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Performance Modeling of Networked Control Systems

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***Abstract*—Networked Control Systems (NCS) form a key element of Cyber-Physical Systems (CPS) where sensors, actuators (plants) and controller functions are spatially distributed and interconnected by communication networks. In this contribution models are developed where significant properties of communication networks such as stochastic delays, error control protocols and shared-network influences are embedded in a closed-loop control system. The resulting integrated systems are analyzed by methods of classical system-theoretic and discrete-time state analysis methods as well as by computer simulations. Fundamental insights in the properties of NCSs can already be gained through system-theoretic studies on basic architectures which provide closed-form expressions which are verified and extended by more detailed studies based on tool-supported discrete-time system state analysis and simulation-tool results. The approach allows for a more detailed detection of network protocol impacts on, e.g., the real-time behavior of NCSs when certain Service Level Agreements (SLA) have to be guaranteed as prescribed percentiles of control reaction times.**

Keywords—distributed control systems; discrete-time analysis; embedded systems; network protocol performance evaluation; simulation system theory .

I. Introduction

In classical control applications the components of a system under control ("plant") and its controller, sensors and actuators were located adjacently, interconnected by short wires and implemented mostly by hardware. In modern control systems the controller function is often implemented by computer software and backed by data base support where its components are spatially distributed and interconnected by communication networks. In most cases these networks are not dedicated to that specific application and have to be shared with other applications and where feedback signals and control commands are exchanged by information packets. Typical examples of such "Networked Control Systems" (NCS) are found in so-called "Cyber-Physical Systems" (CPS) for applications within the Smart Grid to control the feeding of renewable energies into the electric power network, for applications of machine-to-machine (M2M) communications in automated production lines, in car-to-car or car-to-environment applications for traffic control, or in medical applications of health surveillance. Many new challenges have appeared with these types of applications as reliability and dependability, security and safety, and, last but not least, real-time performance with respect to guarantees of so-called Service-Level Agreements (SLA) for meeting, e.g., strict reaction threshold levels.

Control system theories are well developed since more than 80 years, described by classical closed-loop transfer function and system-theoretic methods in the time and in the frequency domain. Applications are manifold and have led to enormous extensions with respect to multiple inputs/outputs (MIMO), nonlinear systems, stochastic control, predictions and optimizations. Similarly, communication networks are well developed in particular for integrated services (voice, video, data) based on packet-oriented operation through the Internet, Local Area Networks (LAN), Wireless LANs (WLAN), and Mobile Communication Networks based on standardized multi-layered protocol architectures. Networked Control Systems have gained a high interest in recent development of applications as indicated above requiring distributed control systems. Studies on NCSs have appeared in the technical literature for more than a decade.

Many studies on NCSs represent the network influence by a dedicated single channel and its properties with respect to interference and noise figures, specific protocol functions and shared network loads. For overviews on the general state-of-the-art in this area we refer to [1-3]. In references [4-11] NCSs are analyzed regarding network properties more specifically for different types of networks by the method of discrete-time state analyses under various assumptions on the embedded network by probabilistic packet delays and dropouts resulting mainly on theorems or statements about the stability of the NCS. In [12] a study on multiple control systems is presented which share the up-link channel of a Radio Access Network (RAN) and devise a scheduling algorithm for radio resource allocation. In [13] multiple event-triggered NCSs are studied considering the ALOHA MAC protocol with respect to the control stability. The review of the literature on NCSs reveals that there exists already substantial knowledge on NCSs but consider mostly specific networks which are dedicated to the specific application problem. In this contribution a different approach to the NCS problem will be addressed originating from applications where typical link-layer communication protocols are applied which are modeled precicely by task graphs which are mathematically reduced to a random packet transit time which appears finally as a virtualized service time of a queuing model being embedded in the control loop [14].

The contribution is structured as follows: In Section II the NCS is modeled and the methods of network representation are outlined. In Section III analytic analyses on NCSs are presented for various network models without and with protocol control yielding closed-form results. In Section IV computational performance studies are reported for more realistic control algorithms by discrete-time state analyses and computer simulations, based on the MATLAB tool system Simulink, respectively, to show the effects of networks on the real-time performance of NCSs. The paper concludes with an outlook on possible extensions and ongoing further studies.

II. Modeling Of Networked Control Systems

A. System Model

In Fig. 1 the principal model of a NCS is sketched consisting of the Plant to be controlled by the Actuator A, the Controller C, the Network N (in forward and in backward direction) and a Decider which generates the Controller input *e(t)* from the Reference Signal *r(t)* and the transmitted output signal y*(t).* The controller result is fed back through the reverse path of the network to the Actuator to be applied at the Plant. We assume that both directions of the shared network have identical properties (which can be easily generalized).

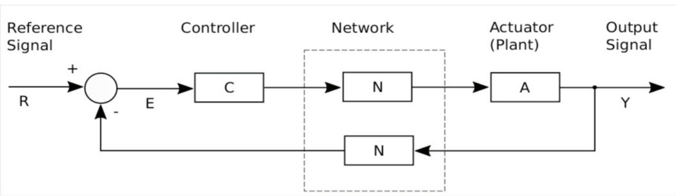

Fig. 1. Principal Model of a Networked Control System

We assume a packet-oriented network, i.e., the output signal *y(t)* is either scanned periodically ("time-driven control") or ignites a state report ("event-driven control") and the signal value is transported digitally within a packet; same happens for the transport of the output signal of the controller to the plant. Output signal values and controller output values are held constant during the sample period *c* (time-driven control) or between event reports by a sample-and-hold function. When the network is represented as a stochastically varying delay all other signals will also become stochastic variables.

