OPTIMAL DESIGN OF ALTERNATE ROUTING SYSTEMS

by

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Synopsis

Most of the known methods for the economically optimal design of alternate routing systems neglect the variance of overflow traffic, especially in case of gradings (for reasons of simplicity). Therefore in some cases these methods yield results which differ remarkably from the actual economic optimum group sizes.

Some methods - more accurate and laborious ones up to simplified and easier ones - are presented for the calculation of such networks. They regard the variance of overflow traffic, too, and are suitable for groups with full access as well as for gradings.

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1. Introduction

In long distance telephone networks, alternate routing is of great advantage and therefore used in many countries. In these alternate routing systems, long distance calls can be switched via several connecting paths.

A very plain example is shown in Fig. 1. Calls from A to C are first offered to the direct route (high usage route, first route) AC . Calls which cannot be switched via this direct route are overflowing to the alternate route ABC. In this plain

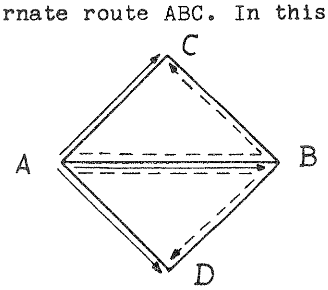

Fig. 1 Example for an alternate routing system

example, there is only one alternate route for calls from A to C, i. e. the alternate route (second route) ABC is also the final route. Similarly, calls from A to D are first offered to the direct group AD. Calls overflowing from this group are offered to the alternate route ABD. Furthermore, calls from A to B may only be switched via the group AB. Then the route AB is an alternate route with respect to calls from A to C or D, but also the only route for calls from A to B. The corresponding graded multiple scheme is shown in Fig. 2.

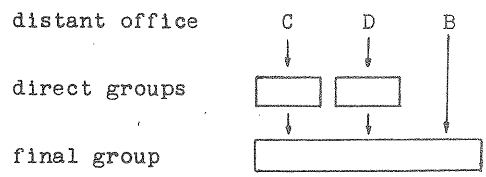

Fig. 2 Graded multiple scheme, corresponding to the system shown in Fig. 1

As can be seen from this example, not only the traffics overflowing from high usage groups can be offered to a final route, but also a direct traffic, for which this route is the only switching possibility. Therefore, if a certain grade of service is required, the call congestion of final groups must not exceed a certain prescribed value, e.g. $B_{fin} = 0{,}01 \mathrel{\hat{=}} 1\ \%$. As a second condition, it can be postulated that the network is as economic as possible, i.e. that the total cost of all trunks is a minimum.

The overflow traffic which is offered to the alternate (final) group has statistical properties different from those of random traffic. Overflow traffic is more peaked than pure chance traffic. The usual loss formulae for random traffic are not adequate for overflow traffic.

In case of full access, groups with offered overflow traffic can be calculated with methods of Wilkinson /6/ and Bretschneider /7/. A method of Lotze and Herzog /1, 2, 4, 5/ also permits the calculation of groups with limited access to which overflow traffic is offered.

In these methods, the mean of the overflow traffic as well as its variance is applied for the calculation of loss. However, the non-randomness of overflow traffic influences not only the calculation of alternate groups but also the sharing of the traffic among the direct and alternate routes.

In most of the known methods for the economically optimal design of alternate routing systems, the variance of overflow

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traffic is neglected, especially in case of gradings. The methods shown here are suitable for the design of alternate routing systems with full or limited access with regard to the variance of overflow traffic, too.

Chapter No. 2 deals with the calculation of groups with overflow input, regarding the variance of this overflow traffic.

Chapter No. 3 in its part A treats engineering methods for the design of alternate routing networks if all offered random traffics are given. In part B it is assumed that only measurements of carried loads are available. By means of tables the offered random traffics as well as non-random overflows can be determined. From that point of the calculation the further design can again proceed according to one of the methods in part A .

2. Calculation of Trunk Groups to which Overflow Traffic Is Offered

For a proper calculation of secondary groups to which overflow traffic is offered, besides the value A of this overflow traffic itself the variance V resp. the variance coefficient D must be considered /13, 14, 15/.

If the traffic offered to a secondary group is composed of several overflow and random traffics, the sum of all means of these traffics be named A_{tot} . The sum of all variances of these traffics be named V_{tot} . Instead of V_{tot} the use of the variance coefficient $D_{tot} = V_{tot} - A_{tot}$ is more convenient for practice.

The following idea holds true for full available groups investigated by R. I. Wilkinson /6/ and G. Bretschneider /7/ as well as for groups with limited access investigated by U. Herzog and A. Lotze /1, 2, 4, 5/ :

Firstly, one single appropriate "equivalent primary group" (A^*, n^*, $k^* \leq n^*$) has to be found, which yields an overflow traffic having the same parameters $\{A_{tot}, D_{tot}\}$.

Secondly, one considers this (fictitious) primary group and the following secondary group (n_{sec}, $k_{sec} \leq n_{sec}$, B_{sec}) to be one single total group only with $n_{tot} = (n^* + n_{sec})$ and $k_{tot} = (k^* + k_{sec})$.

In case of full access for primary and secondary groups obviously holds $k^* = n^*$ and $k_{sec} = n_{sec}$. In case of limited access k^* and n^* are chosen such that n_{tot} , k_{tot} yield a good progressively graded group. By means of such equivalent primary groups one can calculate n_{sec}, if $\{B_{sec}, k_{sec}\}$ are prescribed. Vice versa for prescribed pair $\{n_{sec}, k_{sec}\}$ one can determine B_{sec} . Details hereto can be found in the publications mentioned above, furthermore in the [illegible] ITC paper of U. Herzog /19/ .

