ON THE CALCULATION OF FULL ACCESS GROUPS FOR INTERNAL AND EXTERNAL TRAFFIC WITH TWO TYPES OF SOURCES

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ABSTRACT

In this paper, loss probabilities of switching systems with full access groups for internal and external traffic and with two types of traffic sources are investigated.

The calculations are carried out for two traffic models. In model No. 1, call attempts directed to busy subscribers are interpreted as usual calls, whereas in model No. 2 these call attempts to busy subscribers are neglected.

For the loss probabilities according to model No. 1 an exact, numerical solution is presented, whereas for model No. 2 an exact, explicit formula is derived. Besides these exact calculations, approximation methods are presented for both models. It is shown that the results according to these approximate calculations are in good accordance with the exact values. Results are presented by means of examples and diagrams.

1. INTRODUCTION

In the calculation of loss probabilities in communication networks, the offered random traffic can in many cases be regarded to be of Poisson type. This can e.g. be seen from the well-known measurements by Hayward and Wilkinson [1]. In various calculation methods for small switching systems, the finite number of traffic sources is taken into account. Usually these methods hold true for systems in which all traffic sources (subscribers) have the same calling rate.

In special cases, the traffic sources of a switching system may have different calling rates. Calculation methods for such systems with sources of different calling rates are also existing [2,3]. In these methods [2,3], however, internal traffic (i.e. traffic between sources of the considered switching system) can not be taken into account.

This paper deals with the calculation of loss probabilities in switching systems with full access groups for internal and external traffic and with two different types of traffic sources. The calculations have been carried out for two different traffic models. In model No. 1, call attempts directed to busy subscribers are interpreted as usual calls, whereas in model No. 2 these call attempts directed to busy subscribers are neglected.

Section 2 deals with the structure of the switching system investigated here. In section 3 the two considered traffic models are explained. For model No. 1 an exact, numerical solution is presented in section 4, whereas for model No. 2 an exact, explicit formula is derived in section 5. Furthermore, approximation formulae are derived in section 6. The results according to these approximation formulae are in good agreement with exact values. Finally the results for various examples are compared with the aid of diagrams in section 7.

2. SYSTEM CONFIGURATION

In many switching networks or parts of switching networks, as e.g. in concentrators and in certain stages of link systems, internal and external traffic are switched via the same group of trunks (or links, respectively). An external call in this case occupies one trunk, whereas an internal call occupies two trunks. For such systems several calculation methods are known [4 - 9].

In some cases, switching systems with internal and external traffic may have a different structure, as is indicated in fig. 1. In such a system, external calls are switched via external trunks whereas internal calls are switched via internal junctors (as indicated in fig. 1). In this case an internal call occupies two paths through the switching network but only one internal junctor.

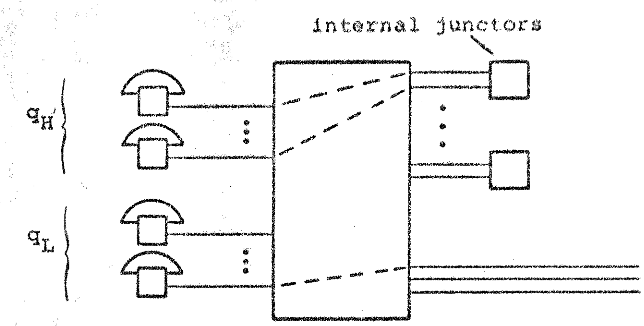

Fig. 1: Switching system with internal and external traffic and with two types of sources (schematically):
q_H sources with higher calling rates
q_L sources with lower calling rates

This paper deals with the calculation of full access switching networks with a structure as indicated in fig. 1 and with two types of sources. It is assumed that there are q_H sources with higher calling rates α_H and q_L sources with lower calling rates α_L. The total number of sources $q_H + q_L$ is denoted by q:

$$q_H + q_L = q \quad (1)$$

For reasons of simplicity, sources with higher calling rates will henceforth be denoted as H-sources, and sources with lower calling rates as L-sources.

3. TRAFFIC MODELS

In this paper two traffic models are considered. These models are based on different assumptions concerning call attempts which are directed to busy subscribers. Usually, the mean holding time of the calls directed to busy subscriber will be rather small as compared with the mean holding time of calls directed to idle sources. In principle, this fact could be regarded explicitly in the loss calculations. In order to reduce the amount of numerical computations required, only two special (limiting) cases are considered in this paper:

Model No. 1 is based on the assumption that the mean holding time of call attempts directed to busy subscribers equals the mean holding time of calls directed to idle subscribers. (I.e., call attempts directed to busy subscribers are considered as "usual calls".)

Model No. 2 is based on the assumption that call attempts directed to busy subscribers are rather short and can therefore be neglected. (I.e., these call attempts are supposed to have zero holding times.) This assumption is, however, only applied to call attempts which are directed to subscribers belonging to the considered switching system (i.e. to internal calls and incoming external calls directed to busy sources).

The accuracy of these two models in describing the traffic in real switching systems depends, e.g., on the type of the considered switching system, and especially on details of the procedures applied in switching a call. In most cases, however, it can be expected that model No. 2 is more realistic than model No. 1.

4. LOSS PROBABILITIES ACCORDING TO MODEL NO. 1

4.1. SWITCHING SYSTEMS WITH TWO TYPES OF TRAFFIC SOURCES (GENERAL CASE)

This section deals with the calculation of loss probabilities in switching systems as shown in fig. 1 according to model No. 1 (i.e., call attempts directed to busy sources are considered as usual calls). The number of internal junctors be denoted by n_i and the number of external trunks by n_e. The holding times are assumed to be negative exponentially distributed with the mean holding time h.

The calling rates of the H-sources are defined as follows:

α_{iHH} for calls to H-sources,
α_{iHL} for calls to L-sources,
α_{eH} for outgoing external calls.

The (total) internal calling rate α_{iH} of H-sources is obtained as

$$\alpha_{iH} = \alpha_{iHH} + \alpha_{iHL}. \qquad (2)$$

The calling rates of the L-sources are defined in the following way:

α_{iLL} for calls to L-sources,
α_{iLH} for calls to H-sources,
α_{eL} for outgoing external calls.

