

ON A QUEUING PROBLEM IN TDMA NETWORKS

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ABSTRACT

This paper deals with a queuing problem in time division systems with multiple access (TDMA systems) in which the subscribers are connected to a branched network of TDM highways. In the nodes of the network considered here, arriving information blocks which can not be transmitted directly can be stored.

The topic of this paper is the calculation of the loss probability in such TDMA systems, i.e. the probability for an arbitrary information block to be lost due to storage overflow.

For the calculation of the loss probability in a node of such a system, an exact formula is derived here. Furthermore, a simple approximation method for calculating the loss probability is presented which is well suited for practical applications. The results according to this approximation method are compared with exact values and simulation results.

1. INTRODUCTION

In most cases it is advantageous in integrated networks to use time division systems with multiple access (TDMA systems). In such systems the subscribers are connected to a network of TDM highways which consist of wideband transmission channels (e.g. optical waveguides).

In this paper TDMA systems with branched networks containing no closed loops (as shown in fig. 1) are

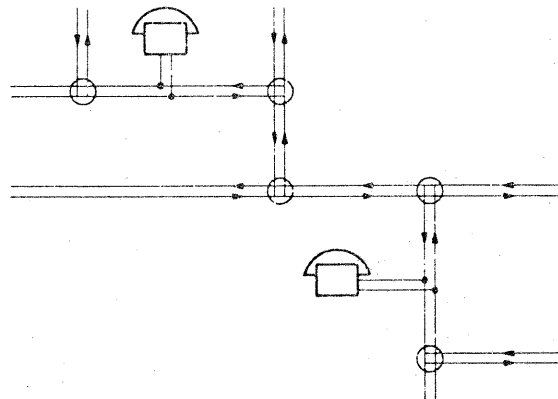


Fig. 1: Branched TDMA network

investigated. Each branch consists of a pair of highways (one highway for each transmitting direction). The number of TDM channels on a highway (i.e. the number of time slots in a frame) is in the order of magnitude of about $N = 10^4$.

In most of the usual TDM systems the receiver address assigned to a transmitted information is closely related to the position of the time slot used during transmission. In the TDMA system considered here, the receiver address is always transmitted together with the corresponding information. I.e., each block transmitted in a time slot consists of address bits and information bits, so that every subscriber is able to pick up the blocks containing his own address from the highway. Therefore such a system represents a decentralized switching systems.

As each subscriber may be connected to the network at an arbitrary point, all blocks must be transmitted through the whole network (unless special routing facilities are provided). Therefore all blocks arriving on an incoming highway of a node must be transmitted to the outgoing highways of all other directions (as indicated in fig. 2). If two (or more) blocks arrive in a node at the same time slot (on different highways), only one block can be transmitted to the outgoing highway immediately whereas the other(s) must be stored. For this purpose a

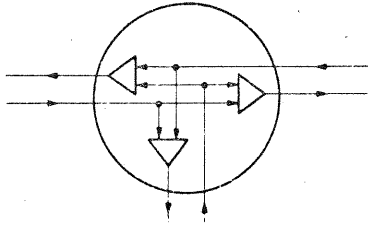


Fig. 2: Transmission of arriving blocks in a node

finite number of block stores (waiting places) is provided. When all waiting places are occupied a storage overflow may occur and information blocks get lost. In this case established connections are disturbed.

The topic of this paper is the calculation of the loss probability p_L in nodes of such TDMA systems, i.e. the probability that an arbitrary block arriving at a node is lost because of storage overflow.

In section 2 an exact formula for the loss probability p_L is derived. The numerical evaluation of this exact formula is, however, rather lengthy and time consuming, even if a digital computer is used. Therefore in section 3 a simple approximation method for calculating the loss probability is presented which is well suited for practical applications. The results according to this approximation method are compared with exact values and with simulation results which are shortly referred to in section 4.

These investigations are based on the condition that not more than 3 pairs of highways meet at each node of the network and that the blocks arriving on the incoming highways are distributed at random within the frames.

2. EXACT CALCULATION OF THE LOSS PROBABILITY

2.1. WAY OF SOLUTION

Considering a node of such a transmitting network it can be seen that the blocks of any two incoming highways must be transmitted on an outgoing highway which is not allocated to these incoming highways. Thus, for reasons of symmetry, it is sufficient to consider one outgoing trunk which transmits the blocks from two incoming trunks. Such a simplified system is shown schematically in fig. 3. Here the blocks

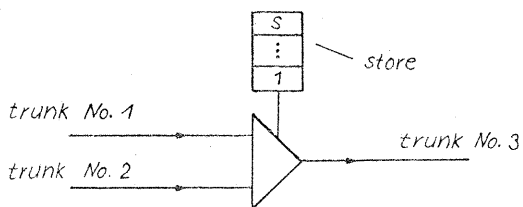


Fig. 3: Connecting element with a finite store

arriving on trunk No. 1 and on trunk No. 2 are commonly transmitted on the outgoing trunk No. 3. For storing blocks which can not be transmitted directly, a store consisting of s waiting places is provided.

