{(n)}$ and j successive and independent packet service times T_{H_i} by

$$\Phi_{THT_i,j}^{(n)}(s) = \Phi_{TRT_i}^{(n)}(s) \cdot \Phi_{H_i}^j(s) \quad (2)$$

in the Laplace domain and comparing $T_{THT_i,j}^{(n)}$ with the constant threshold T_{TTRT} . Thus, the maximum number of packets to be served can be expressed as follows:

$$\begin{aligned}
b_i^{(n)}(j) &= P\{B_i^{(n)} = j\} \\
&= \begin{cases} P\{T_{TRT_i}^{(n)} > T_{TTRT}\} & j = 0 \\ P\{T_{TRT_i}^{(n)} + \sum_{k=1}^{j-1} T_{H_i} < T_{TTRT} < T_{TRT_i}^{(n)} + \sum_{k=1}^j T_{H_i}\} & 0 < j < m_i \\ P\{T_{TRT_i}^{(n)} + \sum_{k=1}^{m_i-1} T_{H_i} < T_{TTRT}\} & j = m_i \end{cases} \quad (3) \\
&= \begin{cases} 1 - F_{THT_i,0}^{(n)}(T_{TTRT}) & j = 0 \\ F_{THT_i,j-1}^{(n)}(T_{TTRT}) - F_{THT_i,j}^{(n)}(T_{TTRT}) & 0 < j < m_i \\ F_{THT_i,m_i-1}^{(n)}(T_{TTRT}) & j = m_i \end{cases}
\end{aligned}$$

In general, it is not easy to determine the distribution function $F_{T_{HT,i,j}}^{(n)}(t)$ from $\Phi_{T_{HT,i,j}}^{(n)}(s)$ given by equ. (2). Therefore, we use an approach, where the first two moments of $T_{T_{HT,i,j}}^{(n)}$ are derived from equ. (2), and the distribution $F_{T_{HT,i,j}}^{(n)}(t)$ is approximated by the two-moments-approximation suggested in [12], which generally yields quite good values. However, it is also possible to consider higher moments or to use a standard numerical technique.

Note that in equ. (3) values $j > m_i$ need not be considered since the service discipline permits at most m_i packets to be served. The actual number of packets to be served $C_i^{(n)}$ can now be determined by the discrete random variable

$$C_i^{(n)} = \text{Min}\{A_i^{(n)}, B_i^{(n)}\} . \quad (4)$$

Assuming independence between $A_i^{(n)}$ and $B_i^{(n)}$, the distribution of $C_i^{(n)}$ follows from (4) as

$$c_i^{(n)}(j) = \begin{cases} 1 - (1 - a_i^{(n)}(0)) \cdot (1 - b_i^{(n)}(0)) & j = 0 \\ 1 - (1 - \sum_{k=0}^j a_i^{(n)}(k)) \cdot (1 - \sum_{k=0}^j b_i^{(n)}(k)) - \sum_{k=0}^{j-1} c_i^{(n)}(k) & j = 1, \dots, m_i \end{cases} \quad (5)$$

Note that

$$\lim_{T_{TRT} \rightarrow \infty} b_i^{(n)}(j) = \begin{cases} 0 & 0 \leq j < m_i \\ 1 & j = m_i \end{cases} \quad (6)$$

which yields $C_i^{(n)} = A_i^{(n)}$ and ordinary gated service. The number of packets that cannot be served in this cycle due to the fact that the Token Holding Timer expires is given by

$$\begin{aligned} D_i^{(n)} &= A_i^{(n)} - C_i^{(n)} = A_i^{(n)} - \text{Min}\{A_i^{(n)}, B_i^{(n)}\} \\ &= \text{Max}\{A_i^{(n)} - B_i^{(n)}, 0\} \end{aligned} \quad (7)$$

with

$$d_i^{(n)}(j) = \begin{cases} \sum_{k=j}^{m_i} a_i^{(n)}(k) b_i^{(n)}(k-j) & 0 < j \leq m_i \\ 1 - \sum_{k=1}^{m_i} d_i^{(n)}(k) & j = 0 \end{cases} \quad (8)$$