The internal calling rate α_{iL} of L-sources is

$$\alpha_{iL} = \alpha_{iLL} + \alpha_{iLH}. \qquad (3)$$

The products of these calling rates with the mean holding time h are denoted by corresponding β values:

$$\left.\begin{matrix} \beta_{iHH} = \alpha_{iHH}\cdot h, & \beta_{eH} = \alpha_{eH}\cdot h, \\ \beta_{iHL} = \alpha_{iHL}\cdot h, & \beta_{eL} = \alpha_{eL}\cdot h, \\ \beta_{iLL} = \alpha_{iLL}\cdot h, & \beta_{iH} = \alpha_{iH}\cdot h, \\ \beta_{iLH} = \alpha_{iLH}\cdot h, & \beta_{iL} = \alpha_{iL}\cdot h. \end{matrix}\right\} \qquad (4)$$

The offered incoming external traffic is assumed to be of Poisson type. The incomming external traffic offered to the H-sources be denoted by A_{ecHO} and the incoming external traffic offered to the L-sources by A_{ecLO}. For the total offered incoming external traffic A_{ecO} holds

$$A_{ecO} = A_{ecHO} + A_{ecLO}. \qquad (5)$$

The momentary number of external calls to or from H-sources be denoted by x_{eH}, and the number of internal calls among H-sources by x_{iH}. Analogously, the number of external calls to or from L-sources be denoted by x_{eL}, and the number of internal calls among the L-sources by x_{iL}. The number of internal calls between two different sources ("intermediate" or "mixed" calls) be named x_{iM}. The probability for a state $\{x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}\}$ be denoted by $p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})$. Accordingly, $p_i(x_i)$ denotes the probability that x_i internal junctors are busy and $p_e(x_e)$ the probability that x_e external trunks are busy.

For the number z_H of idle H-sources and the number z_L of idle L-sources, the following equation hold true:

$$z_H = q_H - 2x_{iH} - x_{iM} - x_{eH}, \qquad (6)$$

$$z_L = q_L - 2x_{iL} - x_{iM} - x_{eL}. \qquad (7)$$

With equations (6) and (7), the following equations of state are obtained

$$\begin{aligned} & p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})\cdot \\ & \cdot[x_{iH}+x_{iL}+x_{iM}+x_{eH}+x_{eL}+A_{ecHO}+A_{ecLO} \\ & +(\beta_{iHH}+\beta_{iHL}+\beta_{eH})\cdot z_H+(\beta_{iLL}+\beta_{iLH}+\beta_{eL})\cdot z_L] \\ = {} & p(x_{iH}+1, x_{iL}, x_{iM}, x_{eH}, x_{eL})\cdot(x_{iH}+1) \\ & +p(x_{iH}, x_{iL}+1, x_{iM}, x_{eH}, x_{eL})\cdot(x_{iL}+1) \\ & +p(x_{iH}, x_{iL}, x_{iM}+1, x_{eH}, x_{eL})\cdot(x_{iM}+1) \\ & +p(x_{iH}, x_{iL}, x_{iM}, x_{eH}+1, x_{eL})\cdot(x_{eH}+1) \\ & +p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}+1)\cdot(x_{eL}+1) \\ & +p(x_{iH}-1, x_{iL}, x_{iM}, x_{eH}, x_{eL})\cdot\beta_{iHH}\cdot(z_H+2) \\ & +p(x_{iH}, x_{iL}-1, x_{iM}, x_{eH}, x_{eL})\cdot\beta_{iLL}\cdot(z_L+2) \\ & +p(x_{iH}, x_{iL}, x_{iM}-1, x_{eH}, x_{eL})\cdot[\beta_{iHL}\cdot(z_H+1) \\ & \qquad +\beta_{iLH}\cdot(z_L+1)] \\ & +p(x_{iH}, x_{iL}, x_{iM}, x_{eH}-1, x_{eL})\cdot[\beta_{eH}\cdot(z_H+1)+A_{ecHO}] \\ & +p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}-1)\cdot[\beta_{eL}\cdot(z_L+1)+A_{ecLO}] \end{aligned} \qquad (8a)$$

with

$x_{iH}\geq 0$, $x_{iL}\geq 0$, $x_{iM}\geq 0$,
$x_{eH}\geq 0$, $x_{eL}\geq 0$,
$x_{iH}+x_{iL}+x_{iM}\leq n_i$, $x_{eH}+x_{eL}\leq n_e$.

Such an equation is obtained for each of the states $\{x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}\}$. It can be shown that the system (8a) consists of $(n_i+1)\cdot(n_i+2)\cdot(n_i+3)\cdot(n_e+1)\cdot(n_e+2)/12$ equations. For marginal values (e.g. for $x_{iH}=0$), individual terms of equation (8a) may be vanishing. In switching systems with $q_H<2n_i+n_e$ or $q_L<2n_i+n_e$, all state probabilities with $2x_{iH}+x_{iM}+x_{eH}>q_H$ or $2x_{iL}+x_{iM}+x_{eL}>q_L$, respectively, are zero.

Furthermore, with the abbreviation

$$p' = p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}) \qquad (8b)$$

the normalizing condition

$$\sum_{x_{iH}=0}^{n_i}\;\sum_{x_{iL}=0}^{n_i-x_{iH}}\;\sum_{x_{iM}=0}^{n_i-x_{iL}-x_{iH}}\;\sum_{x_{eH}=0}^{n_e}\;\sum_{x_{eL}=0}^{n_e-x_{eH}} p' = 1 \qquad (8c)$$

is obtained (i.e. the sum of all probabilities p' equals zero).