This system is now considered during a time interval in which the total number of existing calls (and thus the number of blocks in a frame) is constant. During such a period the pattern of occupied time slots is the same in all frames.

The number of time slots per frame be denoted by N , and the number of blocks (i.e. the number of occupied time slots) per frame on trunk No. 1 and trunk No. 2 be denoted by x_1 and x_2 , respectively. As the x_1 blocks on trunk No. 1 are supposed to be distributed at random within the frame, the probability p_1 of an arbitrary time slot to be occupied on trunk No. 1 is obtained as

$$p_1 = \frac{x_1}{N} \quad (1)$$

Analogously the probability p_2 that an arbitrary time slot is occupied on trunk No. 2 is obtained,

$$p_2 = \frac{x_2}{N} \quad (2)$$

For the probability $p(2B)$ that 2 blocks arrive on an arbitrary time slot it follows

$$p(2B) = p_1 \cdot p_2 \quad (3)$$

Let $p(s|2B)$ be the conditional probability that all s waiting places are occupied when 2 blocks are arriving on the same time slot. Then the overflow probability p_{ov} (i.e. the probability that a store overflow occurs on an arbitrary time slot) is obtained as

$$p_{ov} = p(2B) \cdot p(s|2B) \quad (4)$$

The expectation of the number of storage overflows occurring in a frame (and thus the number of lost blocks) is therefore $N \cdot p_{ov}$. As there are $x_1 + x_2$ blocks in a frame in total, the loss probability p_L (i.e. the probability that an arbitrary block is lost) is obtained as

$$p_L = \frac{N}{x_1 + x_2} \cdot p_{ov}$$

With equations (1), (2), (3) and (4) follows

$$p_L = \frac{x_1 \cdot x_2}{N \cdot (x_1 + x_2)} \cdot p(s|2B) \quad (5)$$

In eq. (5) the loss probability p_L is expressed as a function of the conditional probability $p(s|2B)$. For determining this probability $p(s|2B)$ a time slot is considered on which two blocks are arriving at the node. (Subsequently two blocks which arrive on the same time slot will be referred to as a "double block".) On the remaining $N - 1$ time slots of the frame there are $x_1 - 1$ blocks on trunk No. 1 which can be arranged in

$$n_1 = \binom{N-1}{x_1-1} \quad (6)$$

possible patterns. For each of these patterns there are

$$n_2 = \binom{N-1}{x_2-1} \quad (7)$$

possible patterns of the blocks on trunk No. 2. Thus the total number n_{tot} of possible patterns is

$$n_{tot} = n_1 \cdot n_2 \quad (8)$$

Among these n_{tot} patterns there is a number n_{ov} of patterns with a storage overflow at the considered time slot. (Henceforth such patterns will be called "overflow patterns".) Thus for the probability $p(s|2B)$ holds

$$p(s|2B) = \frac{n_{ov}}{n_{tot}} \quad (9)$$

From eqs. (5), (6), (7), (8) and (9) follows

$$p_L = \frac{x_1 \cdot x_2}{N(x_1 + x_2)} \cdot \frac{1}{\binom{N-1}{x_1-1} \cdot \binom{N-1}{x_2-1}} \cdot n_{ov} \quad (10a)$$

or

$$p_L = c \cdot n_{ov} \quad (10b)$$

where

$$c = \frac{x_1 \cdot x_2}{N(x_1 + x_2)} \cdot \frac{1}{\binom{N-1}{x_1-1} \binom{N-1}{x_2-1}} \quad (10c)$$

In these equations the calculation of the loss probability p_L is reduced to the number of overflow patterns n_{ov} which will be determined in the following sections.

2.2. THE LOSS PROBABILITY IN THE CASE $s = 1$

For reasons of simplicity, the number of overflow patterns and the loss probability are first calculated for the special case that there is only $s = 1$ waiting place.

