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Institute of Communication Networks and Computer Engineering
University of Stuttgart
Pfaffenwaldring 47, D-70569 Stuttgart, Germany
Phone: ++49-711-685-68026, Fax: ++49-711-685-67983
Email: mail@ikr.uni-stuttgart.de, http://www.ikr.uni-stuttgart.de

A novel sampling method for the spatial frequencies of sinusoid-based shadowing models

Siegfried Klein
Bell Labs Germany
Stuttgart, Germany
siegfried.klein@alcatel-lucent.de

Serkan Uygungelen and Christian M. Mueller
Universität Stuttgart, Institute of Communication Networks
and Computer Engineering, Stuttgart, Germany
serkanuygungelen@yahoo.com, christian.mueller@ikr.uni-stuttgart.de

***Abstract*—Careful modeling of the radio channel characteristics is an important issue for system level simulations. While a number of detailed path loss, shadowing and fast fading models can be found in literature, many of them entail large computational efforts and hence lead to long simulation times, or require huge amounts of memory. In this paper, we focus on shadowing models with two-dimensional correlation properties. We present handover performance evaluations as a use case from which requirements to the modeling of the shadow fading are derived. For different implementations of the shadow fading in system level simulations, we discuss their pros and cons and then focus on the sum of sinusoid model proposed by Cai and Giannakis [1]. Our main contribution lies in a new frequency sampling method to determine the coefficients of the sinusoidal waveforms. We show that this new method, denoted as Power Sampling Method (PSM), allows for a significant reduction of the number of sinusoids while the correlation properties of the shadow fading channel are preserved.**

I. Introduction

Radio propagation models are an important component in system level simulations of cellular networks. In this paper, we focus on the modeling of the shadow fading properties of the radio channel. Shadow fading is a spatial phenomenon caused by large obstacles in the radio propagation paths, such as terrain features, buildings, trees, etc. Its effect can be characterized by a lognormally distributed variation of the channel gain with zero mean [2], [3]. Therefore, it is also referred to as *lognormal shadowing*. In a logarithmic scale, this translates to a Gaussian normal distribution. For 3GPP Long Term Evolution (LTE) simulations, typical values for a macro-cell scenario are a standard deviation of 8 dB and a correlation coefficient of 0.5 at a distance of 50m [4].

From the perspective of a moving user, shadow fading leads to a variation of the channel gain over time. Therefore, it is often modeled as a one-dimensional function of time with certain correlation properties for adjacent shadow fading values. A well-known correlation model is the Gudmundson model [5], which has been recommended by the Next Generation Mobile Networks (NGMN) alliance for system level simulations [6].

However, a one-dimensional model does not reflect the spatial correlation properties of the shadowing values. If a terminal moves and thereby enters the shadow area of an obstacle, then the signal strength of its associated base station might decrease, which in-turn might lead to a handover to another cell. If another terminal now comes to a very close position, it is reasonable to expect a similar behavior. In a one-dimensional (time-based) shadowing model, these two terminals might experience completely different shadowing processes, which is not realistic. The authors of [7] have reported a significant difference for results of handover studies, depending on whether a one-dimensional shadowing model or a spatially correlated model is used. Among the list of use cases for self-organizing networks published by the NGMN [8], further problems can be identified where the spatial correlation properties have to be taken into account. For instance, algorithms to detect and compensate for radio coverage problems or algorithms for Neighbor Cell List optimization in LTE will likely be based on feedback of mobile terminals and thus require an accurate modeling of the spatial characteristics of the shadow fading processes. Using spatially-correlated shadowing models however involves computationally complex calculations and can significantly increase simulation time. If traces are used to speed up the actual simulation, memory requirements can become a limiting factor.

The remainder of the paper is organized as follows: In section II, we briefly discuss different implementation possibilities of spatially correlated shadowing models for system level simulations. We then present system level simulations of handover performance as a use case, which shows how the implementation of the shadow fading can impact the channel measurements. The requirements derived from this use case favor an implementation of the shadow fading as a continuous process. Therefore, in section III, we briefly summarize the sum of sinusoids (SoS) model proposed by Cai and Giannakis [1]. Our main contribution is contained in section IV, where we present a new method to determine the parameters of the SoS model. This method allows for a significant reduction of the number of sinusoids and hence reduces computation time. Finally, in section V, we quantify this reduction and show that the correlation properties of the shadowing values are preserved.

II. Implementation aspects of shadow fading models

On the way from the shadow fading's abstract representation as a stochastic process to a computer executable program, the main issues are the compliance of the computer program

with its abstract model, the availability of computing resources as memory or processing power, and the ease of use of the generated program.

A. Shadow fading models for system level simulations

In principle, there are two different models for the implementation of spatial correlated shadowing into a simulation, the matrix model and the SoS model. In the matrix model, e.g. as proposed in [9], a map or grid of shadowing values with a given resolution is generated. The grid values are the result of a two-dimensional convolution of a matrix of independent identical distributed random variables and another matrix containing the auto-correlation coefficients, such that the resulting matrix exhibits the desired correlation properties. In a multi-site scenario, usually one shadowing map is generated per base station, to account for their different (but correlated) shadowing properties. As it will be shown in the next section, the resolution of these maps has to be carefully selected. On the one hand, a coarse grid might not provide sufficient accuracy. Furthermore, there are undesired effects due to discontinuities at the borders of the grid elements. Interpolation is difficult because it changes the correlation properties. On the other hand, the memory requirements of a very fine grid might not be feasible anymore.