For the analytical performance evaluation all elements of the NCS are described in the time domain and in the frequency domain by their Laplace transforms (LT). The *Transfer Function* is defined in the time domain by its impulse response *h(t)* and in the frequency domain by the fraction *H(s)* of the output signal and the input signal LTs, where *s* denotes the complex plane variable. We consider the Reference signal *R(s)*, the system output signal *Y(s)*, and the Controller input signal *E(s)* in the frequency domain and their correspondences in the time domain *r(t)*, *y(t)*, and *e(t)*, respectively. For the Controller we will assume its transfer function *C(s)* of a basic PID controller.

$$C(s) = P + I \cdot s^{-1} + D \cdot s \quad (1a)$$
$$c(t) = LT^{-1}\{C(s)\} \quad (1b)$$
$$H(s) = Y(s)/R(s) \quad (2a)$$
$$h(t) = LT^{-1}\{H(s)\} \quad (2b)$$

The transfer function of the plant *A(s)* can be defined by a second order type network and a constant delay for a backlash. Finally, the network transfer function *N(s)* will be assumed to be consisting of a constant part *d* and a stochastic part. As stochastic part we can apply either a uniform, a negative-exponential, a single-or multi-hump distribution as the Erlang-k, the normal distribution or hyper-deterministic distribution.

B. Network Models

B1. Single Channel without a Logical Link Control Protocol

When the interconnection between the controller and the actuator and between the plant and the decider is by a single channel without an LLC control (but possibly across a shared subnetwork as a LAN, WLAN or Mobile Communication Network without LLC control) the network delay T_N can either be modeled by a constant (propagation) delay T_{PD} when there is no influence of shared use of a subnetwork, or by a series of the constant phase T_{PD} and an independent stochastic phase T_S with probability density function (PDF) $f_S(t)$ for buffering delays with PDF $f_N(t)$ and LT $F_N(s)$, respectively:

$$T_N = T_{PD} + T_S \quad (3a)$$
$$f_N(t) = \delta(t - T_{PD}) \otimes f_s(t) \quad (3b)$$
$$F_N(s) = exp(- T_{PD} \cdot s) \cdot LT\{f_s(t)\} \quad (3c)$$

where $\otimes$ denotes the operation of a convolution integral.

B2. Bi-directional Channel with Link-Layer Protocol

Link layers of communication networks are subdivided into the connection-less Media Access Control (MAC) sublayer (2a) without error control and the connection-oriented Logical Link Control (LLC) sublayer (2b) with error control. For NCS applications in the CPS context we assume that error control is performed at the Link Layer, i.e., by the LLC layer operating over a multi-access network as a LAN, WLAN or Mobile Communication Network. The MAC layer is typically a high-speed channel shared by many users with rather short access delays to save longer connection establishment overhead times. Error control is subjected to the LLC layer where a permanent connection is established for the respective control application. Data units ("Packets") are exchanged at the link layer by encapsulation within data units at the LLC layer ("Frames").

Note 1: For NCSs across larger distances than in case of local networks the communication is likely across the connection-less Internet through "Packets" at the Network Layer where error control is performed through the dominating TCP protocol for Byte-streams at the Transport Layer. In that case the network has to be modeled accordingly, but delays are much larger and amount up to hundred milliseconds and more depending on the currently existing network load. For that reason, the error control by the Transport Layer is not real-time efficient. We suggest to establish an LLC end-to-end control within a permanently established TCP connection. In the current application we use two separate connections in either direction with individual acknowledgements, i.e., no "piggibacking" is used for ACKs.

For LLC error control sliding window protocols are applied as the Stop-and-Wait (SW) protocol with positive/negative acknowledgements (ACK/NAK), with ACK/Timeout (TO) recovery or Selective Repeat (SR). ACK/NAK control may cause deadlocks through frame losses. We assume that link-layer connections are established between the plant and the controller operating under the SW- Protocol with ACK/TO. In [14] a novel method has been developed for the analysis of this class of protocols resulting in an equivalent *Transit time* T_X for a frame, i.e., the time measured from the instant when an arriving or waiting frame is scheduled for transmission until the instant when the reception of an error-free copy is delivered at the Receiver and being acknowledged to the Sender.

Fig. 2 presents a bi-directional communication network model for the interconnection between the Controller and the Plant and between the Plant and the Controller in the reverse direction under ACK/TO control, respectively, from [14].

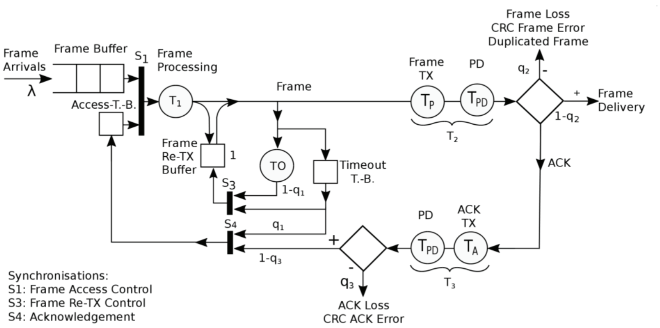

Fig.2. Link Layer Protocol Model for ACK/Timeout (TO) Control

The meaning of the various elements are as follows:

* Frame Buffers for arriving frames and for re-transmissions

* Token Buffers for frame access and for Token return

* Circle Symbols for durations of frame processing (T_1) at the sending side, frame transmission time (T_P), propagation delay (T_{PD}), ACK transmission (T_A), ACK propagation delay (T_{PD}).