The station time $T_{E_i}^{(n)}$ is composed of $C_i^{(n)}$ independent packet service times T_{H_i} and the switchover time T_{U_i} , which yields

$$\Phi_{E_i}^{(n)} = \Phi_{U_i} \cdot G_{C_i}^{(n)}[\Phi_{H_i}(s)] \quad (9)$$

where $G_{C_i}^{(n)}(z)$ is the generating function of $C_i^{(n)}$. The cycle time $T_{C_i}^{(n+1)}$ measured by station i is defined as the time between the n -th and the $(n+1)$ -th token arrival at station i . It is given by

$$T_{C_i}^{(n+1)} = \sum_{k=i}^N T_{E_k}^{(n)} + \sum_{k=1}^{i-1} T_{E_k}^{(n+1)} . \quad (10)$$

Since the station times depend on each other, equ. (10) can be evaluated only approximately by

$$\Phi_{C_i}^{(n+1)}(s) \approx \prod_{k=i}^N \Phi_{E_k}^{(n)}(s) \cdot \prod_{k=1}^{i-1} \Phi_{E_k}^{(n+1)}(s) . \quad (11)$$

The token rotation time in equs. (2),(3) is not equal to the cycle time, since the token rotation timer of the FDDI MAC protocol is not always reset and restarted upon token arrival. However,

$$\Phi_{TRT_i}^{(n+1)}(s) \approx \Phi_{C_i}^{(n+1)}(s) \quad (12)$$

is a good approximation unless the system is under overload and the packet service times are extremely large. This approximation means that the token rotation timer is always reset to zero upon token arrival, which does not alter the basic cycle time properties of the protocol [9].

From the cycle time the queue state after the next token arrival characterized by $A_i^{(n+1)}$, $i = 1, 2, \dots, N$, can be derived by exploiting the state equations of the imbedded Markov chain:

$$a_i^{(n+1)}(j) = a_i^{(n)}(j) \otimes q_{C_i}^{(n+1)}(j) \quad (13)$$

where \otimes stands for the discrete convolution of a finite and an infinite distribution

$$p(j) \otimes q(j) = \begin{cases} \sum_{k=0}^i p(k) \cdot q(j-k) & 0 \leq j < m \\ \sum_{k=0}^m \sum_{l=m-k}^{\infty} p(k) \cdot q(l) & j = m \end{cases} \quad (14)$$

Herein $q_{C_i}^{(n+1)}(j)$ is the probability that j packets arrive at station i during the $(n+1)$ -th cycle

$$q_{C_i}^{(n+1)}(j) = \int_0^{\infty} \frac{(\lambda_i t)^j}{j!} e^{-\lambda_i t} f_{C_i}^{(n+1)}(t) dt \quad j = 0, 1, 2, \dots \quad (15)$$

which can be calculated by using a two-moments-approximation for $T_{C_i}^{(n+1)}$. Due to the complexity of the analysis presented here, closed form expressions for the steady state distributions of $A_i^{(n)}$ and $T_{C_i}^{(n)}$ cannot be found. This is the reason why we have employed an iterative algorithm very similar to that one in [12]. It starts with initial values for $A_i^{(0)}$ and $T_{C_i}^{(0)}$ (typically the values of an empty system). Equs. (1)-(12) yield an improved $T_{C_i}^{(1)}$, and equs. (13)-(15) in turn yield a new $A_i^{(1)}$. These two steps are repeated alternately until the system is stable, i.e.

$$\sum_{i=1}^N \frac{E[A_i^{(n)}] - E[A_i^{(n-1)}]}{E[A_i^{(n-1)}]} < \varepsilon \quad (16)$$

The parameter ε determines the accuracy of the results. After the iteration, the steady state distributions of the queue lengths A_i at token arrival times and the cycle time T_C are known approximately:

$$A_i = \lim_{n \rightarrow \infty} A_i^{(n)} \quad i = 1, 2, \dots, N \quad (17)$$

$$T_C = \lim_{n \rightarrow \infty} T_{C_i}^{(n)} \quad i = 1, 2, \dots, N \quad (18)$$

Note that during the iteration T_{C_i} may depend on i which is not the case for the limiting distributions.