The general structure of a pattern with an overflow on a considered time slot at time t_1 is shown in fig. 4. Such an overflow pattern consists mainly of

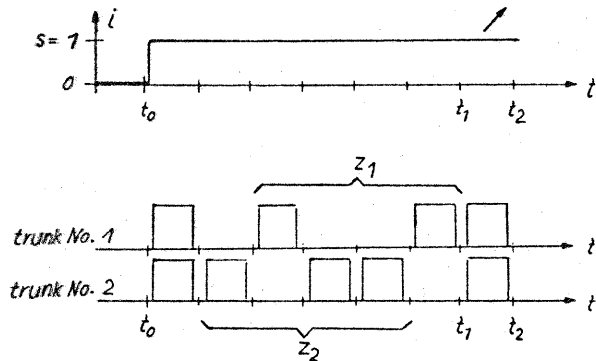


Fig. 4: General structure of an overflow pattern in the case $s = 1$

two double blocks (at time t_0 and at time t_1) with a series of single blocks in between. On the time slots outside the double blocks (i.e. at times $t < t_0$ and $t > t_2$) further blocks may be distributed arbitrarily.

In the upper diagram of fig. 4 the number i of occupied waiting places is shown as a function of time. Supposing that the waiting place has been free at time t_0 , it is occupied at the arrival of the first double block at time t_0 . During the series of single blocks the waiting place remains occupied. Then the second double block arriving at time t_1 causes a storage overflow which is indicated by an arrow in fig. 4. (A storage overflow is also caused by the second double block at time t_1 if the waiting place is already occupied when the first double block is arriving at time t_0 .)

The number of time slots between the double blocks in fig. 4 be denoted by z . From these z time slots, z_1 are supposed to be occupied by a block on trunk No. 1 and z_2 on trunk No. 2; thus

$$z_1 + z_2 = z \quad (11)$$

For the distribution of the z_1 blocks on the z time slots there are

$$\binom{z}{z_1}$$

possibilities (the remaining $z_2 = z - z_1$ time slots being occupied by blocks on trunk No. 2). On the $N - z - 2$ time slots outside the two double blocks there are further $x_1 - z_1 - 2$ blocks on trunk No. 1 which can be arranged in

$$\binom{N-z-2}{x_1-z_1-2}$$

possible patterns and $x_2 - z + z_1 - 2$ blocks on trunk No. 2 which can be arranged in

$$\binom{N-z-2}{x_2-z+z_1-2}$$

possible patterns.

Thus, for arbitrary values (z, z_1) there are

$$n_{ov}^i = \binom{z}{z_1} \cdot \binom{N-z-2}{x_1-z_1-2} \cdot \binom{N-z-2}{x_2-z+z_1-2} \quad (12)$$

possible overflow patterns. The total number of overflow patterns is obtained by summing up n_{ov}^i according to eq. (12) for all possible values of z and z_1

$$n_{ov} = \sum_{z=0}^{x_1+x_2-4} \sum_{z_1=0}^{x_1-2} \binom{z}{z_1} \cdot \binom{N-z-2}{x_1-z_1-2} \cdot \binom{N-z-2}{x_2-z+z_1-2} \quad (13)$$

Now the loss probability p_L can be calculated according to eq. (10a)

$$p_L = c \cdot \sum_{z=0}^{x_1+x_2-4} \sum_{z_1=0}^{x_1-2} \binom{z}{z_1} \cdot \binom{N-z-2}{x_1-z_1-2} \cdot \binom{N-z-2}{x_2-z+z_1-2} \quad (14)$$

where c is determined according to eq. (10c).

2.3. GENERAL FORMULA FOR THE LOSS PROBABILITY

In this section the loss probability p_L is calculated for the general case that s waiting places are provided. For this case the general structure of an overflow pattern is shown schematically in fig. 5.

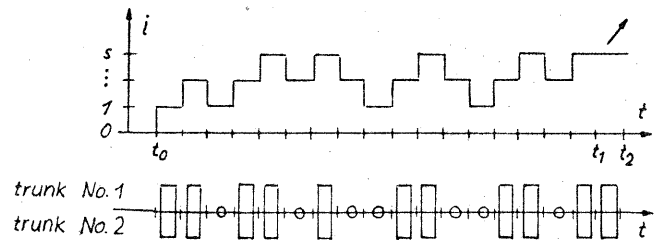


Fig. 5: General structure of an overflow pattern

Between the two double blocks at time t_0 and time t_1 there are $s - 1$ further double blocks, z single blocks (from which z_1 are situated on trunk No. 1 and z_2 on trunk No. 2) and, furthermore, s pairs "double block plus free time slot". (In such a pattern an overflow occurs at time t_1 in any case, no matter how many waiting places are occupied at time t_0 .) In fig. 5 the single blocks are omitted for reasons of simplicity, whereas free time slots are indicated by small circles.