For the numerical solution of the set of equations (8a,c) the so-called method of successive overrelaxation (SOR) is suitable [11]. When the state probabilities p' have been calculated, the probabilities $p_i(x_i)$ and $p_e(x_e)$ can be obtained by summations, respectively:

$$p_i(x_i) = \sum_{x_{eH}=0}^{n_e} \sum_{x_{eL}=0}^{n_e-x_{eH}} \sum_{x_{iH}=0}^{x_i} \sum_{x_{iL}=0}^{x_i-x_{iH}} p'' \qquad (9a)$$

with

$$p'' = p(x_{iH}, x_{iL}, x_i-x_{iH}-x_{iL}, x_{eH}, x_{eL}) \qquad (9b)$$

and

$$p_e(x_e) = \sum_{x_{iH}=0}^{n_i} \sum_{x_{iL}=0}^{n_i-x_{iH}} \sum_{x_{iM}=0}^{n_i-x_{iH}-x_{iL}} \sum_{x_{eH}=0}^{x_e} p''' \qquad (10a)$$

where

$$p''' = p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_i-x_{eH}). \qquad (10b)$$

For the traffic Y_i carried by the internal junctors (i.e. the average number of busy internal junctors) holds

$$Y_i = \sum_{x_i=0}^{n_i} x_i \cdot p_i(x_i). \qquad (11)$$

Analogously, the traffic Y_e carried by the external trunks is

$$Y_e = \sum_{x_e=0}^{n_e} x_e \cdot p_e(x_e). \qquad (12)$$

The total carried traffic Y (i.e. the average number of calls in progress) is obtained as

$$Y = Y_i + Y_e . \qquad (13)$$

The offered internal traffic be denoted by A_i and the offered outgoing external traffic by A_{eg}. It holds

$$A_i = \sum{}'[(\beta_{iHH}+\beta_{iHL})\cdot z_H+(\beta_{iLL}+\beta_{iLH})\cdot z_L]\cdot p' \qquad (14)$$

and

$$A_{eg} = \sum{}'(\beta_{eH}\cdot z_H+\beta_{eL}\cdot z_L)\cdot p' . \qquad (15)$$

The summations $\sum'$ in the equations (14) and (15) comprise all combinations of the values x_{iH}, x_{iL}, x_{iM}, x_{eH} and x_{eL} as in the multiple summation in equation (8c). For the total offered incoming external traffic A_{ecO} holds

$$A_{ecO} = A_{ecHO} + A_{ecLO} \qquad (16)$$

according to equation (5).
The total offered external traffic A_e (incoming and outgoing) is obtained as

$$A_e = A_{eg} + A_{ecO}. \qquad (17)$$

For the total offered traffic A (internal plus external offered traffic) one obtains

$$A = A_i + A_e . \qquad (18)$$

The loss probability for internal calls (i.e. the probability that an arbitrary internal call can not be switched) be denoted by B_i. This loss probability for internal calls is defined as

$$B_i = \frac{A_i - Y_i}{A_i} . \qquad (19)$$

Analogously, the definition of the loss probability B_e for external calls is

$$B_e = \frac{A_e - Y_e}{A_e} . \qquad (20)$$

Finally, the total loss probability B (i.e. the probability that an arbitrary internal or external call can not be switched) is defined as

$$B = \frac{A - Y}{A} . \qquad (21)$$

Now all characteristic values of interest are known.

EXAMPLE NO. 1

As an example, a switching system with n_e=5 external trunks, n_i=5 internal junctors, q_H=15 H-sources (with higher calling rates) and q_L=25 L-sources (with lower calling rates) be considered. For the calling rates and for the offered external traffic values the following values have been chosen:

α_{eH} = 0.2/h ,	α_{eL} = 0.05/h ,
α_{iH} = 0.2/h ,	α_{iL} = 0.05/h ,
α_{iHH} = 0.138272 /h,	α_{iLL} = 0.014286 /h,
α_{iHL} = 0.061728 /h,	α_{iLH} = 0.035714 /h,

A_{ecHO} = 0.705882 Erlangs,
A_{ecLO} = 0.294118 Erlangs,
A_{ecO} = 1 Erlang.

(As these α values are referred to a time interval which is equal to the mean holding time h, the results according to this example are valid for arbitrary mean holding times h.)

In this example, a set of 1176 equations has to be solved in the calculation of the state probabilities $p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})$. From these probabilities one obtains the offered traffic values

A_i = 2.959 Erlangs, A_e = 3.959 Erlangs

and the loss probabilities

B_i = 0.0795 , B_e = 0.1866

4.2. SWITCHING SYSTEMS WITH ONE TYPE OF TRAFFIC SOURCES (SPECIAL CASE)

This section deals with the calculation of loss probabilities according to model No. 1 in the special case that all sources have the same calling rates $\alpha_i=\alpha_{iH}=\alpha_{iL}$ and $\alpha_e=\alpha_{eH}=\alpha_{eL}$, respectively.

With

$$x_i = x_{iH} + x_{iL} + x_{iM} \qquad (22)$$

and

$$x_e = x_{eH} + x_{eL}, \qquad (23)$$

the equations (8a) can be simplified to the following equations of state for the probabilities $p(x_i, x_e)$

$$\begin{aligned} & p(x_i, x_e)\cdot[x_i+x_e+(\alpha_i+\alpha_e)\cdot h\cdot(q-2x_i-x_e)+A_{ecO}] \\ & =(x_i+1)\cdot p(x_i+1, x_e) \\ & + (x_e+1)\cdot p(x_i, x_e+1) \\ & + \alpha_i\cdot h\cdot(q-2x_i-x_e+2)\cdot p(x_i-1, x_e) \qquad (24a) \\ & +[\alpha_e\cdot h\cdot(q-2x_i-x_e+1) +A_{ecO}]\cdot p(x_i, x_e-1), \\ & 0 \le x_i \le n_i, \\ & 0 \le x_e \le n_e. \end{aligned}$$

The system (24a) consists of only $(n_i+1)\cdot(n_e+1)$ equations in this case. The normalizing condition (8c) can be reduced to

$$\sum_{x_i=0}^{n_i} \sum_{x_e=0}^{n_e} p(x_i, x_e) = 1 \qquad (24b)$$

For the probabilities $p_i(x_i)$ and $p_e(x_e)$ one obtains

$$p_i(x_i) = \sum_{x_e=0}^{n_e} p(x_i, x_e), \qquad (25)$$

$$p_e(x_e) = \sum_{x_i=0}^{n_i} p(x_i, x_e). \qquad (26)$$

In this special case, the offered internal traffic A_i and the offered outgoing external traffic A_{eg} can easily be determined according to the formulae

$$A_i = \alpha_i \cdot (q - Y_e - 2Y_i)\cdot h, \qquad (27)$$

$$A_{eg} = \alpha_e \cdot (q - Y_e - 2Y_i)\cdot h. \qquad (28)$$

The (total) offered external traffic A_e, the total offered traffic A, the carried traffic values Y_i, Y_e and Y and the loss probabilities B_i, B_e and B can now be calculated from the equations (17),(18),(11),(12),(13),(19),(20) and (21), respectively.