3.3 Improved Cycle Time Analysis

Unfortunately, equ. (11) is not exact because the dependence of the station times on each other is not considered. This can be explained as follows: Suppose that station i is allowed to send a large number of packets. Then the token rotation time measured by the next station $i + 1$ will be large and the number of packets that may be transmitted will be small. Thus station times T_{E_i} and $T_{E_{i+1}}$ are correlated with a negative covariance. In this section, we improve our analysis by taking into account these covariances. In order to support the readability, we suppress the superscript (n) denoting the n-th iteration step.

The cycle time is given by

$$T_C = \sum_{i=1}^N T_{E_i} \quad (19)$$

which yields the following expressions for its mean and variance

$$E[T_C] = \sum_{i=1}^N E[T_{E_i}] \quad (20)$$

$$VAR[T_C] = \sum_{i=1}^N VAR[T_{E_i}] + 2 \sum_{i=1}^{N-1} \sum_{k=i+1}^N COV[T_{E_i}, T_{E_k}] , \quad (21)$$

where in the general case the covariances are defined as follows

$$COV[T_{E_i}, T_{E_k}] = E[T_{E_i} \cdot T_{E_k}] - E[T_{E_i}] \cdot E[T_{E_k}] \neq 0 . \quad (22)$$

In our analysis, we consider only the dependence between the station times of adjacent stations, which means we assume

$$COV[T_{E_i}, T_{E_k}] \approx 0 \quad |k - i| \geq 2 . \quad (23)$$

This leads to

$$VAR[T_C] = \sum_{i=1}^N VAR[T_{E_i}] + 2 \sum_{i=0}^{N-1} COV[T_{E_i}, T_{E_{i+1}}] , \quad T_{E_0} = T_{E_N} . \quad (24)$$

In order to determine the unknown covariances, we consider the following two-dimensional Laplace-Stieltjes transform of two subsequent station times:

$$\Phi_E(s_i, s_{i+1}) = \int_{t_i=0}^{\infty} \int_{t_{i+1}=0}^{\infty} e^{-s_i t_i} e^{-s_{i+1} t_{i+1}} f_E(t_i, t_{i+1}) dt_i dt_{i+1} \quad (25)$$

$$= \sum_{x_i=0}^{m_i} \sum_{x_{i+1}=0}^{m_{i+1}} p_{x_i, x_{i+1}} \Phi_{H_i}^{x_i}(s_i) \Phi_{U_i}(s_i) \Phi_{H_{i+1}}^{x_{i+1}}(s_{i+1}) \Phi_{U_{i+1}}(s_{i+1}) \quad (26)$$

with

$$\begin{aligned} p_{x_i, x_{i+1}} &= P\{C_i = x_i, C_{i+1} = x_{i+1}\} \\ &= P\{C_{i+1} = x_{i+1} | C_i = x_i\} \cdot P\{C_i = x_i\} \end{aligned} \quad (27)$$

which yields

$$COV[T_{E_i}, T_{E_{i+1}}] = E[T_{H_i}] \cdot E[T_{H_{i+1}}] \cdot COV[C_i, C_{i+1}] \quad (28)$$

where

$$COV[C_i, C_{i+1}] = \sum_{x_i=0}^{m_i} \sum_{x_{i+1}=0}^{m_{i+1}} x_i x_{i+1} p_{x_i, x_{i+1}} - E[C_i] \cdot E[C_{i+1}] \quad . \quad (29)$$

The probabilities $p_{x_i, x_{i+1}}$ can be calculated by introducing conditional cycle times conditioned on $C_i = x_i$ and evaluating the conditional distribution of C_{i+1} using equ. (3)-(5). The advantage of the improved variance in equ. (24) is that it yields a more accurate distribution of the $B_i^{(n)}$ in equ. (3) and thus the accuracy of the analysis presented in Section 3.2 is improved.