* Synchronization Symbols (chosen from Petri Net method) S1 for frame access control, S2 for Token Release after an ACK

* Rhombus Symbol for decision reasons, indicated by probabilities q_2 for frame loss or error, and q_3 for Access Token return, respectively.

* Synchronization symbol S3 for frame retransmission control after a Timeout *TO*.

* Probability q_1 for the expiration of the Timer.

Note 2: The analysis of the LLC protocol in this paper is based on the Transit Time T_X of a frame which ends at the Token return after a successful frame transmission. For the current application the frame is already delivered after a successful reception of the frame at the receiver side, denoted by the frame *Delivery time* T_D, i.e. $T_D = T_X - T_3$. For the delay calculation we use T_X as virtual service time of an equivalent queuing system, while T_D is used within the NCS control loop.

The model describes the whole path of a packet from its admission for transmission out of the Frame Buffer until the successful delivery at the receiving side, including all possible choices of frame retransmissions in case of frame errors, frame losses or Timer expirations and acknowledgements.. We denote this duration as *"Transit time"* T_X. In the considered SW protocol with ACK and Timeout recovery only *one* packet is in transit simultaneously. We will derive the probability density function (PDF) $f_X(t)$ in the next Section and consider this time as a "virtual service time" of a queuing model.

B3. Shared Communication Infrastructures

In most distributed NCSs many applications have to share a common communication infrastructure as a LAN with a high-speed transmission channel at the MAC Layer of a wired or wireless channel, where the channel utilization is composed by the sum of *all* applications and where access delays are caused through the applied MAC channel access protocol. In specific cases as for WLANs acc. to the IEEE Standard 802.11b a "Point Coordination Function" (PCF) is provided by which stations are polled periodically for frame transmissions which could serve for our NCS application. For specific cases a permanently established channel may be provided through the mobile communication networks (MCN). For long distance networks such possibilities don't exist except for the case of a more expensive permanent "Leased Line" connection. For larger network extensions NCSs have to be based on either individual connectionless packet exchanges or on pre-established virtual channel connection between the distant locations by the MPLS (Multi-Protocol Label Switching) method; despite the faster packet forwarding, additional delays have to be suffered due to the dynamic channel sharing. In any case, the network model becomes more complex. In [15] the method of "Reduced Occupancy Approximation" (ROA) has been successfully applied for modeling queuing networks with different priority classes where the shared network is modeled by its remaining average or state-dependent throughput capacity of a virtual single server, which can be applied to model shared use of communication infrastructures in NCSs.

III. Analysis Of Networked Control Systems

A. System-theoretic Solutions

In the classical system-theoretic solution approach each system component is described by its Transfer Function $H(s)$ only. For linear and time-invariant systems the output signal $Y(s)$ is the product of the LT of the input signal $R(s)$ and $H(s)$. The characteristic "Impulse Response" $h(t)$ is the response of the system to the input delta function $\delta(t)$. The System Function. $H(s)$ of the NCS acc. to Fig. 1 amounts to

$$H(s) = \frac{Y(s)}{R(s)} = \frac{P(s)\cdot N(s)\cdot A(s)}{1 + P(s)\cdot N^2(s)\cdot A(s)} \quad (4)$$

Random time delays in NCSs have to be related to the sampling period c for "time-driven control" which depends on the requirements of the specific application. A high sampling rate increases the accuracy of the control but causes a higher communication load and (possibly) delay, and vice versa. The Network Transfer Function $N(s)$ can be modeled either by the Delivery time T_D only or by the the Flow (Response or Sojourn) time T_F of a queuing system of the type D/G/1, where D indicates the constant ("deterministic") arrival process of samples (sample rate λ), G the service time of the network channel, i.e., by T_X, and 1 server for the communication link, respectively. For "event-driven control" the arrival process has to be adapted to the the characteristics of control events and can be modeled in the queuing system either by a type GI when events occur independently, or by type G with correlated events, e.g., by a proper phase-type model [16]. The flow time T_F is the sum of the waiting time T_W in the queue and the delivery time T_D.

A1. System-theoretic Solution for a Channel without LLC

We will illustrate the method for the simplest case of a P-controller with proportional parameter $P(s) = p$ and a linear plant with $A(s) = a$. The network transfer function is either a constant value d, a negative-exponentially distributed random variable with mean $1/\varepsilon$, or the sum of both, i.e., the network transfer functions are either

$$N(s) = \begin{cases} exp(-sd) & (5a) \\ \varepsilon/(s+\varepsilon) = \beta & (5b) \\ \varepsilon/(s+\varepsilon)\cdot exp(-sd) = \beta\cdot exp(-sd) & (5c) \end{cases}$$