5. LOSS PROBABILITIES ACCORDING TO MODEL NO. 2

5.1. SWITCHING SYSTEMS WITH TWO TYPES OF TRAFFIC SOURCES (GENERAL CASE)

The loss probabilities according to model No. 2 can be calculated in analogy to section 4.1. It must, however, be regarded that in model No. 2 internal calls and incoming external calls are taken into account only if the called subscriber is idle. I.e., the probabilities that an H-source (or L-source) is idle when called via an external trunk (or by an H-source or L-source, respectively) must be taken into account in this case. Then the following equations of state (corresponding to the equations (8a) in case of model No. 1) are obtained:

$$\begin{aligned}
&p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})\cdot\\
&\cdot[x_{iH}+x_{iL}+x_{iM}+x_{eH}+x_{eL}\\
&+\beta_{iHH}\cdot z_H\cdot(z_H-1)/(q_H-1)+\beta_{iHL}\cdot z_H\cdot z_L/q_L\\
&+\beta_{iLL}\, z_L\cdot(z_L-1)/(q_L-1)+\beta_{iLH}\cdot z_L\cdot z_H/q_H\\
&+\beta_{eH}\cdot z_H+A_{ecHO}\cdot z_H/q_H+\beta_{eL}\cdot z_L+A_{ecLO}\cdot z_L/q_L]\\
&= p(x_{iH}+1, x_{iL}, x_{iM}, x_{eH}, x_{eL})\cdot(x_{iH}+1)\\
&+p(x_{iH}, x_{iL}+1, x_{iM}, x_{eH}, x_{eL})\cdot(x_{iL}+1)\\
&+p(x_{iH}, x_{iL}, x_{iM}+1, x_{eH}, x_{eL})\cdot(x_{iM}+1) \qquad (29)\\
&+p(x_{iH}, x_{iL}, x_{iM}, x_{eH}+1, x_{eL})\cdot(x_{eH}+1)\\
&+p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}+1)\cdot(x_{eL}+1)\\
&+p(x_{iH}-1, x_{iL}, x_{iM}, x_{eH}, x_{eL})\cdot\\
&\qquad\cdot\beta_{iHH}\cdot(z_H+2)\cdot(z_H+1)/(q_H-1)\\
&+p(x_{iH}, x_{iL}-1, x_{iM}, x_{eH}, x_{eL})\cdot\\
&\qquad\cdot\beta_{iLL}\cdot(z_L+2)\cdot(z_L+1)/(q_L-1)\\
&+p(x_{iH}, x_{iL}, x_{iM}-1, x_{eH}, x_{eL})\cdot\\
&\qquad\cdot[\beta_{iHL}\cdot(z_H+1)\cdot(z_L+1)/q_L\\
&\qquad+\beta_{iLH}\cdot(z_L+1)\cdot(z_H+1)/q_H]\\
&+p(x_{iH}, x_{iL}, x_{iM}, x_{eH}-1, x_{eL})\cdot\\
&\qquad\cdot[\beta_{eH}\cdot(z_H+1)+A_{ecHO}\cdot(z_H+1)/q_H]\\
&+p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}-1)\cdot\\
&\qquad\cdot[\beta_{eL}\cdot(z_L+1)+A_{ecLO}\cdot(z_L+1)/q_L],
\end{aligned}$$

$x_{iH}\ge 0,\quad x_{iL}\ge 0,\quad x_{iM}\ge 0,$
$x_{eH}\ge 0,\quad x_{eL}\ge 0,$

$x_{iH} + x_{iL} + x_{iM} \le n_i,\quad x_{eH} + x_{eL} \le n_e.$

Furthermore, equation (8c) holds true.

In analogy to a formula by Bazlen [8], an exact, explicit solution of the equations (29, 8c) can be derived in this case. In a first step, all probabilities $p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})$ are successively expressed by the probability p(0,0,0,0,0). Then this value p(0,0,0,0,0) can be determined with the aid of the condition (8c). This leads to the following exact, explicit formula for the probabilities $p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})$

$$p(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}) = r(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL})/S, \qquad (30a)$$

where

$$S = \sum_{x_{iH}=0}^{n_i}\ \sum_{x_{iL}=0}^{n_i-x_{iH}}\ \sum_{x_{iM}=0}^{n_i-x_{iH}-x_{iL}}\ \sum_{x_{eH}=0}^{n_e}\ \sum_{x_{eL}=0}^{n_e-x_{eH}} r' \qquad (30b)$$

and

$$r' = r(x_{iH}, x_{iL}, x_{iM}, x_{eH}, x_{eL}) = \frac{\left(\frac{\beta_{iHH}}{q_H-1}\right)^{x_{iH}}\cdot\left(\frac{\beta_{iLL}}{q_L-1}\right)^{x_{iL}}\cdot\left(\frac{\beta_{iHL}}{q_L}+\frac{\beta_{iLH}}{q_H}\right)^{x_{iM}}}{x_{iH}!\cdot x_{iL}!\cdot x_{iM}!\cdot x_{eH}!\cdot x_{eL}!} \qquad (30c)$$

$$\cdot\frac{\left(\beta_{eH}+\frac{A_{ecHO}}{q_H}\right)^{x_{eH}}\cdot\left(\beta_{eL}+\frac{A_{ecLO}}{q_L}\right)^{x_{eL}}}{(q_H-2x_{iH}-x_{iM}-x_{eH})!\cdot(q_L-2x_{iL}-x_{iM}-x_{eL})!}$$

The fact, that the formulae (30a-c) fulfil the equation of state (29) can be easily proved by inserting equations (30a-c) in equation (29).

Now the probabilities $p_i(x_i)$ and $p_e(x_e)$ can be determined according to the equations (9a,b) and (10a,b), respectively. For the offered internal traffic A_i and the offered external traffic A_e one obtains then

$$\begin{aligned}
A_i = \textstyle\sum'[&\beta_{iHH}\cdot z_H\cdot(z_H-1)/(q_H-1)\\
&+\beta_{iLL}\cdot z_L\cdot(z_L-1)/(q_L-1)\\
&+\beta_{iHL}\cdot z_H\cdot z_L/q_L\\
&+\beta_{iLH}\cdot z_L\cdot z_H/q_H]\cdot p'
\end{aligned}$$

and

$$\begin{aligned}
A_e = \textstyle\sum'[&\beta_{eH}\cdot z_H + \beta_{eL}\cdot z_L\\
&+A_{ecHO}\cdot z_H/q_H + A_{ecLO}\cdot z_L/q_L]\cdot p' \qquad (32)
\end{aligned}$$

where z_H, z_L and p' are given by the equations (6), (7) and (8b), respectively. The summations $\sum'$ in the equations (31) and (32) comprise all possible states like in the equations (14) and (15).