If the loss B_{sec} is prescribed, a good approximate value for n_{sec} can easily be obtained by means of graphs found by calculation of a great many cases with the "equivalent primary group" method /1, 2, 16, 19/. It holds (for groups with full or limited access)

$$n_{sec} = n_{rand} + \Delta n \quad , \qquad (1a)$$

where n_{rand} means the number of trunks necessary in case of offered random traffic and Δn the amount of trunks needed additionally (because of the non-randomness of the overflow traffic). The value for n_{rand} can be drawn from usual loss tables (e.g. MPJ tables /2/), for Δn holds

$$\Delta n = \frac{D}{A} \cdot \left[C_1 \; (A - 20) + C_2 \right] . \qquad (1b)$$

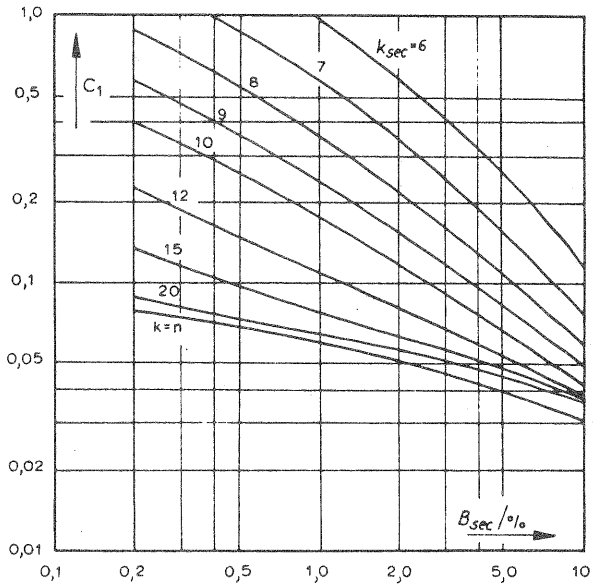

Fig. 3 The coefficient C_1 in (1b)

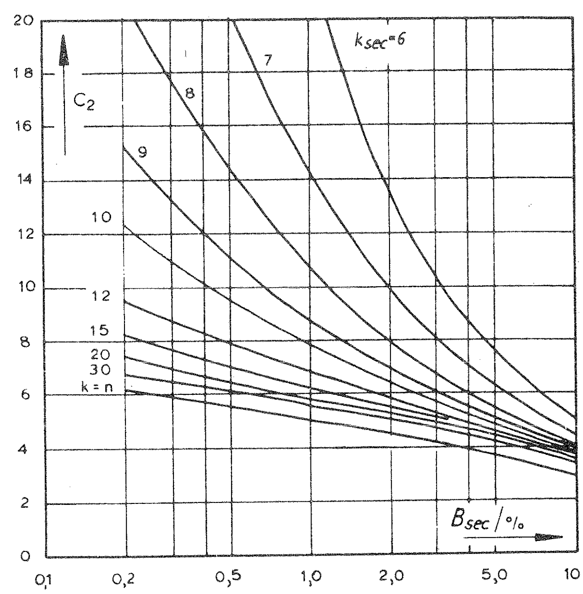

Fig. 4 The coefficient C_2 in (1b)

The coefficients C_1 and C_2 can be drawn from the diagrams shown in Fig.3 and Fig.4 as a function of the loss B_{sec} and the availability k_{sec}.

If the prescribed loss has always the same value, e.g. B_{sec} = 1 per cent, it is convenient to draw n_{sec} directly from a table as shown in Fig.5 instead of calculating with the equations (1a) and (1b). E.g., for A = 25,8 and D = 7,7

k=10	B=1%

n \ D/A	0	0,1	0,2	0,3	0,4	..
..	..	..	..	..	..	..
47	27,1	..	..	25,3	..	..
48	27,7	..	..	25,8	..	..
49	28,4	..	..	26,5	..	..
50	29,0	..	..	27,1	..	..
51	29,6	..	..	27,8	..	..
..	..	..	..	..	..	..

Fig. 5 Table for calculating groups with offered overflow traffic

(thus D/A ≈ 0,3) in case of B_{sec} = 1 % and k_{sec} = 10 the number of trunks necessary is

$$n_{sec} = 48 \quad .$$

3. Methods for Calculating Alternate Routing Systems

A. Methods for Calculating Alternate Routing Systems if the Offered Random Traffics Are Given

Often the random traffics offered to the high usage groups and final groups are given. Then the methods shown in this section can be used. To begin with, the group sizes of all first routes are determined as well as the traffics overflowing from these groups and the corresponding variance coefficients. Now the traffic A_{tot} offered to a second route is known, being the sum of the first routes' overflow traffics and sometimes of additional random traffics. In the same way the corresponding variance coefficient D_{tot} can be easily determined by addition. Knowing these values, the number of trunks in each second route can be determined, and so on.

According to the grade of simplification which is desirable for practical engineering, several methods are presented in the following sections.

AI. Optimal Design of Single-Stage Alternate Routing Systems

This section refers to systems as shown in Fig. 2 or, more generally, Fig. 6 . There are several high usage groups, numbered from 1 to m . To the high usage group No. i (with the availability k_i) a random traffic A_i is offered (i = 1 .. m). R_i represents the traffic overflowing from high usage group No. i to the final group (with the availability k_{fin}), and D_i the variance coefficient of this overflow traffic. Furthermore, a random traffic denoted by A_0 may be offered to the final group directly. B_{fin} is the loss prescribed for the final group. The costs of a traffic channel in the high usage routes are denoted

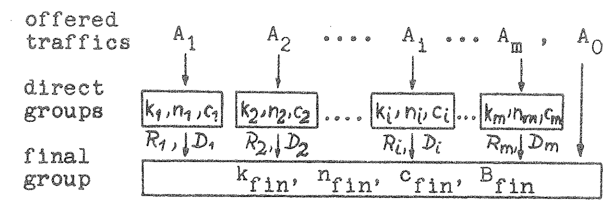

Fig. 6 Graded multiple scheme for a general single-stage alternate routing system.

as c_i ,and the cost of a traffic channel in the final group as c_{fin} . These costs are supposed to include the costs of terminal equipments. All these values are assumed to be given.