3.4 Queueing Analysis

The queueing analysis can be performed separately for each queue. The influence of the other queues is contained in the cycle time. Therefore we suppress the index i denoting the station number in order to improve the readability.

Let us first define the following conditional probability

$$p_{z,l}(j) = P\{K = j | t = t_z^-, C = l\} \quad 0 \leq j \leq m, \quad 0 \leq l \leq m, \quad 0 \leq z \leq l, \quad (30)$$

where K is the number of packets in the queue, t_z^- is the instant just after the z -th service of a service period at the considered station ($z = 0$ denotes the token arrival instant) and C is the total number of packets served during this service period. Conditioning on $C = l$ means introducing conditional cycle times. We call a cycle where $C = l$ packets are served at the considered station a "type- l cycle". Note that once the number of packets waiting upon token arrival A and the maximum number to be served B are known, the actual number C of packets to be served as well as the behaviour during a service period can be derived in a deterministic manner. For $z = 0$ we get

$$p_{0,l}(j) = \begin{cases} 0 & 0 \leq j < l \\ \frac{1}{c(l)} \sum_{k=j}^m a(j) \cdot b(k) & j = l \\ \frac{1}{c(l)} a(j) b(l) & l < j \leq m \end{cases} \quad . \quad (31)$$

Note that with (5) $\sum_{j=0}^m p_{0,l}(j) = 1$.

One packet service time later we have one packet less in the queue and some new arrivals characterized by $q_H(j)$, the probability of j arrivals during a packet service time, which can be calculated analogously to equ. (15). This yields for $z > 0$

$$p_{z,l}(j) = p_{z-1,l}(j+1) \otimes q_H(j), \quad 1 \leq z \leq l \quad (32)$$

where \otimes stands for the convolution defined in equ. (14). At arbitrary time instants during the z -th interval of a type- l cycle we get

$$p_{z,l}^*(j) = \begin{cases} p_{0,l}(j) \otimes q_{V,l}^R(j) & l = 0, z = 0 \\ p_{z,l}(j) \otimes q_H^R(j) & 0 < l < m, 0 \leq z < l \\ p_{l,l}(j) \otimes q_{V,l}^R(j) & 0 < l < m, z = l \end{cases} \quad (33)$$

where $q_H^R(j)$ and $q_{V,l}^R(j)$ represent the number of arrivals during the backward recurrence time of the service time and the backward recurrence time of the conditional type- l vacation

time, respectively. The conditional type- l vacation times are given by

$$T_{V_i,l} = T_{U_i} + \sum_{\substack{j=1 \\ j \neq i}}^N T_{E_j,l} . \quad (34)$$

For the state distribution at an arbitrary instant of a type- l cycle we get

$$p_l^*(j) = \begin{cases} \frac{E[T_H] \cdot \sum_{z=0}^{l-1} p_{z,l}^*(j) + E[T_{V,l}] \cdot p_{l,l}^*(j)}{E[T_{V,l}] + l \cdot E[T_H]} & 0 < l \leq m \\ p_{0,l}^*(j) & l = 0 . \end{cases} \quad (35)$$

Unconditioning on $C = l$ finally yields the state distribution at arbitrary instants

$$p^*(j) = \sum_{l=0}^m c(l) \cdot p_l^*(j) . \quad (36)$$

Since we have assumed Poisson arrivals, the loss probability of the packets equals the blocking probability of the queue

$$p_L = p^*(m) . \quad (37)$$

The mean queue length at arbitrary time instants is given by

$$E[L] = \sum_{j=0}^m j p^*(j) , \quad (38)$$

and the mean waiting time can be derived by using Little's law

$$E[T_W] = \frac{E[L]}{\lambda(1 - p_L)} . \quad (39)$$

4 Results

For the validation of the analytical results against results obtained by a simulation program we consider a relatively small system in order to show the properties of the analysis. Ten stations are connected to an FDDI ring with 100 Mbps bandwidth and 100 km ring length. Every queue has a limited buffer space of $m_i = 5$. The traffic is supposed to be symmetric, i.e. the traffic parameters are the same for all stations. We assume a constant packet length of 1000 bit. The total offered traffic is varied by increasing the packet arrival rate. In order to be able to show the effects of the target token rotation time, we chose a small value of 1 ms for T_{TTRT} .