In the upper diagram of fig. 5 the number i of occupied waiting places is shown as a function of time (for the case that $i = 0$ waiting places are occupied at time t_0). It can be seen that this is a step function which may ascend and descend several times.

For a fixed value of ν there are, in general, several possible types of step functions. The number of these types be denoted by $t_s(\nu)$. For a convenient calculation of $t_s(\nu)$ these step functions are considered to consist of a (monotonously ascending) basic step function on the individual steps of which there are various humps. In fig. 6 the basic step function

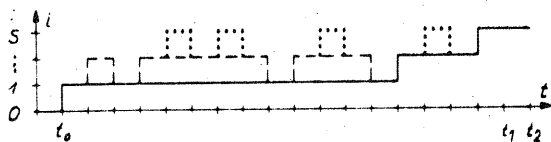


Fig. 6: Decomposition of a step function into a basic step function and various humps
 — basic step function
 - - humps on the level $i = 1$
 humps on the level $i = 2$

corresponding to the upper diagram of fig. 5 is indicated by a solid line; the ν_1 humps which are situated on the first step of this basic function are indicated by dashed lines (in this example $\nu_1 = 3$). In the new function obtained in this way the level $i = 2$ consists of $\nu_1 + 1$ different sections, namely the second step of the basic function and ν_1 humps. On these $\nu_1 + 1$ sections of the level $i = 2$ a number ν_2 of humps is situated which are indicated by a dotted line in fig. 6, etc. ($\nu_2 = 4$ in the example shown in fig. 6).

The number of humps on the level i be denoted as ν_i . Then it can be shown that there are

$$\mu_i = \binom{\nu_i + \nu_{i-1}}{\nu_{i-1}} \quad (15)$$

possible arrangements of these ν_i humps on the $\nu_{i-1} + 1$ sections of level i . With the aid of eq. (15) it is possible to calculate the number μ of types of step functions for fixed values $(\nu_1, \nu_2, \dots, \nu_{s-1})$. Regarding that $\mu_1 = 1$, one obtains

$$\mu = \mu_2 \cdot \mu_3 \cdot \mu_4 \cdot \dots \cdot \mu_{s-1} \quad (16)$$

The total number $t_s(\nu)$ of possible types of step functions with ν humps (i.e. ν pairs "double block plus free time slot") is obtained by summing up μ according to eq. (16) for all possible values $\nu_1, \nu_2, \dots, \nu_{s-1}$ (ν_{s-1} being determined by the condition that the sum of all ν_i values ($i = 1, 2, \dots, s-1$) is equal to ν):

$$t_s(\nu) = \sum_{\nu_1=0}^{\nu} \sum_{\nu_2=0}^{\nu-\nu_1} \sum_{\nu_3=0}^{\nu-\nu_1-\nu_2} \dots \sum_{\nu_{s-2}=0}^{\xi_3} \mu_2 \cdot \mu_3 \cdot \dots \cdot \mu_{s-1} \cdot \binom{\xi_3}{\nu_{s-2}}, \quad (17a)$$

$s \geq 2$

where

$$\xi_j = \nu - \sum_{i=1}^{s-j} \nu_i, \quad j=1, 2, 3 \quad (18)$$

The formula (17a) holds true for $s \geq 2$. The following formula

$$t_s(\nu) = \sum_{\nu_1=0}^{\nu} \sum_{\nu_2=0}^{\nu-\nu_1} \sum_{\nu_3=0}^{\nu-\nu_1-\nu_2} \dots \sum_{\nu_{s-1}=0}^{\xi_2} \mu_2 \cdot \mu_3 \cdot \dots \cdot \mu_{s-1} \cdot \binom{0}{\xi_1}, \quad (17b)$$

$s \geq 1,$

which is a little more complicated is also valid for the case $s = 1$ and thus for $s \geq 1$.