We have to calculate the economically optimal numbers of trunks in the high usage routes, n_i , and the number of trunks in the final group, n_{fin}.

The traffic carried on a high usage group with constant offered traffic A_i (and constant availability k_i) increases from Y_i to $Y_i + \Delta Y_i$ if one trunk is added. On the other hand, the carried traffic on a final group with constant loss B_{fin} and constant availability k_{fin} (but not constant A_{tot}) increases from Y_{fin} to $Y_{fin} + \Delta Y_{fin}$ if one trunk is added. The cost ratios be defined as

$$q_i = \frac{c_{fin}}{c_i} \quad , \quad i = 1 .. m \quad .$$

Then the optimum condition, which is found by differentiation, leads to

$$\Delta Y_i = \frac{\Delta Y_{fin}}{q_i} \quad , \quad i = 1 ... m \quad . \qquad (2),(16)$$

This optimization method is shown in detail in the appendix.

The equations (2) yield conditions for determining the high usage group sizes. If ΔY_{fin} is known, the numbers of trunks n_i in the high usage routes can easily (by means of tables) be found such that all ΔY_i have the optimum values $\Delta Y_{fin}/q_i$. The overflowing traffics R_i and their variance coefficients D_i can be drawn from the same table in the same read out.

Now the total traffic $A_{tot} = \{\Sigma R_i + A_0\}$, which is offered to the final group, and its variance coefficient $D_{tot} = \Sigma D_i$ can be calculated.

Knowing A_{tot} and D_{tot}/A_{tot}, the number of trunks necessary for the final group (with prescribed loss B_{fin}) can be drawn from a table as shown in Fig.5 .

Because ΔY_{fin} is not exactly known before the numbers of trunks n_i and n_{fin} have been calculated, an iteration would be necessary for perfect calculation of the group sizes. For practical purposes, however, an iteration is not wanted.

In order to avoid this iteration, an approximate value for ΔY_{fin} can be used. Then the resulting numbers of trunks in many cases will be not the very optimum. But this in practice can always be accepted if the value ΔY_{fin} is carefully chosen, because the cost minimum is very flat (thus, the total costs are very close to the actual optimum).

In the following the procedure of calculating the optimum group sizes with a tabulated value ΔY_{fin} will be treated in detail.

AI1. Most Accurate Method (No.1)

Calculation of ΔY_{fin} . As stated above, the first problem is to get a good approximate value for ΔY_{fin} , i.e. the increase of load on the final group if one trunk is added there. The load Y_{fin} carried on the final group - and therefore ΔY_{fin} , which stands for its derivative - depends on the constant loss B_{fin} , the availability k_{fin} , the number of trunks n_{fin} in the final route as well as on the ratio D_{tot}/A_{tot} of the offered traffic. Examining these dependences, we find:

The loss B_{fin} is a constant value and doesn't need to be regarded. The influence of n_{fin} on ΔY_{fin} is small, especially in larger groups; the availability k_{fin} , however, should be considered (see Fig.7).

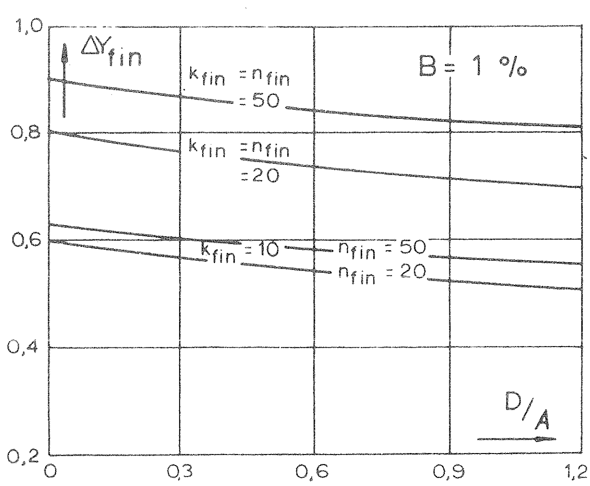

Fig. 7 The value ΔY_{fin}

The D/R ratio of overflow traffic behind gradings (for usual overflow probabilities, e. g. 5 % ÷ 30 %) is mainly influenced by the availability k (of the high usage group), as shown in Fig. 8.

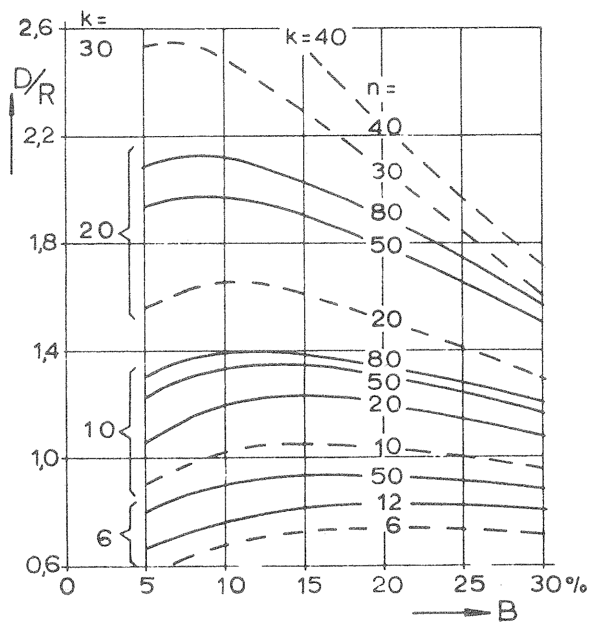

Fig. 8 The D/R ratio

Thus, the ΔY_{fin} values are mainly dependent on the availabilities k_i of the high usage groups and on the availability k_{fin} in the final group.