The total offered traffic A, the carried traffic values Y_i, Y_e and Y and the loss probabilities B_i, B_e and B can be determined according to the equations (18), (11), (12), (13), (19), (20) and (21), respectively.

EXAMPLE NO. 2

The switching system considered in example No.1 (section 4.1) is calculated again here, however for model No. 2. In this case the following offered traffic values (A_i, A_e) and loss probabilities (B_i, B_e) are abtained:

A_i = 2.306 Erlangs, B_i = 0.0248,

A_e = 3.859 Erlangs, B_e = 0.1761.

A comparison of these results with the corresponding values of example No. 1 (section 4.1) shows that the offered traffic values (and, as a consequence, also the loss probabilities) in example No. 2 are a little smaller than those in example No. 1. This can be explained by the different traffic models applied: That part of the offered traffic which corresponds to call attempts directed to busy subscribers is contained in the offered traffic values in case of model No. 1 but not in model 2. This effect concerns all of the internal traffic A_i, but only a part of the external traffic, namely the offered incoming external traffic A_{ec} (or A_{eco}, respectively). This explains the fact that the difference between the offered external traffic values A_e of example No. 1 and example No. 2 is smaller than the difference between the offered internal traffic values A_i.

A more comprehensive comparison of results is made in section 7.

5.2. SWITCHING SYSTEMS WITH ONE TYPE OF TRAFFIC SOURCES (SPECIAL CASE)

This section deals with the special case that all sources have the same calling rates (according to model No. 2).

Regarding equations (22) and (23), the explicit formula (30a,b,c) can be simplified to

$$p(x_i,x_e) = \frac{\frac{c_i^{x_i}}{x_i!}\cdot\frac{c_e^{x_e}}{x_e!}\cdot\frac{1}{(q-2x_i-x_e)!}}{\sum_{z_i=0}^{n_i}\sum_{z_e=0}^{n_e}\frac{c_i^{z_i}}{z_i!}\cdot\frac{c_e^{z_e}}{z_e!}\cdot\frac{1}{(q-2z_i-z_e)!}} \qquad (33a)$$

with

$$c_i = \alpha_i\cdot h/(q-1)\ , \qquad (33b)$$

$$c_e = \alpha_e\cdot h+A_{ecO}/q\ . \qquad (33c)$$

From these values $p(x_i,x_e)$ the probabilities $p_i(x_i)$ and $p_e(x_e)$ and the carried traffic values Y_i, Y_e and Y can be determined according to the equations (25), (26), (11), (12) and (13), respectively. For the offered internal traffic A_i the following formula is obtained

$$A_i = \frac{1}{q-1}\sum_{x_i=0}^{n_i}\sum_{x_e=0}^{n_e} p(x_i,x_e)\cdot(p-2x_i-x_e)\cdot(q-2x_i-x_e-1). \qquad (34)$$

For the (actual) total offered external traffic A_e holds

$$A_e = (\beta_e+\frac{A_{ecO}}{q})\cdot(q-Y_e-2Y_i)\ . \qquad (35)$$

The total offered traffic A and the loss probabilities B_i, B_e and B can be determined according to the equations (18), (19), (20) and (21).

6. APPROXIMATION METHOD

The exact calculation methods described in sections 4 and 5 may be rather time-consuming or even impracticable in case of larger switching systems. Therefore in this section an approximation method is presented which can be applied for model No. 1 as well as for model No. 2.

The following calculations are based on the assumption that, according to the different calling rates, H-sources are called more frequently than L-sources. This assumption can approximately be taken into account by means of the conditions

$$\frac{\alpha_{iHH}}{\alpha_{iHL}} = \frac{\alpha_{iH}\cdot(q_H-1)}{\alpha_{iL}\cdot q_L}\ , \qquad (36)$$

$$\frac{\alpha_{iLL}}{\alpha_{iLH}} = \frac{\alpha_{iL}\cdot(q_L-1)}{\alpha_{iH}\cdot q_H}\ , \qquad (37)$$

$$\frac{A_{ecHO}}{A_{ecLO}} = \frac{\alpha_{eH}\cdot q_H}{\alpha_{eL}\cdot q_L}\ . \qquad (38)$$

For the total calling rates of H-sources and L-sources (which are denoted by α_H and α_L, respectively) the following equations hold true

$$\alpha_H = \alpha_{iH} + \alpha_{eH} \qquad (39)$$

$$\alpha_L = \alpha_{iL} + \alpha_{eL} \qquad (40)$$

The quotient of these calling rates α_L/α_H be denoted by s:

$$\alpha_L/\alpha_H = s. \qquad (41)$$

For this quotient the following condition holds true

$$0 \le s \le 1. \qquad (42)$$

In the special cases s=0 and s=1 (in which all sources are equal) the loss probabilities can be calculated exactly according to sections 4 and 5. The approximation method derived in this section represents an interpolation between these two special cases s=0 and s=1.

Let us assume that, besides the values n_i, n_e, q_H, q_L, s and A_{ecO}, the carried traffic values Y_i and Y_e are given and that the loss probabilities B_i, B_e and B are to be calculated.

First, the special case s=0 (correspondending to α_L=0) is considered. In this fictitious case the loss probabilities (denoted as B_{iO}, B_{eO} and B_O) refer to a switching system having q_H equal sources with the (unknown) calling rates α_i and α_e. These calling rates α_i and α_e are now iteratively determined such that the carried internal traffic and the carried external traffic have the given values Y_i and Y_e, respectively. Then the loss probabilities B_{iO}, B_{eO} and B_O can also be calculated.

In a second step, the special case s=1 (corresponding to $\alpha_L=\alpha_H$) is considered which refers to a switching system having $q_H + q_L$ equal sources. The loss probabilities B_{i1}, B_{e1} and B_1 are determined in the same way as the values B_{iO}, B_{eO} and B_O above.