From (4) and (5a-c) we get for the three cases (5a,b,c):

$$H(s) = (pa)\cdot exp(-sd)/[1 + (pa)\cdot exp(-s2d)] \quad (6a)$$

$$= (pa)\cdot[exp(-sd) - (pa)\cdot exp(-s3d) + (pa)^2\cdot exp(-s5d) -+ ...]$$

$$= pa\beta / [1 + pa\beta^2] = \frac{pa\varepsilon\,(s+\varepsilon)}{s^2 + 2\varepsilon s + \varepsilon^2(1+pa)}$$

$$= pa\varepsilon\cdot(s+\varepsilon)/[(s-s_1)(s-s_2)] \quad (6b1)$$

where $s_{1,2} = -\varepsilon +/- \varepsilon\sqrt{-pa} = -\varepsilon +/- j\varepsilon\sqrt{pa}$, $\sqrt{-1} = j$
Alternatively, $H(s)$ can also be developed by power expansion resulting in

$$H(s) = ap\beta[1 - \beta^2(ap) + \beta^4(ap)2 - \beta^6(ap)3 + -...] \quad (6b2)$$

$$H(s) = \frac{pa\beta\cdot\cdot exp(-sd)}{1 + pa\beta^2\cdot exp(-s2d)} \quad (6c)$$

$$= pa\beta\cdot exp(-sd) - (ap)^2\beta^3\cdot exp(-s3d) + (ap)^3\beta^5\cdot exp(-s5d)$$

The impulse responses $h(t)$ follow from (6a-c) through the inverse LT, see Eqs. (7a-c), and the unit-step function responses $y(t)$ in Eqs.(8a-c) follow either as response to the input $R(s) = 1/s$ from $H(s)/s$ by the inverse LT or as the convolution of $h(t)$ with the unit step function $u(t)$ by integration which appears as

$$y(t) = \int_{\tau=0}^{\tau=t} h(\tau)d\tau.$$

$$h(t) = (pa)\cdot\delta(t-d) - (pa)^2\cdot\delta(t-3d) + (pa)^3\cdot\delta(t-5d) +-.. \quad (7a)$$

$$h(t) = (pa\varepsilon)\cdot exp(-\varepsilon t)\cdot cos(\varepsilon\sqrt{ap}\,t) \quad (7b1)$$

$$h(t) = (pa)\varepsilon\cdot exp(-\varepsilon[t-d]) - (pa)^2\varepsilon^3(t-3d)^2 exp(-\varepsilon[t-3d])/2$$
$$+ (pa)^3\varepsilon^5(t-5d)^4 exp(-\varepsilon[t-5d])/24 - + ... \quad (7c)$$

$$y(t) = (pa)\cdot u(t-d) - (pa)^2\cdot u(t-3d) + (pa)^3\cdot u(t-5d) - +. \quad (8a)$$

$$y(t) = pa/(1+pa)^{-1}\cdot\bullet \quad (8b1)$$
$$\bullet\{1 + exp(-\varepsilon t)\cdot[cos(\varepsilon\sqrt{ap}\cdot t) - \sqrt{pa}\cdot\cdot sin(\varepsilon\sqrt{pa}\,t)]\}$$

$$y(t) = (pa)\cdot\{1-exp(-\varepsilon[t-d])\}\cdot u(t-d)$$
$$- (pa)^2\{1-exp(-\varepsilon[t-3d] - \varepsilon(t-3d)\cdot exp(-\varepsilon(t-3d)$$
$$- \varepsilon^2(t-3d)^2\cdot exp(-\varepsilon[t-3d])/2\}\cdot u(t-3d) + - ... \quad (8c)$$

A2 System-theoretic Solution for a Channel with LLC

Following [14] the random Transit time $T_X(n)$ for the Stop-and-Wait (SW) protocol with positive acknowledgement (ACK) and Timeout Control for $n-1$ frame re-transmissions (as in Fig. 2) is

$$T_X(n) = T_1 + (n-1)\cdot TO + (T_2 + T_3 | T_2 + T_3 \le TO) \quad (9a)$$

where n indicates the random number of frame transmission cycles appearing with probability p_n and average $E[N]$

$$p_n = P\{N=n\} = q_F^{n-1}(1-q_F) \quad \text{for } n = 1,2,... \tag{9b}$$

$$E[N] = 1/(1-q_F) \tag{9c}$$

where $q_F = 1 - (1-q_2)(1-q_3)$; q_2 and q_3 are the error probabilities for a packet frame and an acknowledgement frame, respectively. A frame has to be retransmitted after a Timeout with probability q_1

$$q_1 = P\{T_2 + T_3 > TO\} = 1 - \int_{t=0}^{TO} f_2(t) \otimes f_3(t)dt \tag{9d}$$

Regarding the rules for conditional probability density functions (PDF) the PDF $f_x(t)$ of T_X amounts to

$$f_x(t) = f_1(t) \otimes \sum_{n=1}^{\infty} p_n \cdot \delta(t-[n-1]\cdot TO) \otimes$$

$$\otimes f_2(t) \otimes f_3(t) \cdot [1-u(t-TO)] /(1-q1)$$

$$= (1-q_F) \cdot \sum_{i=1}^{\infty} q_F^{i-1} \cdot \delta(t-[t_1+t_0+(i-1)\cdot TO]) + ... \tag{10a}$$

where $t_0 = t_P + t_A + 2T_{PD}$. In the current application the frame lengths of samples and acknowledgements are constant. Under these assumptions the average and the ordinary moments are:

$$E[T_X] = t_1 + t_0 + TO \cdot q_F /(1-q_F) \tag{10b}$$

$$E[T_X^n] = (1-q_F) \cdot \sum_{i=1}^{\infty} q_F^{i-1}[t_1+t_0+(i-1)TO]^n \tag{10c}$$