If the availabilities k_i of all high usage groups are equal ($k_i = k_h$, h standing for "high usage group"), and if there is no random traffic directly offered to the final group, the value for ΔY_{fin} can be drawn from a table as shown in Fig. 9.

B = 1 %

k_h \ k_{fin}	10	20	$k_{fin}=n_{fin}$
0	0,63	0,75	0,85
6	0,55	0,68	0.78
10	0,52	0,66	0,76
20	0,51	0,65	0,75
$k_h=n_h$	0,48	0,62	0,72

Fig. 9 Table for ΔY_{fin}

Otherwise a weighted mean value for ΔY_{fin} can be taken , assuming an overflow of about 20 %. For this purpose, an additional random traffic directly offered to the final group can be regarded as a traffic overflowing from a high usage group with the availability $k_0 = 0$ and $n_0 = 0$ trunks (i. e., $R_0 = A_0$, $D_0 = 0$). Therewith we get

$$\Delta Y_{fin} = \frac{A_0 \cdot \Delta Y_{fin,0} + 0,2 \cdot \sum_{i=1}^{m} A_i \cdot \Delta Y_{fin,i}}{A_0 + 0,2 \cdot \sum_{i=1}^{m} A_i} \qquad (3)$$

(For quick and easy calculations the arithmetic mean could be taken instead of (3)).

The optimum high usage group sizes. According to (2), the size of a high usage group must be calculated such, that ΔY_i has the value

$$\Delta Y_h = \frac{\Delta Y_{fin}}{q_h} \quad .$$

The proper numbers of high usage trunks can be taken from a table as shown in Fig.10.

$k_h = 6$

A \ ΔY_h		...	0,34	0,36	0,38	...
..	.	...	..	..	..	...
..	.	...	..	..	..	...
30	n	...	38	36	35	...
	R	...	4,2	4,9	5,3	...
	D	...	3,8	4,5	4,8	...
..	.	...	..	..	..	...
..	.	...	..	..	..	...
40	n	...	50	49	48	...
	R	...	5,6	6,0	6,3	...
	D	...	5,2	5,4	5,9	...
..	.	...	..	..	..	...

Fig. 10 High usage group table (for first-hunted groups)

Here the optimum number of trunks n_h in a high usage group is listed as a function of ΔY_h and the random traffic A offered to the high usage group (with the availability k_h of the high usage group as a parameter). In this table, below the number of trunks, the corresponding overflow traffic R and its variance coefficient D can also be found by one single read out.

The size of the final group. Adding all the overflow traffics (found in the high usage group table shown in Fig.10), we get the total traffic offered to the final group

$$A_{tot} = \left\{ A_0 + \sum_{i=1}^{m} R_i \right\}$$

and its variance coefficient

$$D_{tot} = \sum_{i=1}^{m} D_i$$

Furthermore the relative variance coefficient D_{tot}/A_{tot} can be calculated, too. It remains to calculate the number of trunks n_{fin} such that the loss in the final group is equal to B_{fin} . This number of trunks can be drawn from the table shown in Fig. 5 as a funktion of the total offered traffic A_{tot}, its D/A ratio D_{tot}/A_{tot} and the availability k_{fin} .

Now all group sizes are calculated.

Example No.1. Let us consider the following example. Be given a system with two high usage routes, as shown in Fig. 11 .

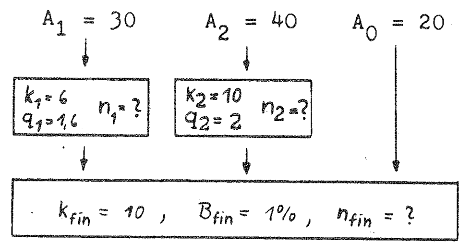

Fig. 11 Graded multiple scheme for example No. 1 and 2

with the values

$A_1 = 30$ Erl., $A_2 = 40$ Erl., $A_0 = 20$ Erl.,
$k_1 = 6$, $k_2 = 10$, $k_0 = 0$,
$q_1 = 1,6$, $q_2 = 2$,
$k_{fin} = 10$, $B_{fin} = 1$ % .

We have to calculate the numbers of trunks n_1 and n_2 in the high usage groups and n_{fin} in the final group.

With these values one can find in the table shown in Fig. 9

$$\Delta Y_{fin,0} = 0,63$$
$$\Delta Y_{fin,1} = 0,55$$
$$\Delta Y_{fin,2} = 0,52 \quad .$$

With equation (3) one obtains

$$\Delta Y_{fin} = \frac{20 \cdot 0,63 + 0,2 \cdot (30 \cdot 0,55 + 40 \cdot 0,52)}{20 + 0,2 \cdot (30 + 40)}$$

$$= 0,59$$

and

$$\Delta Y_1 = 0,59/1,6 = 0,369$$
$$\Delta Y_2 = 0,59/2 = 0,295$$

With $A_1 = 30$ Erl., $k_1 = 6$ and $\Delta Y_h = \Delta Y_1 = 0,36$ we find in the high usage group table shown in Fig. 10

$$n_1 = 36 \; , \; R_1 = 4,9 \; , \; D_1 = 4,5 \; ,$$

and similarly (not shown in Fig.10) for

$k_2 = 10$, $A_2 = 40$ Erl. and $\Delta Y_h = \Delta Y_2 = 0{,}30$

$$n_2 = 52 \quad , R_2 = 2{,}9 \quad , D_2 = 3{,}8 \quad .$$

With these values and with $A_0 = 20$ Erl. we get

$$A_{tot} = 27{,}8 \quad ,$$

$$D_{tot} = 8{,}3 \quad ,$$

$$D_{tot}/A_{tot} \approx 0{,}3 \quad .$$

With $A_{tot} = 27{,}8$, $D_{tot}/A_{tot} = 0{,}3$, $k_{fin} = 10$, $B_{fin} = 1$ % the value n_{fin} can be drawn from the table shown in Fig. 5 as

$$n_{fin} = 51 \quad .$$

(The total cost for these trunks is

$$c_{tot} = c_{fin} \cdot (n_{fin} + \frac{n_1}{q_1} + \frac{n_2}{q_2})$$
$$= 99{,}5 \quad . c_{fin} \quad)$$