The approximation values for the given switching system (with to types of sources) are now determined by an interpolation. The investigation of various examples has shown that a function of second order is well suited for this purpose. This leads to the approximation formula

$$B = B_1 - (B_1 - B_O)\cdot(1 - \frac{s\cdot(q_L+q_H)}{s\cdot q_L+q_H})^2\ . \qquad (43)$$

Analogously, the loss probabilities B_i and B_e and the state probabilities $p_i(x_i)$ and $p_e(x_e)$ can be determined. For the offerd traffic values A_i, A_e and A the equations

$$A_i = Y_i/(1 - B_i)\ , \qquad (44)$$

$$A_e = Y_e/(1 - B_e)\ , \qquad (45)$$

$$A = (Y_i + Y_e)/(1 - B) \qquad (46)$$

hold true.

If the offered traffic values are given instead of the carried traffic values, the same method can be applied. In this case the values A_i and A_e are regarded as constants (instead of Y_i and Y_e). If, however, the loss probabilities are given, a further iteration is necessary.

A comparison shows that the approximation values are in good accordance with exact results. This can, e.g., be seen from the following example.

EXAMPLE NO. 5

In this example, a switching system with the values n_i=5, n_e=5, q_H=20 and q_L=20 is calculated according to model No. 2. Besides the quotient of the calling rates $s=\alpha_L/\alpha_H$=0.5, the traffic values Y_i=0.8761 Erlangs, Y_e=0.7893 Erlangs and A_{ecO}=0.3434 Erlangs be given. (These traffic values are obtained with the calling rates α_{iH}=0.033744/h, α_{eH}=0.016872/h, α_{iL}=0.016872/h and α_{eL}=0.008436/h).