Analogously, the expressions for the Delivery time T_D can be found for n and for *all* frame transitions, respectively:

$$T_D(n) = T_1 + (n-1)\cdot TO + (T_2 \mid T_2 + T_3 < TO) \tag{11a}$$

$$f_D(t) = f_1(t) \otimes f_2(t) \otimes \cdot \sum_{n=1}^{\infty} p_n \cdot \delta([n-1]\cdot TO) \tag{11b}$$

$$\Phi_D(s) = \Phi_1(s) \cdot \Phi_2(s) \cdot (1-q_F) \cdot \sum_{n=1}^{\infty} q_F^{n-1} \cdot exp(-[n-1]TO \cdot s)$$

$$= (1-q_F) \cdot exp(-[t_1+t_2]s) \;/\; [1 - exp(-TO \cdot s)] \tag{11c}$$

$$= (1-q_F) \cdot \{exp(-[t_1+t_2]s) + q_F exp(-[t_1+t_2+TO]s) + ...\}$$

$$f_D(t) = (1-q_F) \cdot \{\delta(t-[t_1+t_2]) + q_F \cdot \delta(t-[t_1+t_2+TO])$$

$$+ q_F^2 \cdot \delta(t-[t_1+t_2+2TO]) + ...\}$$

$$= (1-q_F) \cdot \sum_{i=0}^{\infty} q_F^{\,i} \delta(t-[t_1+t_2+i \cdot TO]) \tag{11d}$$

$$F_D(t) = (1-q_F) \cdot \sum_{i=0}^{\infty} q_F^{\,i} u(t-[t_1+t_2+i \cdot TO]) \tag{11e}$$

$$E[T_D^n] = (1-q_F) \cdot \sum_{i=0}^{\infty} q_F^{\,i} \cdot [t_1+t_2+i \cdot TO]^n \tag{11f}$$

Finally, the flow time T_F of an arriving frame until its delivery at the Receiver is the sum of the waiting time T_W and the Delivery time T_D; as the delivery time is independent of the waiting time its PDF is the convolution between the PDFs of the waiting time and the delivery time.

Based on the results for the frame transit time T_X and the frame delivery time T_D , which reflect the specific protocol effects, the delay analysis of the network channel can now be analyzed by a queuing system of the Type GI/G/1, where GI indicates the type of arrival process (*G*enerally distributed, ndependent) of system state samples, G the type of the virtual service time T_X of a single server model *(n=1)* as derived above. In case of "time-driven control" GI is a deterministic (D) process with constant inter-arrival times, while in the case of an "event-driven control" GI is replaced by G which reflects properties of the control application. This process can be of any type GI (when events occur independently of each other). In control applications it is likely that successive samples are correlated; in that case the arrival process can be modeled by a phase-type stochastic process which is well-known from queuing theory [16]..

The queuing model analysis is based on the virtual service time T_X and frame delivery time T_D as derived above either exactly or by two-moment approximations and provides typical performance metrics as

- W probability of waiting of an arriving frame
- $w = E[T_W]$ mean of the stochastic waiting time T_W
- $t_W = w/W$ mean waiting time of delayed frames
- $W^c(>t)/W = P\{T_W > t\}$ complementary DF of the waiting time of waiting frames $(T_W \mid T_W > 0)$
- $f_W(t)$ PDF of the waiting time T_W
- $F^c(t) = P\{T_F > t\}$ complementary DF of the flow time, where
- $f_F(t) = f_w(t) \otimes f_D(t)$ PDF of the flow time.

A3. Discussion of Analytical Results

Analytical results are advantageous as the influence of system model parameters can be directly valued, while results of computational algorithms (as, e.g., control system state analysis in discrete time and simulations) are more compute time intensive and require systematic parametric studies. The classical control system analysis in the frequency domain depends heavily on the feasibility of the inverse Laplace transformation. We have, therefore, limited our first study to linear controller and plant functions and leave the analysis of more complex systems to tool-supported examples (Sect. IV).

A3.1 Asymptotic Control Performance

Based on the NCS *System function* $H(s) = s\cdot Y(s)$ of eq.(4) and eqs.(6a-c) the two asymptotic theorems of the Laplace Transform provide an immediate inside of the NCS behavior of the unit-step function response $y(t)$ for $t \rightarrow 0$ and $t \rightarrow \infty$:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} H(s) \quad \text{Final Value Theorem} \quad (12a)$$

$$\lim_{t\to 0} y(t) = \lim_{s\to\infty} H(s) \quad \text{Initial Value Theorem} \quad (12b)$$

From Eqs.(12a,b) we can immediately find the parameter conditions of the asymptotic stability and on the closeness of the output from the unit-step function response.

A3.2 Unit Step Response Settling

From the explicit unit-step response functions of eqs.(8a-c) we can immediately see how quickly the amplitudes attenuate around the steady-state level with increasing time which gives an insight into the response overshooting and the settling behavior, which is important when real-time service-level agreements (SLA) exist as delay percentiles on the reaction of the control to sudden changes of the plant by external impacts or on the reference value.