AI2. Simplified Method (No.2)

From the table in Fig.9 can be seen that ΔY_{fin} depends mainly on k_{fin} and not so much on k_h. Using only the row of this table for $k_h = 10$ (as a medium value), the value ΔY_{fin} can be combined with the high usage group table shown in Fig.10 . Then a table as shown in Fig. 12 is obtained in which the calculation of the quotient $\Delta Y_h = \Delta Y_{fin}/q$ is also contained implicitly.

$k_h = 6$, $B_{fin} = 1$%

k_{fin}		cost ratio q					
10		..	1,2	1,4	1,6	1,8	..
20		..	1,6	1,8	2,0		..
k=n		..	1,8	2,0		2,5	..
A	.	..	...	...	...	...	..
30	n	..	...	36	38	...	..
	R	..	...	4,9	4,2	...	..
	D	..	...	4,5	3,8	...	..
..	.	..	...	...	...	...	..
40	n	..	...	48	51	...	..
	R	..	...	6,3	5,3	...	..
	D	..	...	5,9	4,9	...	..
..	.	..	...	...	...	...	..

Fig. 12 High usage group table for simplified method No.2

From this table in Fig. 12 the number of trunks in a first hunted high usage group can be read out directly as a function of the high usage group availability k_h, the final group availability k_{fin} , the cost ratio q and the offered random traffic A. The corresponding overflow traffic R and its variance coefficient D can also be drawn in the same read out from the table in Fig. 12. The easy application of this method is shown in the following example.

Example No 2. For comparison, the example for method No. 1 will be calculated once again with method No. 2.

From the high usage group table shown in Fig. 12 with $k_1 = 6$, $k_{fin} = 10$, $q_1 = 1{,}6$, $A_1 = 30$ Erl. we obtain

$$n_1 = 38 \quad , R_1 = 4{,}2 \quad , D_1 = 3{,}8 \quad ,$$

and from a similar sheet with parameter $k_h = 10$ we get with $k_{fin} = 10$ and $q_2 = 2$ and $A_2 = 40$ Erl. the results

$$n_2 = 54 \quad , R_2 = 2{,}3 \quad , D_2 = 3{,}0 \quad .$$

In the next step we obtain

$$A_{tot} = 4{,}2 + 2{,}3 + 20 = 26{,}5 \text{ Erl.} \quad ,$$

$$D_{tot} = 3{,}8 + 3{,}0 = 6{,}8 \quad \text{and}$$

$$D_{tot}/A_{tot} \approx 0{,}3 \quad .$$

Finally we find the number of trunks n_{fin} from the table shown in Fig. 5 as

$$n_{fin} = 49 \quad .$$

The total cost of all trunks turns out to be

$$c_{tot} = c_{fin} \cdot (n_{fin} + \frac{n_1}{q_1} + \frac{n_2}{q_2})$$
$$= 99{,}75 \; c_{fin}$$

(whereas with the method No. 1 $c_{tot} = 99{,}5 \; . \; c_{fin}$ was obtained).

It can be seen that the simplification of method No. 2 is acceptable, because the resulting costs are nearly the same. Therefore, method No. 2 is recommended for single-stage alternate routing systems.

A II. Optimization of Multi - Alternate Routing Systems

In multi-alternate routing systems, calls often can be switched alternately via three or more connecting paths. An example is shown in Fig. 13 . Systems of

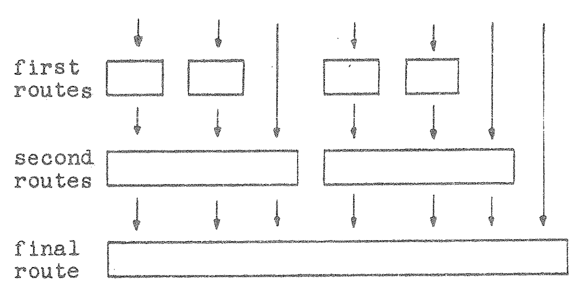

Fig. 13 Multi-alternate routing system

this kind can become very complex. The very optimum of group sizes in such systems cannot be found with reasonable amount of computation. In fact, multi-alternate routing systems are not only used for economics' sake but also for other reasons (flexibility, reliability). Therefore, in this section A II a first way of approximate optimization will be shown. In section AIII a second way is presented yielding economic group sizes.

AII1. Method Using the Actual D/R Ratio of Overflow Traffic

This method is similar to the simplified method No. 2 shown in section A I2. Here the condition is used that the load increase ΔY_h on any high usage route (first route, second route etc. be

$$\Delta Y_h = \Delta Y_{fin} \cdot \frac{c_h}{c_{fin}} = \frac{\Delta Y_{fin}}{q_h}$$

Thus, the calculation of the group sizes of first routes is done exactly as shown in section AI2 by means of the table shown in Fig. 12. This table is valid for offered r a n d o m traffic only.

For the calculation of second, third etc. routes (to which overflow traffic is offered) this table must be modified. The sheets which must be added to the table are shown in Fig. 14. Each sheet

D_{tot}/A_{tot} = 0,2	k_h = ...
	B_{fin} = 1 %

k_{fin}	cost ratio q_h			
10	.. ...	...	...	..
20	.. 1,2	1,4	1,6	..
k=n	.. ...	...	...	..
A	$\left\{ \begin{matrix} n_h \\ R_{ov} \\ D_{ov} \end{matrix} \right\}$			

and similar sheets for D_{tot}/A_{tot}=0,4 ; 0,6 .. 1,8; 2 ; 2,5 and 3 .