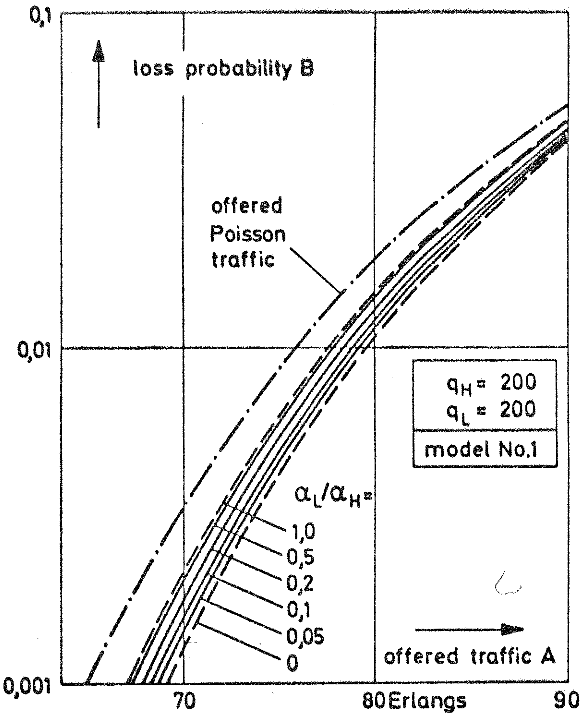

Fig. 2: Loss probability B = f(A)
q_H = 200, q_L = 200
model No. 1

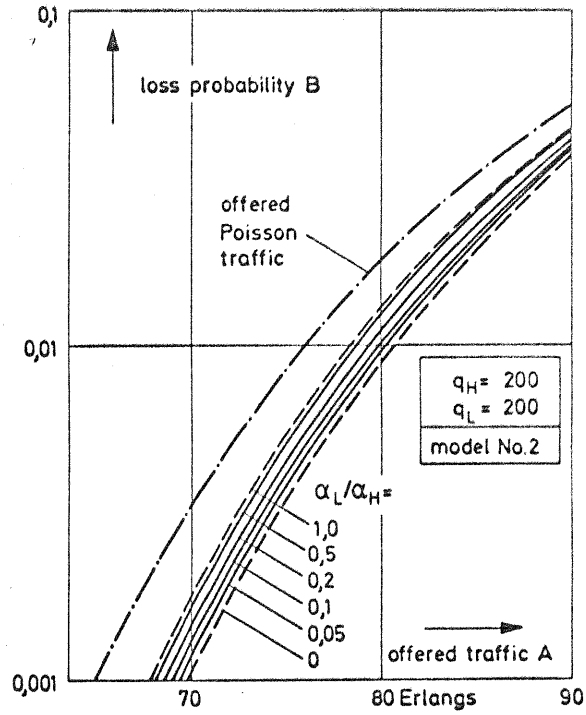

Fig. 3: Loss probability B = f(A)
q_H = 200, q_L = 200
model No. 2

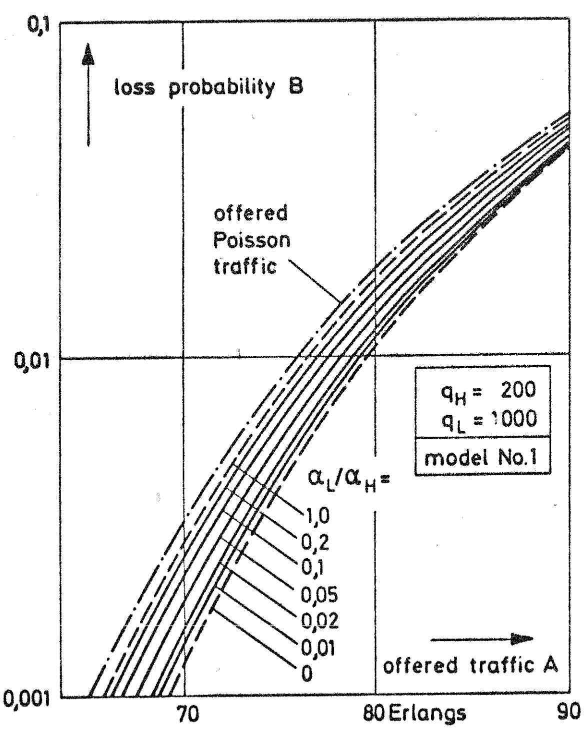

Fig. 4: Loss probability B = f(A)
q_H = 200, q_L = 1000
model No. 1

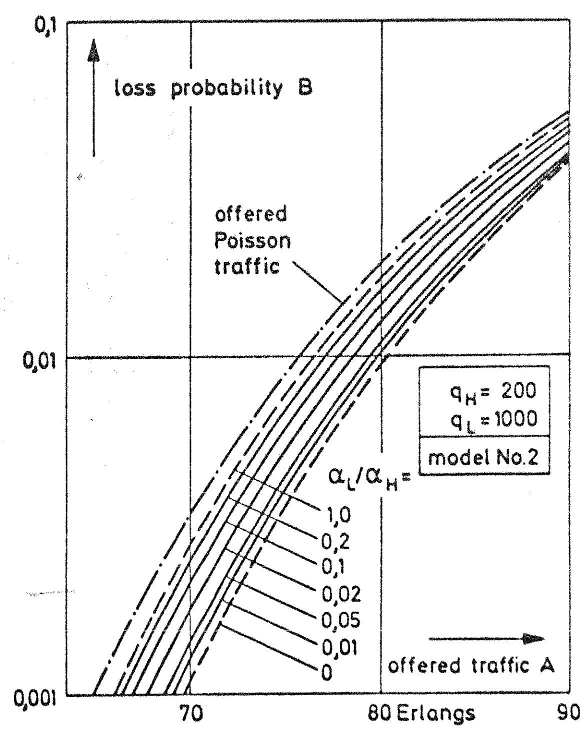

Fig. 5: Loss probability B = f(A)
q_H = 200, q_L = 1000
model No. 2

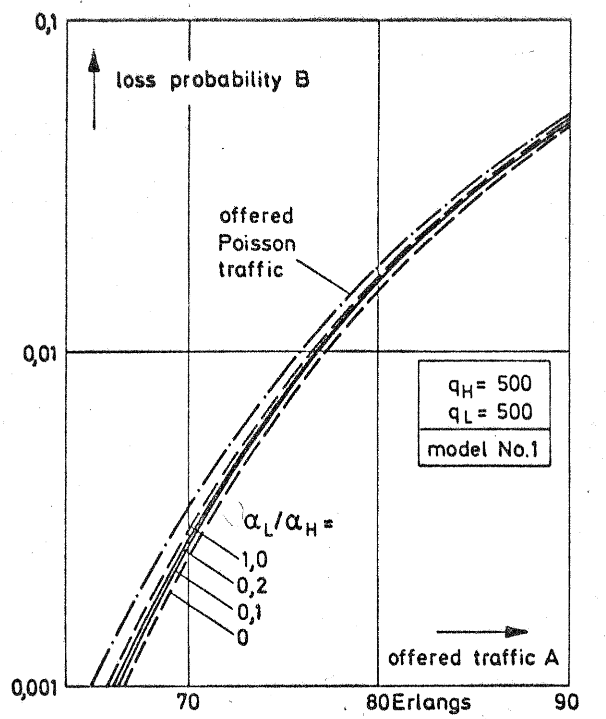

Fig. 6: Loss probability B = f(A)
q_H = 500, q_L = 500
model No. 1

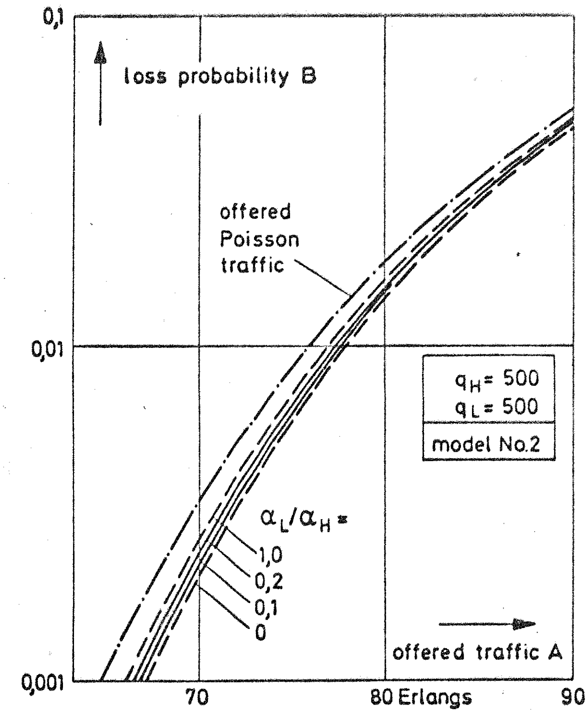

Fig. 7: Loss probability B = f(A)
q_H = 500, q_L = 500
model No. 2

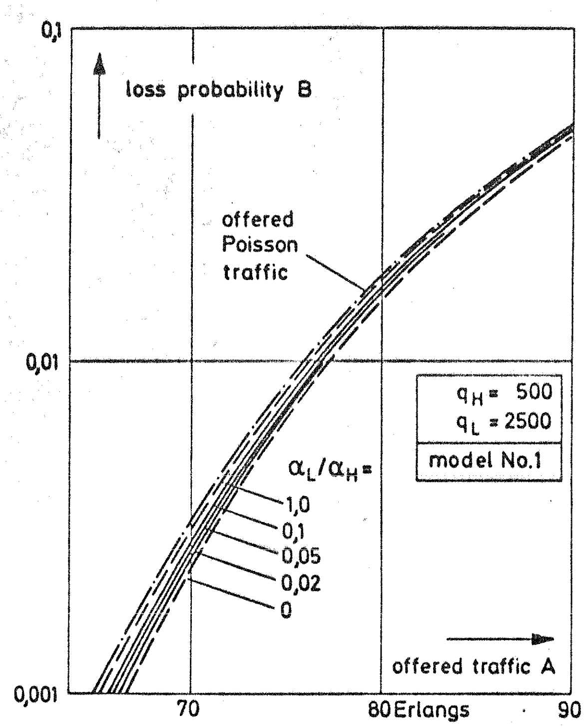

Fig. 8: Loss probability B = f(A)
q_H = 500, q_L = 2500
model No. 1

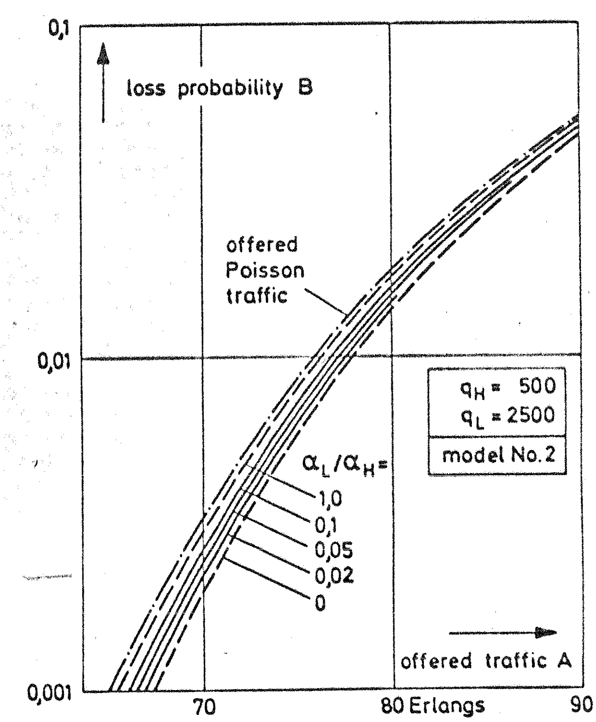

Fig. 9: Loss probability B = f(A)
q_H = 500, q_L = 2500
model No. 2

The exact loss probabilities (according to section 5.2) are in this example

B_i=0.000482, B_e=0.000866, B=0.000664.

For the special case s=0 (lower limit) one obtains (with α_i=0.057250/h and α_e=0.028069/h) the values

B_{i0}=0.000097, B_{e0}=0.000368, B_0=0.000664.

For the case s=1 (upper limit) one obtains (for α_i=0.024963/h and α_e=0.012504/h) the values

B_{i1}=0.000545, B_{e1}=0.000889, B_1=0.000708.

The interpolation according to equation (43) leads to the following results (the relative deviations from the exact values are given in parentheses)

B_i=0.000495 (+2,74%),
B_e=0.000864 (-0,23%),
B=0.000670 (+0,94%).

The investigation of further examples has shown that the accuracy of the approximation formula (43) increases with growing loss probabilities. This can be explained by the fact that the difference between the upper and lower limits (e.g. between B_{e1} and B_{e0}) decreases with growing loss.

7. RESULTS

In this section results according to the approximation formula (43) for model No. 1 and No. 2 are presented. In the diagrams in fig. 2 to fig. 9 the loss probability B is shown as a function of the offered traffic A for several values of the quotient α_L/α_H. All diagrams refer to systems with n_i=50 internal junctors and n_e=50 external trunks. For reasons of simplicity, A_i=A_e=A/2 and A_{ec0}=0 has been chosen in all diagrams. The curves for the limiting cases α_L/α_H=0 and α_L/α_H=1 are shown as dashed lines, the other curves as solid lines. As an upper limit, the loss probabilities for offered Poisson traffic (according to the Erlang formula) are shown (-·-·-) for comparison.

From fig. 2 (q_H=200, q_L=200, model No. 1) it can be seen that the loss probability B increases with growing α_L/α_H and that the quotient α_L/α_H of the calling rates has only a slight influence on the loss probabilities in this case.

In fig. 3 (q_H=200, q_L=200, model No. 2) the loss probabilities are smaller than those of fig. 2 because the smoothing effect in model No. 1 is smaller than in model No. 2.