A3.3 Protocol Influence on Network Delays

We will consider a link-layer connection operating under the Send-and-Wait Protocol with ACK/TO Recovery as treated in Section IIIA2. If the communication channel is not shared with other applications, we can restrict ourselves on individual delivery times T_D without waiting, at least when the sample time c is significantly larger than the network delay. Eq.(11e) provides the DF of the network delay T_D. The main protocol influence parameters are the constant times t_1, t_2, and TO for frame processing, frame forwarding, and Timer, respectively, and the probability q_F for frame errors or losses. From Eq.(11e) can be concluded that the settling pattern follows a periodic repetition at multiples of the Timer value TO, where the amplitude is attenuated by a power law (q_F^i) which results mainly from the constant delay parts. In case of an exponential network delay the amplitude attenuation would follow the quicker exponential law, cf. Eq.(8b).

IV. NCS PERFPRMANCE STUDIES

In this Section two examples will be discussed to see how our suggested methods can be applied. In the first example we study more complex control and system functions using MATLAB Simulink tools [18]. In the second example we study the influences of network protocol effects an the real-time behavior of the NCS [17].

A. Network without LLC Control

We consider a NCS for a DC Servo Engine with second-order system function $A(s) = 1000/(s^2+s)$ and a PID-controller with system function $C(s) = P + I/s + D\cdot s$, where $P = 0.21$, $I = 0.344$ and $D = 0.03$. The network is modeled by (a) a constant delay $\tau = \{2.5, 5.0, 7.5, 10.0\ ms\}$ and (b) by four different delay distributions of the types "constant", "uniform" between $0 \ldots 20\ ms$, "negative-exponential", and "shifted exponential" with constant delay 5 and $E[T_N] = 10\ ms$.

Figs. 3 and 4 show the unit-step responses of the NCS for various constant delays, and with respect to four different network delay distributions, respectively, from which we can draw the following Conclusions:

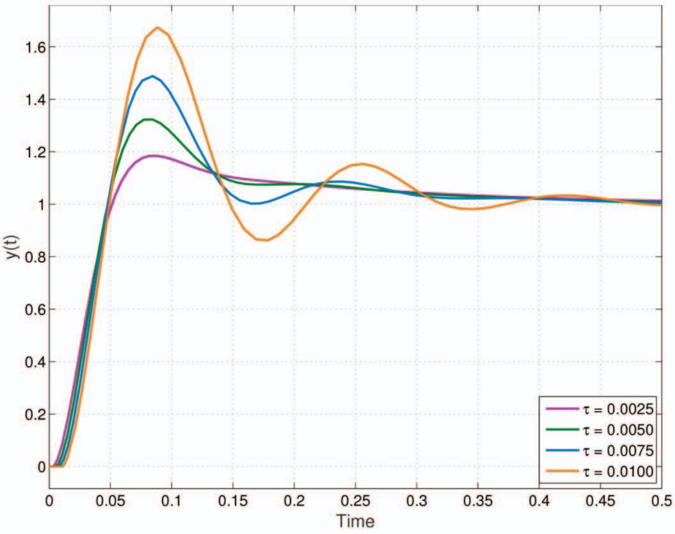

Fig. 3. Unit-step Responses for Four Constant Delays Indicated in the Insert (Unit of Time = 1 ms)

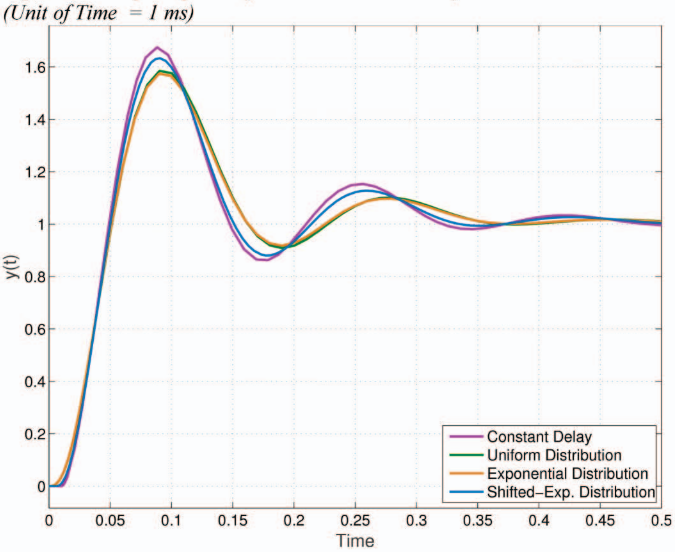

Fig. 4. Unit-step Responses for Four Network Delay Type Distributions Indicated in the Insert (Unit of Time = 1 ms)

Discussion:

1. The *mean network delay* is the dominant factor for the NCS responses as it determines the system response with respect to the overshooting and the settling time (Fig.3).
2. The network *delay distributions* have a minor effect on the control performance (Fig.4).

B. Network with LLC Protocol SW with ACK/Timeout Control

In this example we consider the LLC protocol with ACK/TO Control according to the model of Fig.2 and its analysis in Section IIIA2. The analysis provides exact results for this protocol. In the control context only the aggregated flow time T_F is of importance measured between the instant of either, when a new sensor sample arrives (either as time-triggered or as event-triggered signal) from the plant and the instant of delivery of that signal at the Decider/Controller or from the Controller to its delivery at the Actuator, respectively. The LLC protocol analysis results are expressed by the mean flow time $t_F = E[T_F]$ and the coefficient of variation c_F, where c_F is defined by $c_F^2 = E[T_F^2]/ E[T_F]^2 - 1$.