In the diagrams the simulation results are depicted with their 95 % confidence intervals which are suppressed if they are smaller than 1 % of the absolute value.

Figure 2 shows the mean cycle time $E[T_C]$ versus the total offered traffic ρ , which is calculated according to

$$\rho = \sum_{i=1}^N \lambda_i \cdot E[T_{H_i}] \quad (40)$$

It can be seen clearly that $E[T_C]$ is limited above by T_{TTRT} which has been proved in [6, 9]. The analysis underestimates slightly the upper limit of the mean cycle time because the

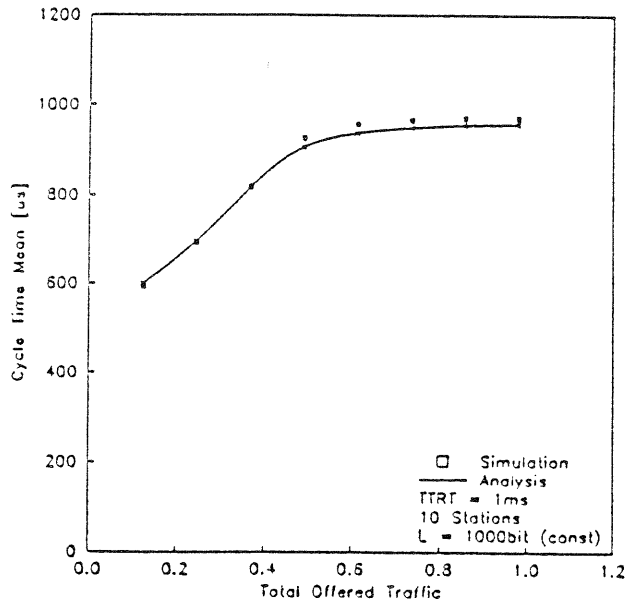


Figure 2: Mean Cycle Time

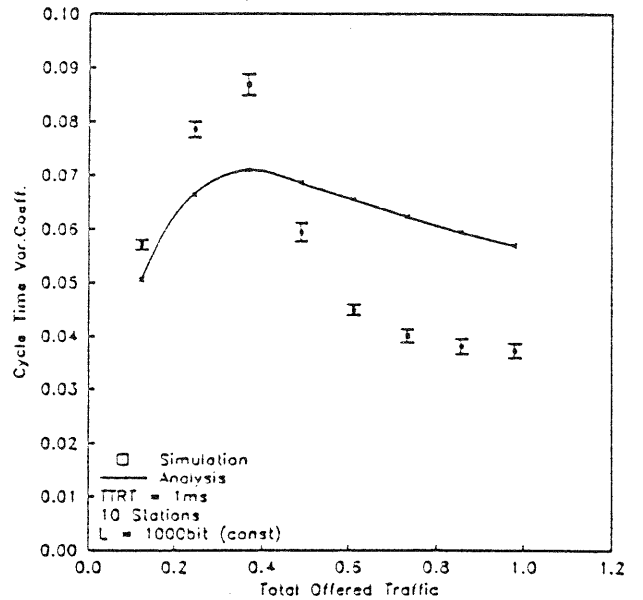


Figure 3: Coefficient of Variation of the Cycle Time

timer controlled exhaustive service of the FDDI MAC protocol is approximated by a timer controlled gated service discipline in our model. However, the error is less than 3 %.