For each of these $t_s(\nu)$ types there exists a number $k_s(\nu)$ of possible step functions according to the location of the z single blocks. It holds

$$k_s(\nu) = \binom{z + 2\nu + s - 1}{z} \quad (19)$$

For each of these $t_s(\nu) \cdot k_s(\nu)$ possible step functions there are, in analogy to the case $s = 1$ (see section 2.2),

$$\binom{z}{z_1}$$

possible arrangements of the z_1 blocks between the double blocks at time t_0 and time t_1 , and furthermore

$$\binom{N - (s+1) - z - 2\nu}{x_1 - (s+1) - z_1 - \nu}$$

possible patterns for the blocks outside these double blocks on trunk No. 1 and

$$\binom{N - (s+1) - z - 2\nu}{x_2 - (s+1) - z + z_1 - \nu}$$

possible patterns on trunk No. 2. Thus, for arbitrary values (ν, z, z_1) there are

$$n'_{ov} = t_s(\nu) \cdot k_s(\nu) \cdot \binom{z}{z_1} \cdot \binom{N - (s+1) - z - 2\nu}{x_1 - (s+1) - z_1 - \nu} \cdot \binom{N - (s+1) - z - 2\nu}{x_2 - (s+1) - z + z_1 - \nu} \quad (20)$$

possible overflow patterns. By summation of n'_{ov} according to eq. (20) for all values ν, z and z_1 the total number n_{ov} of overflow patterns is obtained

$$n_{ov} = \sum_{\nu=0}^{x_1-1} \sum_{z=0}^{x_1+x_2-2s-2-2\nu} \sum_{z_1=0}^{x_1-1-\nu} n'_{ov} \quad (21)$$

The loss probability, i.e. the probability of an arbitrary block to be lost because of storage overflow, can now be calculated according to eq. (10b). One obtains

$$p_L = c \cdot \sum_{\nu=0}^{x_1-1} \sum_{z=0}^{x_1+x_2-2s-2-2\nu} \sum_{z_1=0}^{x_1-1-\nu} n'_{ov} \quad (22)$$

In eq. (22) the values c and n'_{ov} can be determined according to the eqs. (10c) and (20), respectively.

2.4. NUMERICAL EVALUATION AND EXAMPLE

The loss probability can be calculated exactly according to the formulae derived in section 2.3. As, however, the numerical evaluation of these formulae is rather lengthy, this exact calculation method can only be applied to relatively small systems, even if a digital computer is used.

As an example a system with $s = 2$ waiting places and $N = 100$ time slots per frame is considered during a time interval in which $x_1 = 40$ blocks per frame arrive at the node on trunk No. 1 and $x_2 = 40$ blocks per frame on trunk No. 2. In this case the loss probability

$$p_L = 0.02084$$

is obtained.

As the numerical evaluation of the exact calculation method derived here is rather complicated, a simple approximate formula will be developed subsequently in section 3.

3. APPROXIMATE CALCULATION OF THE LOSS PROBABILITY

3.1. THE EQUATIONS OF STATE

In eq. (5) the loss probability p_L is expressed as a function of the conditional probability $p(s|2B)$, i.e. the probability that all s waiting places are occupied when a double block arrives at a node. For this probability $p(s|2B)$ an approximation formula is derived under the condition that

$$x_1, x_2 \gg s \quad (23)$$

(or, as a limiting case, $x_1, x_2 \rightarrow \infty$).

For this purpose a time slot is considered on which a double block is arriving. This time slot be denoted by h . On the remaining $N - 1$ time slots there are $x_1 - 1$ blocks on trunk No. 1. For the probability p'_1 that an arbitrary one of these $N - 1$ time slots is occupied on trunk No. 1 follows

$$p'_1 = \frac{x_1 - 1}{N - 1} \quad (24)$$

Analogously one obtains for the probability p'_2 of an arbitrary time slot on trunk No. 2 to be occupied

$$p'_2 = \frac{x_2 - 1}{N - 1} \quad (25)$$

Because of the condition (23) the probabilities according to eqs. (24) and (25) are also valid if - in addition to the considered time slot h - further time slots are supposed to be occupied or free.

The probability that there are α blocks ($\alpha = 0, 1, 2$) on an arbitrary time slot (except the considered time slot h) be denoted by q_α . For these probabilities one obtains the values

$$q_0 = (1 - p'_1) \cdot (1 - p'_2) \quad (26)$$

$$q_1 = p'_1 \cdot (1 - p'_2) + p'_2 \cdot (1 - p'_1) \quad (27)$$

$$q_2 = p'_1 \cdot p'_2 \quad (28)$$

with

$$q_0 + q_1 + q_2 = 1 \quad (29)$$

Now it is assumed that i waiting places are occupied at the beginning of a time slot a (at time t_a). If no block arrives on this time slot a , then at the end of this time slot a (i.e. at the beginning of the following time slot b at time t_b) $i - 1$ waiting places are occupied. If one block arrives on time slot a , the number i of occupied time slots remains unchanged, and i increases by 1 if two blocks arrive on time slot a . This is shown schematically in fig. 7. The

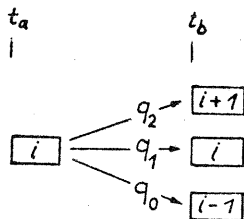


Fig. 7: Possible transitions between succeeding time slots

probabilities for these transitions are also indicated in fig. 7. Completing fig. 7 - by taking into account all possible values of i ($i = 0, 1, \dots, s$) - the state diagram shown in fig. 8 is obtained.