Table 1 provides the numerical results of the performance evaluation of the SW protocol with ACK/TO control for different load levels ranging from λ = 0.1 to 0.75 frames per time unit and 1 server. Frames arrive acc. to a Poisson process as example for an event-driven control application. In the upper line the analytic values are given; in the lower line (*italic* fond) the results of computer simulations show the close agreement between our exact analysis and the independent computer simulation using the time-true event-by-event simulation.

Table 1. Results of the ACK/TO SW Protocol Analysis Together with Computer Simulation Results

SW Protocol with Ack/Timeout Control						
λ/n		0.10	0.30	0.50	0.70	0.75
ρ		0.127	0.380	0.633	0.887	0.950
$E[T_X]$	analytical	1.267	1.267	1.267	1.267	1.267
	simulation	1.266	1.267	1.267	1.267	1.267
$E[T_W]$	analytical	0.108	0.455	1.283	5.812	14.117
	simulation	0.108	0.457	1.290	5.856	15.093
t_D	analytical	0.851	1.198	2.026	6.556	14.860
	simulation	0.853	1.210	2.034	6.660	14.346
c_D	analytical	0.867	0.907	0.946	0.984	0.993
	simulation	0.872	0.911	0.949	0.988	0.995
$E[T_F]$	analytical	0.894	1.241	2.069	6.599	14.903
	simulation	0.894	1.244	2.076	6.642	15.133
c_F	analytical	0.731	0.831	0.912	0.976	0.990
	simulation	0.733	0.833	0.914	0.980	0.992

In the attached Fig. 5 the complementary DF (CDF) of the flow time T_F of the ACK/TO protocol is shown for the 5 load cases in semi-logarithmic scale together with the results of our computer simulation. The CDF is constructed from the first

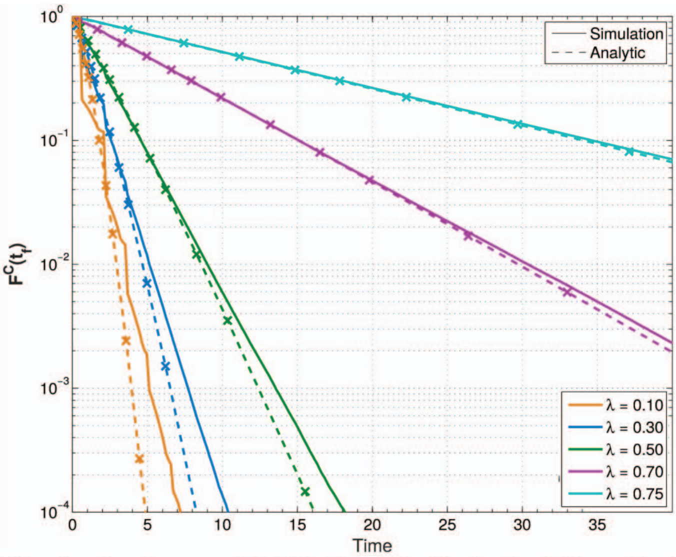

Fig. 5. Complementary DF of the Flow Time T_F for Five Different Load Levels together with Simulation Results (Unit of Time = 1 ms)

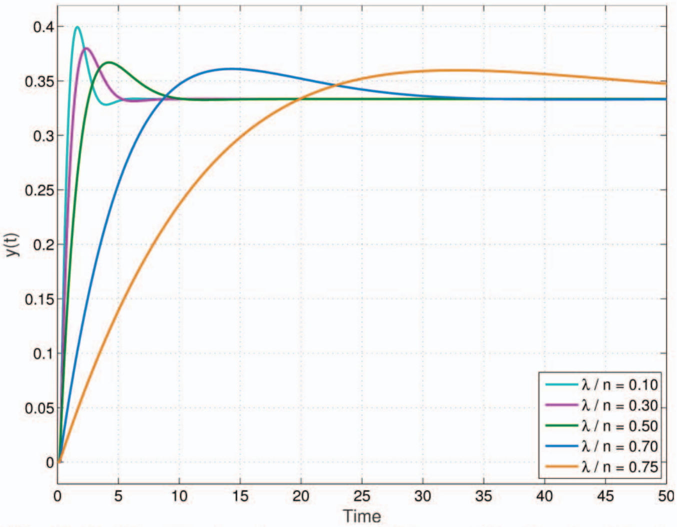

Fig. 6. Unit-Step Function Responses o LLC Protocol for Five Load Levels λ for n = 1 Server (Unit of Time = 1 ms)

and second moments (i.e., by the mean t_F and the coefficient of variation c_F) of the flow time T_F, constructed by use of the Weibull CDF using tabled results from [19]. As before, the results show the close agreements between our theoretical results and simulations. From Fig 5 one can read-off the threshold values for service level agreements on the control reactions as percentiles *p*, e,g., *p = 0.05, 0,02, or 0.01*, i.e., for the threshold times t_{Th} which should be exceeded only with the percentile probability *p*, or, in other words, that a real-time SLA is guaranteed with probability *1-p*. The flow time DF cannot be inserted directly in the Simulink tool. As $c_F^2 < 1.0$ in our example, the DF of the flow time could be approximated by a series of a constant part *d* and an negative-exponential part with average *1/ε*; the two parameters follow from the two parameters t_F and c_F as $d = t_F(1-c_F)$ and $\varepsilon^{-1} = t_F \cdot c_F$. Fig. 6 shows the final results of the aggregated NCS for a network which is represented by a shifted exponential delay, where the constant and the exponential parts agree in the first and second moment with the results of the LLC protocol performance analysis.