The coefficient of variation of the cycle time $c[T_C]$ is depicted in Figure 3. It is defined as

$$c[T_C] = \frac{\sqrt{\text{VAR}[T_C]}}{E[T_C]} \quad (41)$$

In the empty system $c[T_C] = 0$. Up to a certain threshold of the total offered traffic the coefficient of variation increases. If the total offered traffic exceeds this threshold, the cycle time is limited by the token holding timers which yields a decreasing $c[T_C]$. This behaviour could not be achieved with the first approach of the cycle time analysis in Section 3.2, where $c[T_C]$ increases monotonously. The second approach presented in Section 3.3 corrects this error partly, but $c[T_C]$ is still overestimated for high load mainly due to the assumption that only subsequent station times depend on each other.

Figure 4 shows the mean waiting time of the packets $E[T_W]$, which is also overestimated by the analysis. This has two reasons: First, the waiting time increases with the variance of the cycle time which has been overestimated itself, and second, our model overestimates the queue length due to the assumption of timer controlled gated service, where arrivals during the service phase have to wait until the next token arrival. The effect that gated service yields larger waiting times than exhaustive service has also been found in [10, p. 123] for symmetric polling systems with unlimited buffer space and without timer control.

Finally, the loss probability of the data packets is depicted in Figure 5, where the error is as small as the error of the mean cycle time. This can be explained by the fact that in a symmetric system the loss probability can be derived directly from the mean cycle time according to

$$p_L = 1 - \frac{1}{\rho} \left(1 - \sum_{i=1}^N \frac{E[T_{U_i}]}{E[T_C]} \right) \quad (42)$$

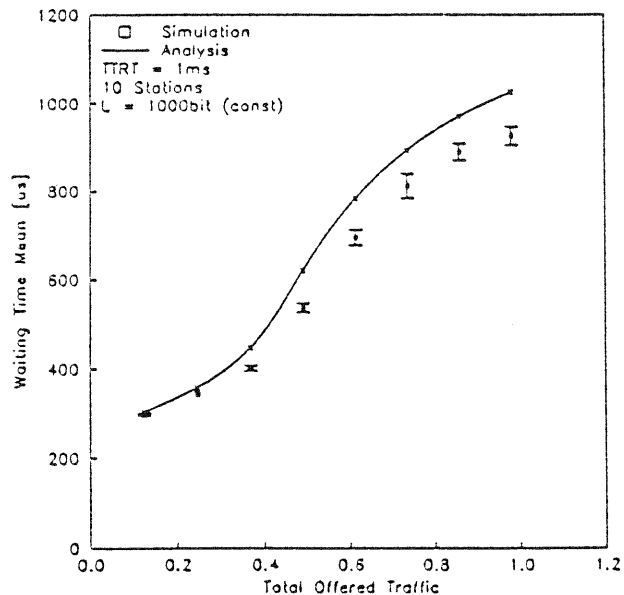


Figure 4: Mean Waiting Time

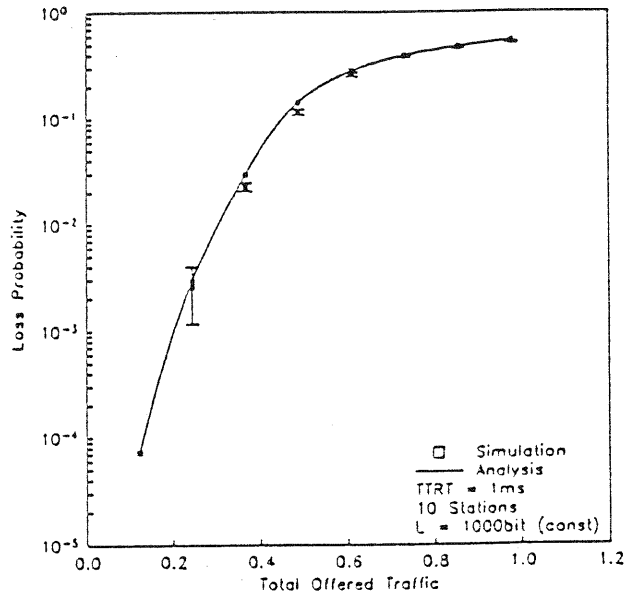


Figure 5: Loss Probability of the Packets

We conclude this section with some remarks concerning the computational effort required for the algorithm. The results presented here have been obtained with a Pascal program of approximately 1500 lines. The complexity of a single iteration step is $O(N \cdot (m^2 + 2m))$. Up to now, the conditions for convergence of this algorithm have not been evaluated theoretically, but it can be stated that the algorithm converges very fast. Here, e.g., usually less than 20 iterations are needed in order to achieve $\varepsilon = 10^{-5}$. However, if the offered load reaches the crucial limit of $(1 - \sum_{i=1}^N T_{U_i} / T_{TTRT})$, the algorithm converges slower and up to 100 iterations are required.