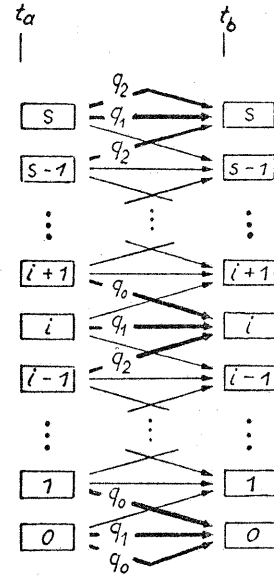


Fig. 8: State diagram

If the probability that i waiting places are occupied at time t_a (or t_b , respectively) is denoted by $p'_a(i)$ (or $p'_b(i)$, respectively), then the state diagram shown in fig. 8 leads to the following linear equations of state

$$p'_b(0) = q_0 \cdot p'_a(0) + q_1 \cdot p'_a(0) + q_2 \cdot p'_a(1) \quad (30a)$$

$$p'_b(i) = q_2 \cdot p'_a(i-1) + q_1 \cdot p'_a(i) + q_0 \cdot p'_a(i+1), \quad (30b)$$

$i = 1, 2, \dots, s-1$

$$p'_b(s) = q_2 \cdot p'_a(s-1) + q_1 \cdot p'_a(s) + q_2 \cdot p'_a(s) \quad (30c)$$

Under the conditions made here, the probabilities $p'_a(i)$ and $p'_b(i)$ are identical; thus

$$p'_a(i) = p'_b(i) = p'(i) \quad (31)$$

With the eqs. (29) and (31) and with the abbreviation

$$\frac{q_2}{q_0} = m \quad (32)$$

the eqs. (30a, b, c) lead to the following set of equations

$$m \cdot p'(0) = p'(1) \quad (33a)$$

$$(1+m) \cdot p'(i) = m \cdot p'(i-1) + p'(i+1), \quad (33b)$$

$i = 1, 2, \dots, s-1$

$$p'(s) = m \cdot p'(s-1) \quad (33c)$$

with the normalizing condition

$$\sum_{i=0}^s p'(i) = 1 \quad (33d)$$

The equations (33a, b, c) are not independent of each other. If, however, one of these equations, e.g. eq. (33c), is omitted and replaced by eq. (33d), an inhomogeneous set of equations (33a, b, d) is obtained which will be solved in the following section.

3.2. THE APPROXIMATION FORMULA

By summing up eq. (33a) and the first i equations (33b) one obtains (after rearranging terms)

$$m \cdot p'(i) = p'(i+1), \quad (34)$$

$$i = 0, 1, \dots, s-1$$

A multiplication of the first i equations (34) leads to

$$p'(i) = m^i \cdot p'(0), \quad (35)$$

$$i = 0, 1, \dots, s$$

From eqs. (33d) and (35) follows

$$p'(0) = \frac{1}{\sum_{i=0}^s m^i} \quad (36)$$

Regarding that the sum in eq. (36) represents a geometric series one obtains

$$p'(0) = \frac{1-m}{1-m^{s+1}} \quad (37)$$

Inserting eq. (37) in eq. (35) yields

$$p'(i) = \frac{1-m}{1-m^{s+1}} \cdot m^i, \quad (38)$$

$$i = 0, 1, \dots, s$$

With the aid of eq. (38) the probabilities $p(i)$ can be calculated. According to the definition, $p(i)$ means the probability that i waiting places are occupied at the beginning of an arbitrary time slot, under the condition that there are two blocks on the considered time slot h . Consequently for the probability $p(s|2B)$ holds

$$p(s|2B) = p'(s) \quad (39)$$

Inserting eqs. (38) and (39) into eq. (5), the loss probability p_L is easily obtained as

$$p_L = \frac{x_1 \cdot x_2}{N \cdot (x_1 + x_2)} \cdot \frac{(1-m)}{(1-m^{s+1})} \cdot m^s \quad (40)$$

where, according to eqs. (24), (25), (26), (28) and (32),

$$m = \frac{(x_1 - 1) \cdot (x_2 - 1)}{(N - x_1) \cdot (N - x_2)} \quad (41)$$

From eq. (40) follows

$$s = \frac{\ln m}{\ln \left(\frac{p(s|2B)}{1 - m + m \cdot p(s|2B)} \right)} \quad (42)$$

where, according to eq. (5),

$$p(s|2B) = \frac{(x_1 + x_2) \cdot N}{x_1 \cdot x_2} \cdot p_L \quad (43)$$

With the aid of eq. (40) the loss probability p_L can easily be calculated as a function of the number s of waiting places provided. In many cases of application, however, it is convenient that the required number s of waiting places as a function of a prescribed loss probability can also be calculated directly according to the eqs. (42) and (43).