In fig. 4 (q_H=200, q_L=1000, model No. 1) the lower curve (for α_L/α_H=0) is the same as in fig. 2. For α_L/α_H>0 the loss probabilities shown in fig. 4 are higher (because of the higher number of sources) than those of fig. 2. Thus the influence of different types of sources in fig. 4 is slightly increased as compared with fig.2.

In fig. 5 (q_H=200, q_L=1000, model No. 2) the curves are similar to those of fig.4. The loss probabilities are, however, slightly smaller than those of fig. 4. The lower limiting curve in fig. 5 is identical to that of fig. 3.

The diagrams in figs. 6 to 9 are similar to those of figs. 2 to 5. The numbers of sources are, however, larger by a factor 2.5 (q_H=500 and q_L=500 or q_L=2500, respectively). This leads to slightly enlarged loss probabilities in fig. 6 to fig. 9.

8. CONCLUSION

In this paper, loss probabilities of switching systems with internal and external traffic and with two types of sources are calculated exactly. Furthermore, an approximation formula is presented. The approximation values are in good accordance with exact results.

The calculations are carried out for two different traffic models.

The results are shown for several examples by means of diagrams.

LITERATURE

[1] Hayward, W.S. and Wilkinson, R.I.: Human factors in telephone systems and their influence on traffic theory, especially with regard to future facilities. 6th ITC München, 1970, pp. 431/1-431/10.

[2] Cohen, J.W.: The generalized Engset formulae. Philips Telecommunication Review 18 (1957), No. 4, pp. 158 - 170.

[3] Bächle, A.: On the calculation of full available groups with offered smoothed traffic. 7th ITC Stockholm, 1873, pp. 223/1-223/6.

[4] Rønnblom, N.: Traffic loss of a circuit group consisting of both-way circuits, which is accessible for the internal and external traffic of a subscriber group. Tele (1959) No. 2, pp. 79 - 92.

[5] Botsch, D.: Die Verlustwahrscheinlichkeit einstufiger Koppelanordnungen der Vermittlungstechnik mit Extern- und Internverkehr. Ph. D. thesis, University Stuttgart, 1966.

[6] Botsch, D.: Die Verlustwahrscheinlichkeit einstufiger Koppelanordnungen der Vermittlungstechnik mit Extern- und Internverkehr. A.E.Ü. 22 (1968) No. 3, pp. 127 - 132.

[7] Botsch, D.: and Weller, H.: Zweistufige Linksysteme mit Extern- und Internverkehr. Institute for switching and data techniques, University Stuttgart, Monograph, 1966.

[8] Bazlen, D.: Mehrstufige doppelt-gerichtete Koppelanordnungen der Vermittlungstechnik mit Intern- und Externverkehr. Ph. D. thesis, University Stuttgart, 1973.

[9] Bazlen, D.: Link systems with both-way connections and outgoing finite source traffic. 7th ITC Stockholm, 1973, pp. 316/1-316/8.

[10] Brockmeyer, E., Halstrom, H. and Jensen, A.: The life and works of A.K. Erlang. Acta Polytech. Scandinavica 1960, No.287.

[11] Young, D.M.: Iterative methods for solving partial difference equations of elliptic type. Trans.Amer.Math.Soc. 76 (1954) pp.92-111.