5 Conclusion

An analysis for the timer-controlled FDDI media access control protocol has been provided. It is an iterative solution based on a cycle time analysis and an imbedded Markov chain approach. The results in terms of the first two moments of the cycle time as well as station characteristics like the waiting time and loss probability have been presented. Using an exemplary FDDI configuration the system behavior has been discussed and the accuracy of the algorithm has been validated by detailed computer simulations.

Currently an extension of the analysis is under work to evaluate the Timer Controlled Exhaustive Service in order to improve the analysis for high load.

Acknowledgements

The programming efforts of M. Weil are greatly appreciated. We would also like to thank Prof. P.J. Kühn and an anonymous referee for their helpful comments.

References

- [1] D. Dykeman and W. Bux, "An Investigation of the FDDI media-access control protocol," in *Proc. EFOC/LAN 87*, Basel, Switzerland, June 1987, pp. 229-236
- [2] D. Dykeman and W. Bux, "Analysis and Tuning of the FDDI Media Access Control Protocol," *IEEE JSAC*, Vol. 6, pp. 997-1010, July 1988
- [3] "FDDI Token Ring Media Access Control," ANSI Standard X3T9.5, 1988
- [4] R. M. Grow, "A timed token protocol for local area networks," presented at *Electro 82*, May 1982
- [5] M. J. Johnson, "Fairness of Channel Access for Non-Time-Critical Traffic using the FDDI Token Ring Protocol," in *Proc. Real-Time LANs 86*, Bandol, 1986, pp. 145-157
- [6] M. J. Johnson, "Proof that Timing Requirements of the FDDI Token Ring Protocol are Satisfied," *IEEE Trans. Commun.*, Vol. 35, pp.620-625, June 1987
- [7] M. J. Johnson, "Analysis of FDDI Synchronous Traffic Delays," in *Proc. Systems Design + Networks Conference*, Santa Clara, California, April 1988, pp. 65-72
- [8] J. W. M. Pang and F. A. Tobagi, "Throughput Analysis of a Timer-Controlled Token-Passing Protocol Under Heavy Load," in *Proc. INFOCOM 88*, New Orleans, March 1988, pp. 796-804
- [9] K. C. Sevcik and M. J. Johnson, *Cycle Time Properties of the FDDI Token Ring Protocol*, Technical Report CSRI-179, Computer Systems Research Institute, University of Toronto, Canada, April 1986
- [10] H. Takagi, *Analysis of Polling Systems*, The MIT Press, 1986
- [11] M. Tangemann, "Analytische Leistungsuntersuchung des Timer-gesteuerten Medienzugriffsprotokolls von FDDI", Internal Report, Institute of Communications Switching and Data Technics, University of Stuttgart, Federal Republic of Germany, October 1989
- [12] P. Tran-Gia and T. Raith, "Multiqueue Systems with Finite Capacity and Nonexhaustive Cyclic Service," in *Proc. of the Int. Seminar on Computer Networking and Performance Evaluation*, Tokyo, Japan, 1985
- [13] J. M. Ulm, "A timed token ring local area network and its performance characteristics," in *Proc. 7th Conf. Local Computer Networks*, Minneapolis, 1982, pp. 50-56
- [14] A. Valenzano, P. Montuschi and L. Ciminiera, "On the Behaviour of Control Token Protocols with Asynchronous and Synchronous Traffic," in *Proc. INFOCOM 89*, Ottawa, Ont., Canada, April 1989, pp. 348-355