3.3. ACCURACY OF THE APPROXIMATION METHOD

The accuracy of the presented approximation method has been investigated by comparing the results according to the approximate formulae with exact values. In fig. 9 the loss probability is shown as a

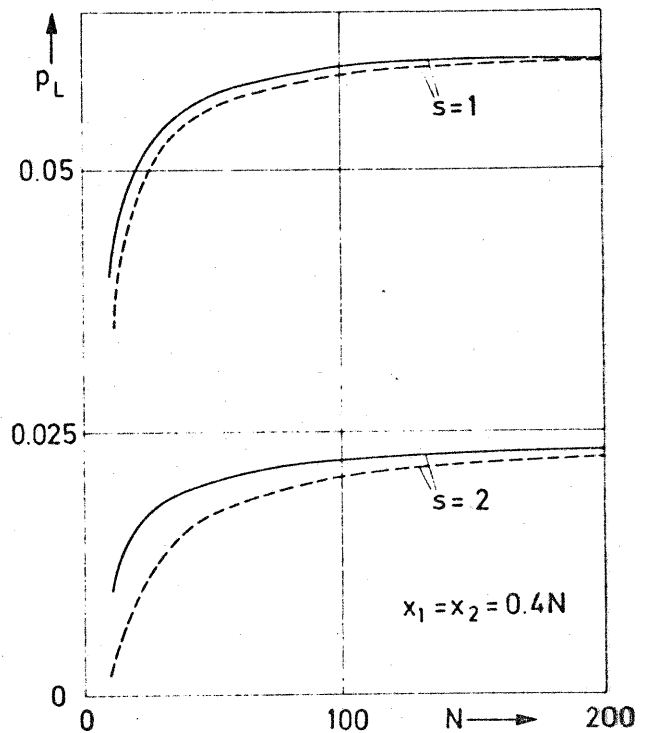


Fig. 9: The loss probability p_L as a function of the number N of time slots per frame for systems with $s = 1$ and $s = 2$ waiting places, respectively. There are $x_1 = x_2 = 0.4 \cdot N$ blocks on each of the incoming trunks in this example; thus about 80 % of all time slots are occupied on the outgoing trunk. In fig. 9 approximate values according to eq. (40) are indicated by a solid line and exact values according to eq. (22) by a dashed line.

From the diagram shown in fig. 9 it can be seen that the approximation method yields results which are a bit too large, i.e. on the "safe side".

For very small systems (e.g. $N < 50$), which are of theoretical interest only, the approximation method is rather rough. Therefore in such cases the use of the exact calculation method is recommended.

For larger systems, however, the approximation method yields results of high accuracy. E.g., in the example illustrated in fig. 9 the relative deviation is less than 5% in case of $s = 2$ and less than 1% in case of $s = 1$ if there are at least $N = 200$ time slots in a frame.

As mentioned in section 1, the main scope for the application of the approximation method derived is represented by systems with rather large frames consisting e.g. of about $N = 10\ 000$ time slots. For systems of this kind the approximation formulae yield results of very high accuracy, as can be seen from comparisons with simulation results (see section 4).

4. SIMULATION

For investigating the accuracy of the approximation method described in section 3 in the case of large systems, a simulation program has been established.

For the example of a system with $N = 10\ 000$ time slots per frame, $s = 2$ waiting places and $x_1 = x_2 = 4\ 000$ blocks on each of the two incoming trunks a simulation yields the loss probability

$$p_L = 0.02357 \pm 0.00149.$$

According to the approximation formula (40) the value

$$p_L = 0.02404$$

is obtained which is in good accordance with the simulation result.