Fig. 14 High usage group table for multi-alternate routing systems

is calculated with a constant relative variance coefficient (belonging to the offered overflow traffic A_{tot}), e. g. D_{tot}/A_{tot} = 0,2; 0,4; 0,6; ... 1,8; 2; 2,5; and 3. The table in Fig. 12 forms the special case D_{tot}/A_{tot} = 0.

The method proceeds in the following manner: First, all groups which are first hunted routes only are determined. The numbers of trunks of these groups, the traffics overflowing and the corresponding variance coefficients can be found in the sheet of the table in Fig. 12 by one single read out, just like in section AI2.

Now the total traffic A_{tot} offered to each second route can be calculated (by addition of the offered overflow and random traffics), and also the corresponding variance coefficients D_{tot} . From A_{tot} and D_{tot} follows the ratio D_{tot}/A_{tot}.

The traffic offered to a second route may have the D_{tot}/A_{tot} ratio 0,8. With A_{tot}, q_h and k_h one reads out the number of trunks n_h and the overflow traffic R_{ov} (and its variance coefficient D_{ov}) from the sheet with the head parameter D_{tot}/A_{tot}= 0,8. When all second routes are calculated, the third, fourth etc. routes are determined in the same manner as the second routes. At last, the final group is calculated exactly as shown in the sections AI1 and AI2 by means of the table shown in Fig. 5.

AII2. Method Using an Estimated D/R Ratio of Overflow Traffic

The procedure of determining the group sizes can be simplified if the relative variance coefficient of overflow traffic is estimated. For example, D/R = 2 can be taken as an approximate average relative variance coefficient of overflow traffic.

The numbers of trunks in all first hunted groups and the corresponding overflow traffics can be taken from a table as shown in Fig. 15 (for 0 % overflow traffic).

k_h = ...	0% overflow traffic
B_{fin} = 1%	100% random traffic

k_{fin}	cost ratio q_h			
10	.. ...	...	...	..
20	.. 1,2	1,4	1,6	..
k=n	.. ...	...	...	..
A	$\left\{ \begin{matrix} n \\ R \end{matrix} \right\}$			

and similar sheets for

25 % / 75 %
50 % / 50 %
75 % / 25 %
100 % / 0 %

Fig. 15 Table for calculating first, second etc. routes (simplified)

The total traffic A_{tot} offered to each second route can be calculated by adding all overflow and random traffics offered to each group. The percentage of overflow traffic contained in the total traffic A_{tot}

can be evaluated, too.

Now for each second group the number of trunks and the corresponding traffic R overflowing from this group can be drawn from the table shown in Fig. 15, as a function of the offered traffic A_{tot}, the availabilities k_h and k_{fin} , the cost ratio q_h and as a function of the percentage of overflow traffic at the offered traffic. Then third, fourth etc. routes are calculated similarly.

The number of trunks in the final route can be taken from a table as shown in Fig. 16.

k_{fin} = ...
B_{fin} = 1 %

n_{fin}	percentage of offered overflow traffic: 0%	25%	50%	75%	100%
		A values			

Fig. 16 **Table for simplified determination of final group sizes** (this table is calculated with a relative variance coefficient D/A = 2 for 100 per cent overflow traffic)

A further simplification for provisional enlargement of groups could be made by using a constant overflow ratio, e. g. 75 per cent overflow traffic in case of second and further routes.

AIII. Economic Design of Multi - Alternate Routing Systems (with Constant Overflow Probability)

As stated above, factors as flexibility and reliability play an important part in multi-alternate routing systems. It is often sufficient to provide a high enough average load per trunk. This will always be the case if an overflow probability of B = 20 per cent is applied.

AIII1. Economic Design of Multi - Alternate Routing Systems Using the Actual D/R Ratio of Overflow Traffic

In this method, for all high usage routes (first, second etc. routes) a constant overflow probability is postulated, e. g. B = 20 %.(This is an average overflow probability in case of optimum single-stage alternate routing systems). The final route is calculated as shown above (e. g. B_{fin} = 1 %).

Then, the high usage groups are economic, because with an overflow probability B = 20 %, the average load per trunk is sufficiently high. Because of B_{fin} = 1 %, a sufficient grade of service is granted.

The numbers of trunks of first, second etc. routes, the overflowing traffics R_{ov} and the corresponding variance coefficients D_{ov} can be drawn from a table as shown in Fig. 17.

k_h = ...
B = 20%

D_{tot}/A_{tot} / n	0	0,2	0,4	..	2	2,5	3
			A_{tot}, R_{ov}, D_{ov} triples				

Fig. 17 Table for first, second etc. high usage routes with B = 20 %

The determination of the high usage group sizes proceeds in the same manner as shown in section AII1, using the table shown in Fig. 17, however, instead of the tables in Fig. 12 resp. 14.

At last, the final group is calculated like in section AII1 by means of the table shown in Fig. 5.

This method is recommended for calculating multi-alternate routing systems. It is easy to employ and does not need too much tables.

AIII2. Economic Design of Multi - Alternate Routing Systems Using an Estimated D/R Ratio of Overflow Traffic

The method can be simplified by using an estimated relative variance coefficient for the overflowing traffics. As in section AII2, D/R = 2 can be taken as an appropriate average relative variance coefficient.

Then, the numbers of trunks in first, second etc. routes and the corresponding overflow traffics R can be determined by means of the table shown in Fig. 18.

The method advances like the procedure shown in section AII2, but instead of the table in Fig. 15, the table in Fig. 18 is used.