Discussion:
1. The network channel load causes additional delays which are directly reflected in the delay of a transmitted sample from the plant with corresponding settling times.
2. Overshooting is highest for the smallest network load, but settles to a stable value fastest. The shape of the response resembles quite close with the principal NCS behavior of Fig.3.

V. Conclusions And Outlook

In this contribution a method has been proposed to extend NCS analyses to more realistic models for the network part through a novel approach to aggregate the network function behavior by an equivalent stochastic delay function which is embedded in the control loop. This method reduces the system complexity and opens the way to extensions and adaptations to other future challenges. Closed-form results were derived for generic network models without specific protocol functions by standard control theoretic approaches which allows a principal explanation of network effects within a control loop. For more complex models the classical control theory analysis depends on the feasibility of inverse Laplace transformation, but is advantageous through tool-supported state-based analysis methods .

Our current studies are continued with respect to a more effective method to consider shared network use for applications where dedicated networks are too expensive, i.e., to shared Local Computer Networks, Mobile 5G Communications, and the Internet where multi-layer architectures and their mutual interactions have to be considered simultaneously. A practical problem has been observed when aggregated results of specific network analyses have to be used as input to control analysis tools which seem to lack on flexibility. Studies on the optimization of protocol *and* control parameter settings for meeting strict real-time service level agreements are in progress.

References

[1] A. Seuret, L. Hetel, J. Daafous, K.H. Johanssen (Eds.), "Delays and Networked Control Systems",Springer Verlag Series "Advances in Delays and Dynamics", Vol. 6, 2016.

[2] R.A. Gupta, M-Y. Chow, "Networked Control System: Overview and Research Trends", IEEE Trans. on Industrial Electronics, Vol. 57, No.7, pp. 2527 - 2535, 2010.

[3] S. Zampieri, "Trends in Networked Control Systems", Proc. 17 th World Congress IFAC, Seoul, Korea, July 6 -11, 2008.

[4] K. Liu, E. Friedman, K.H. Johansson, "Discrete-Time Networked Control under Scheduling Protocols", Chapter 9 of [1], pp. 151 - 165.

[5] Y. Tipsuwan, M-Y. Chow, "Control Methodologies in Networked Control Systems", Control Engineering Practice, Vol. 11, Issue 10, pp.1099 - 1111, 2003.

[6] G.P.Liu, X. Mu, D. Rees, S,C. Chai,"Design and Stability Analysis of Networked Control Systems with Random Communication Time Delay Using the Modified MPC", Int. J. of Control, Vol. 79, No. 4, pp. 288 - 29, 2006..

[7] L. Zhang, D.Hristu-Varsakelis, "Communication and Control Co-design for Networked Control Systems", Elsevier Automatica, Vol. 42, Issue 6, pp. 953 - 958, 2006.

[8] H. Li, Z. Sun, M-Y. Chow, B. Chen, " State feedback Controller Design of Networked Control Systems with Time Delay and Packet Dropout", Proc. 17th World Congress IFAC, Seoul, Korea, July 6 - 11, 2008.

[9] R. Blind, F. Allgöwer, "On the Stabilizability of Continuous-time Systems over a Packet Based Communication System with Loss and Delay", Proc. 18th World Congress IFAC, Cape Town, South Africa, pp. 6466 - 6471, 2014.

[10] J. Nilsson, "Real-Time Control Systems with Delays",PhD Dissertation, Lund Institute of Technology, Lund, Sweden, 1998.

[11] Dong, W-J.Kim, "Markov-Chain-Based Output Feedback Control for Stabilization of Networked Control Systems with Random Time Delays and Packet Losses", Int. J. of Control, Automation, and Systems, 10, pp.1013 - 1022, 2012.

[12] M. Vilgelm, O.Ayan, S.Zoppi, W. Kellerer, "Control-aware Uplink Resource Allocation for Cyber-Physical Systems in Wireless Networks", European Wireless Conf., Dresden, Germany, VDE Verlag, 2017.

[13] M. Vilgelm, M.H.Mamduhi, W. Kellerer, S. Hirche, "Adaptive Decentralized MAC for Event-Triggered Networked ControlSystems", 19th ACM Int. Conf. on Hybrid Systems: Computation and Control (HSCC), Vienna, Austria, April 12 - 14, 2017.

[14] P.J. Kuehn, "Communication Protocol Modeling and Real-Time Performance Analysis by Task Graph Reductions". Forthcoming Paper (submitted).

[15] P.J. Kuehn, W. Schmitt,"Transit Delay Distributions in Priority Queuing Networks", Proc. 11 th International Teletraffic Congress (ITC), Kyoto, Japan, Sept. 4 - 9, Paper 3.4A-3-1, 1985.

[16] H. Kobayashi, B.L. Mark,"System Modeling and Analysis- Foundations of System Performance Evaluation,Pearson Education International, 2009.

[17] Fei Li, "Analysis of Real-Time Performance of Communication Protocols", Master Thesis, University of Stuttgart, 2017.

[18] Siyuan Cao, "Performance Modeling of Networked Control Systems", Master Thesis, University of Stuttgart, 2017.

[19] P.J.Kuehn, "Tables on Delay Systems", Institute of Switching and Data Technics, University of Stuttgart,1976.