5. RESULTS

In fig. 10 the loss probability p_L (according to the approximation formula derived in section 3) is shown as a function of the number of blocks $x_1 + x_2$, with the number s of waiting places as a parameter. This diagram is valid for a system with $N = 1\ 000$ time

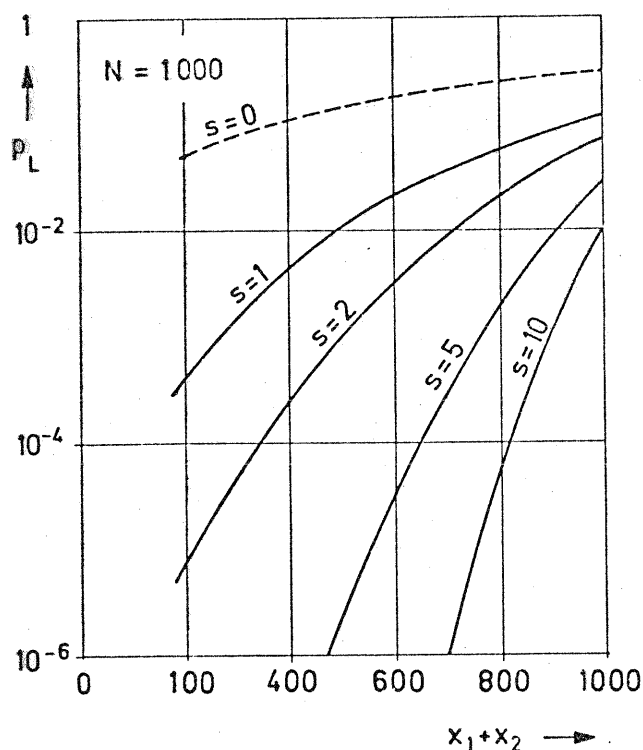


Fig. 10: The loss probability p_L as a function of the total number of blocks $x_1 + x_2$

slots and for the case $x_1 = x_2$ (which is the "worst case" for the loss probability p_L). The formulae (40) and (42), of course, are also valid for the general case $x_1 \neq x_2$.

It can be seen from fig. 10 that about 5 to 15 waiting places are sufficient for realizing a loss probability p_L of about 10^{-6} , if the number of blocks $x_1 + x_2$ per frame is in the range of about $0.5 \cdot N$ to $0.8 \cdot N$.

6. REMARKS

- 1.) The exact and approximate formulae for the loss probability p_L hold true for arbitrary, fixed values x_1 and x_2 , i.e. for a fixed number of blocks on each of the incoming trunks. In many cases of application, however, the expectation p_L^* of the loss probability is also required. For this purpose, first of all the state probabilities $p(x_1, x_2)$ that x_1 time slots are occupied on trunk No. 1 and x_2 on trunk No. 2 are calculated according to usual methods, e.g. the Erlang Formula. Then the expectation value p_L^* can be obtained as a weighted mean by multiplying the loss probabilities p_L for the various pairs of values (x_1, x_2) with the corresponding state probabilities $p(x_1, x_2)$ and summing up all of these products.
- 2.) The formulae derived in section 2 and section 3 are based on the assumption that the blocks on the incoming trunks are distributed at random within the frames. Strictly speaking, this condition is only fulfilled in nodes which are situated in marginal districts of TDMA networks where the arriving blocks have not yet passed any further nodes (and, of course, in networks having only one node).

If the blocks arriving at a node have, however, passed further nodes before, they are no more distributed at random within the frames. This results in a slightly increased loss probability. For such nodes the formulae derived can also be used as an approximation. In this case it is convenient that the approximation method presented (for random distribution of the blocks on the frames) slightly overestimates the loss probability.

- 3.) The calculation methods presented can also be applied to TDMA networks with routing facilities.
- 4.) The total loss probability for blocks which are transmitted through the network on a route consisting of several highway sections can be calculated by summing up the loss probabilities of all nodes which are passed by a considered block. (This is feasible because the loss probabilities of the individual nodes are small, e.g. $p_L \approx 10^{-6}$.)
- 5.) In principle the calculation methods presented here can also be extended to TDMA networks in which more than 3 pairs of highways are interconnected in a node. Such a generalization would, however, lead to formulae which are more complicated.
- 6.) The approximation method derived in section 3 can easily be used in the limiting case $N \rightarrow \infty$. Therefore this approximation method can also be applied for calculating the loss probability in certain synchronized message switching systems with constant holding times (i.e. constant message lengths).

7. CONCLUSION

In this paper a queuing problem in TDMA systems is investigated.

For the loss probability in nodes of such TDMA networks an exact calculation method is derived. As the

numerical evaluation of this method is rather lengthy, an exact calculation is only feasible in case of relatively small systems.

Therefore a simple approximation method for calculating the loss probability is also presented which is well suited for application in practical engineering. The accuracy of the results according to this approximation method is shown by comparison with exact values and simulation results.

8. BIBLIOGRAPHY

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