As in section AII2, a further simplification for the provisional enlargement of groups could be made by using a constant overflow ratio, for instance 75 per cent

k_h = ...
B = 20%

n_h	percentage of offered overflow traffic: 0%	25%	50%	75%	100%
		$\left\{ \begin{matrix} A_{tot} \\ R_{ov} \end{matrix} \right\}$			

Fig. 18 Table for simplified determination of first, second etc. high usage routes (this table is calculated with a relative variance coefficient D/A = 2 for 100 % overflow traffic)

overflow traffic at the total traffics offered to second and further routes.

B. Calculation of the Offered Traffic Values from Measured Loads Carried Only

Designing new systems, the offered random traffics are given in most cases. On the other hand, existing systems must often be enlarged. Then in many cases only the loads carried on the existing system are known by measurements. (The measuring of the offered traffic values is often too difficult or troublesome). Then the following methods can be used for calculating the offered traffics from the measured carried loads.

Having evaluated these offered traffic values, a method of section A can be applied for determining the new group sizes.

B1. Precise Determination of the Offered Traffic Values

It is possible to calculate the offered traffic values precisely, if the numbers of trunks, the availabilities and the carried loads are given. Though this method is somewhat more laborious than the approximate one described below, it can be easily done by means of tables.

Let us assume that the traffics offered to a certain multi-alternate routing system, which has been designed some time ago, have increased in the meantime. Now only the actual (increased) load carried on each group is known by measurements. From these carried loads, from the existing number of trunks and from the availability in each group the offered traffics are to be determined.

The random traffic offered to first routes can be easily drawn from a table as shown in Fig. 19 a. This offered random

k = ...
D_{tot}/n = 0

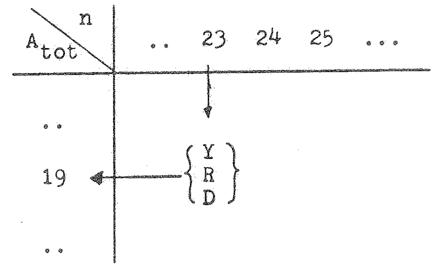

Fig.19a Table for the calculation of offered random traffic from the load carried

Fig.19b Table for the determination of the offered (random plus overflow) traffic A_{tot} from the load carried by means of the D_{tot}/n ratio

Similar tables as in Fig.19a, but with D_{tot}/n = 0,2; 0,4; ... 1,8; 2; 2,5; 3.

traffic is a function of the availability k, the number of trunks n and the carried load Y of this group. The traffic R overflowing from this group and the corresponding variance coefficient D can also be drawn from this table.

The total traffic (A_{tot}, D_{tot}) offered to a second route is

$$A_{tot} = A_0 + \sum^{i} R \qquad (4)$$

$$D_{tot} = 0 + \sum^{i} D \qquad (5)$$

where A_0 be a random traffic offered directly to this second route. This direct traffic A_0 must be determined by means of n, k, $\sum R$, $\sum D$ and the measured carried traffic Y of this route.

This total traffic A_{tot} (random plus overflow) which is offered to a second route can be easily determined by means of the table shown in Fig. 19 b. For each pair (k, D_{tot}/n) one table is provided. With the number n of trunks one looks for the carried load Y in the corresponding column and finds the values R, D of overflow below Y. In the corresponding row the offered traffic A_{tot} (being the sum of random and overflow) can be found (see arrows in Fig. 19).

Then, the random traffic share A_0 of the total traffic A_{tot} can be calculated with (4) as

$$A_0 = A_{tot} - \sum^{i} R$$

By this procedure advancing from the first up to the final route one obtains all the random traffics A_0 offered to the various groups.

Having these values one can apply again the method of section AII1 or AIII1 . In the special case of only one single alternate route the further calculation proceeds as described in section AI1 or AI2 .

B2. Approximate Determination of Offered Traffic Values with Presumed (Constant) D/R Ratio of Overflow Traffic

If the share of overflow traffic at the total offered traffic is estimated, the corresponding total offered traffic value can be drawn from a table as shown in Fig.20 . This table is calculated with a relative variance coefficient D/A = 2 for 100 per cent overflow traffic and can be

k = ...

0% overflow traffic
100% random traffic

A \ n	... 19	20	21	...
..				
23	Y values of (measured) carried traffic			
24				
25				
..				

and similar sheets for

25 % / 75 %
50 % / 50 %
75 % / 25 %
100 % / 0 %

Fig. 20 Table for the determination of the offered combined random and overflow traffic with the (estimated) percentage of overflow traffic

used for first, second etc. as well as for final routes. For first routes, of course, the sheet for 0 per cent overflow traffic and 100 per cent random traffic has to be used.

Subsequently one can use the tables according to Fig.15 in section AII2 (cost ratios regarded) or according to Fig.18 in section AIII2 (constant overflow B = 20 %).

The overflow traffics R in the tables Fig.15 or Fig.18 resp. are not needed for the approximate method described here.

It should be remarked that this simple approximate method dimensions each group separately. From this follows that the new-dimensioning of a group - say hunted in 2nd order - can effect a smaller overflow to another group hunted in 3rd order. Nevertheless this group of 3rd order may possibly be dimensioned according to its measured overloading without respect to the fact that already the enlargement of the 2nd order group could diminish or remove the overload of the following 3rd order group.

As in sections AII2 and AIII2, a further simplification for the provisional enlargement of groups could be made by using a constant overflow ratio, for example 75 per cent overflow traffic in case of second and further routes.

4. Conclusion

This paper gives methods for the design of alternate routing systems, with respect to the variance of overflow traffic in case of full or limited availability.

Several methods with different grades of simplification are presented.

Generally, it is recommended to use those methods which make simplifications on the side of optimizing rather than on the side of regarding the variance.

5. Appendix: Derivation of the Cost Function

The total cost of a network as shown in Fig.6 is

$$c_{tot} = c_{fin} \cdot n_{fin} + \sum_{i=1}^{m} c_i \cdot n_i \qquad (6)$$

This cost function is to be a minimum. The corresponding numbers of trunks in the high usage groups can be obtained by derivation. For an optimal number of trunks n_1 the condition

$$\frac{\partial c_{tot}}{\partial n_1} = 0$$

must be fulfilled. We obtain

$$\frac{\partial c_{tot}}{\partial n_1} = c_{fin} \cdot \frac{\partial n_{fin}}{\partial n_1} + \sum_{i=1}^{m} c_i \cdot \frac{\partial n_i}{\partial n_1} = 0 ,$$

or

$$\frac{\partial c_{tot}}{\partial n_1} = c_{fin} \cdot \frac{\partial n_{fin}}{\partial n_1} + c_1 + \sum_{i=2}^{m} c_i \cdot \frac{\partial n_i}{\partial n_1} = 0.$$

The terms $\frac{\partial n_i}{\partial n_1}$ are equal to zero if i is not equal to 1 , thus

$$\frac{\partial c_{tot}}{\partial n_1} = c_{fin} \cdot \frac{\partial n_{fin}}{\partial n_1} + c_1 = 0 .$$

Similarly, for the optimal numbers of trunks n_i the conditions

$$c_{fin} \cdot \frac{\partial n_{fin}}{\partial n_i} + c_i = 0 \qquad (i=1..m) \qquad (7)$$

must be fulfilled.

Let us now consider the term $\partial n_{fin}/\partial n_i$. It is useful, first to derive n_{fin} with respect to the total traffic

$$A_{tot} = A_0 + \sum_{i=1}^{m} R_i \tag{8}$$

which is offered to the final group:

$$\frac{\partial n_{fin}}{\partial n_i} = \frac{\partial n_{fin}}{\partial A_{tot}} \cdot \frac{\partial A_{tot}}{\partial n_i} \tag{9}$$

(we assume here that n_{fin} is calculated with regard to the correct variance coefficient D_{tot} belonging to the offered traffic A_{tot}, but for the sake of convenience we do not denote this in this derivation).

The load Y_{fin} carried on the final group is

$$Y_{fin} = A_{tot} \cdot (1 - B_{fin}) \quad , \quad \text{or}$$

$$A_{tot} = Y_{fin} / (1 - B_{fin}) \quad .$$

As B_{fin} is constant, holds

$$\frac{\partial A_{tot}}{\partial n_{fin}} = \frac{1}{(1-B_{fin})} \cdot \frac{\partial Y_{fin}}{\partial n_{fin}} \quad , \quad \text{or}$$

$$\frac{\partial n_{fin}}{\partial A_{tot}} = (1-B_{fin}) \cdot \frac{\partial n_{fin}}{\partial Y_{fin}} \tag{10}$$

Concerning the term $\partial A_{tot}/\partial n_i$, it holds

$$\frac{\partial A_{tot}}{\partial n_i} = \underbrace{\frac{\partial R_1}{\partial n_i} + \frac{\partial R_2}{\partial n_i} + \ldots +}_{=0} \frac{\partial R_i}{\partial n_i} \underbrace{+ \ldots + \frac{\partial R_m}{\partial n_i} + \frac{\partial A_0}{\partial n_i}}_{=0}$$

As $\partial R_j/\partial n_i = 0$ if $i \neq j$, we obtain

$$\frac{\partial A_{tot}}{\partial n_i} = \frac{\partial R_i}{\partial n_i} \quad . \tag{11}$$

If the load carried on the high usage group No. i is denoted by Y_i, we get

$$R_i = A_i - Y_i \quad ,$$

and because A_i is constant

$$\frac{\partial R_i}{\partial n_i} = - \frac{\partial Y_i}{\partial n_i} \quad . \tag{12}$$

From the equations (7),(9),(10),(11) and (12) we obtain

$$\frac{\partial c_{tot}}{\partial n_i} = - c_{fin} \cdot (1-B_{fin}) \cdot \frac{\frac{\partial Y_i}{\partial n_i}}{\frac{\partial Y_{fin}}{\partial n_{fin}}} + c_i = 0$$

or

$$\frac{\partial Y_i}{\partial n_i} = \frac{c_i}{c_{fin}} \cdot \frac{1}{(1-B_{fin})} \cdot \frac{\partial Y_{fin}}{\partial n_{fin}} \quad . \tag{13}$$

It is convenient to define the cost ratios q_i as

$$q_i = \frac{c_{fin}}{c_i} \quad , \qquad i = 1 \ldots m \quad . \tag{14}$$

Further, as in the usual loss formulas for random traffic and overflow traffic the numbers of trunks must be integers, it is useful to substitute the differential quotients $\partial Y/\partial n$ in (13) by difference quotients $\Delta Y/\Delta n$, and to set all differences Δn equal to unity. Then we obtain from (13) with (14)

$$\Delta Y_i = \frac{1}{(1-B_{fin})} \cdot \frac{1}{q_i} \cdot \Delta Y_{fin} \tag{15}$$

(ΔY_i means the load increase on high usage group No. i if one trunk is added to n_i, with A_i = const. and k_i = const. ; ΔY_{fin} means the increase of load on the final group if one trunk is added to n_{fin}, with k_{fin} = const. and B_{fin} = const.)

In most cases the loss B_{fin} is very small, e.g. $B_{fin} = 1\%$, thus

$$B_{fin} \ll 1 \quad ,$$

i.e. for practical purposes B_{fin} in (15) can be neglected. In this case we get the following conditions for the optimum numbers of trunks in the high usage groups

$$\boxed{\Delta Y_i = \Delta Y_{fin}/q_i} \quad , \quad i = 1..m \quad (16),(2)$$

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