In the SoS model, the correlated Gaussian random variables are generated by summation of a finite number of sinusoids of different frequencies, amplitudes and phases [1]. The computational cost of this model is high. Depending on the requirements of the application, it requires a large number of sinusoids (around 1000 according to [1], [7]) to be calculated and summed up for the calculation of each shadow fading value. Its main advantage is the continuous representation of the stochastic process and its ease of use.

As it has been suggested by [7], the computational complexity of the SoS model might be overcome if traces are used. This is based on the assumption that the simulation can be split up into two phases, and that the second phase is executed more often than the first. The first phase monitors the movement of the mobiles in the simulation area and records the shadowing values towards all base stations. In the second phase, the traces are replayed and provide the position of the user together with its shadow fading values. The drawback of this approach is that the traces need to be recalculated every time the placement of the users, their mobility parameters or another relevant system parameter changes.

In the main part of this paper, we will therefore focus on the SoS model and assume that traces are not used. Our goal is to decrease the computational effort by a reduction of the number of sinusoids.

B. Effects of grid-based models on channel measurements

This section illustrates the influence of a grid-based shadow fading implementation on the signal of the simulated channel and relates it to handover performance.

The channel model is composed of pathloss, shadow fading and fast fading, with shadow fading being modelled as a 1D process with a standard deviation of $\sigma = 10dB$ and a decorrelation distance $d_c = 20m$, as described in [3] (section D.4). The subcomponents of the named model are sampled individually at a basic time step of 50ms. The signal $S_{w/o}$ is calculated for each sample by adding its associated pathloss, shadow fading and fast fading values and subtracting their sum from the transmission power, which is assumed to be 46dBm. The effect of a mobile moving through a grid of shadow fading values is modeled by extending the update interval of the shadow fading to the grid points. For a mobile moving at a speed of 3km/h and a grid with a sampling distance of 1m, this means an update interval of 1.2s for shadowing value. In simulations, it is common that a signal is not only calculated at the times defined by the sampling distance, but also in between; i.e. interpolation is required here.

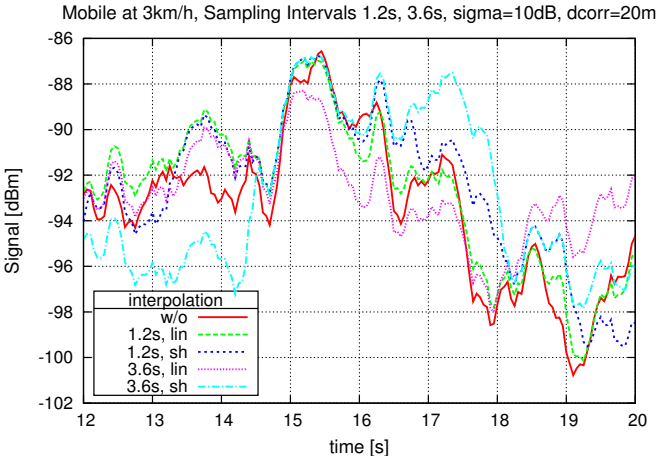

Fig. 1. Comparison of different update intervals and interpolations

Figure 1 shows the behaviour of the standardized ideal 1D process [3] [5], $S_{w/o}$, the measured signal with sample-and-hold interpolation, S_{sh}, and the measured signal for linear interpolation, S_{lin}. All signals are shown for sampling intervals of 1.2s and 3.6s; i.e. a sampling distance of 1m and 3m. In Fig. 1, deviations of the measured signal of more than 2dB for the 1m grid and more than 4dB for the 3m grid from $S_{w/o}$ can be observed. In addition, $S_{w/o}$ also shows the highest dynamics. As the handover procedure usually selects a server for a mobile based on the strongest received signal and due to the observed effect of the sampling interval on the signal, an impact on the handover performance is expected.

III. Sum of Sinusoids Model

In this section we summarize the SoS model presented in [1]. A stationary ergodic stochastic process $s(x, y)$ is characterized by its auto-correlation function (ACF). In the SoS model the sinusoids are selected from the Fourier transform of the ACF. The summation of a finite number of sinusoids N yields an approximation of $s(x, y)$, which can be given to

$$\hat{s}(x,y) = \sum_{n=1}^{N} c_n \cos(2\pi(f_{x,n} \cdot x + f_{y,n} \cdot y) + \theta_n) \quad (1)$$

Here, c_n are the amplitudes of these sinusoids. $f_{x,n}$ and $f_{y,n}$ are the spatial frequencies, and θ_n are uniformly distributed random variables over $[0, 2\pi)$.

In [1], three different sampling methods to determine the amplitudes and spatial frequencies have been proposed, among which the Monte Carlo Method (MCM) yields the highest accuracy. MCM introduces a certain randomness in the selection of the parameters of the sinusoids. The spatial frequencies f_x and f_y are now rewritten to

$$f_x = f_r \cos(\xi), f_y = f_r \sin(\xi), \quad (2)$$

where ξ is a uniformly distributed random variable in $[-\pi/2, \pi/2)$. The spatial frequencies f_r are replaced by a random variable F_r with the cumulative distribution function:

$$\mathrm{CDF}_{F_r}(f_r) = 1 - \frac{a}{\sqrt{a^2 + 4\pi^2 f_r^2}} \quad (3)$$

Here, a is an environment dependent constant. It is derived from the decorrelation distance d_c and it is calculated as $\ln(2)/d_c$. The decorrelation distance denotes the distance at which the value of the auto-correlation function of the stochastic process drops to 0.5. By inverting CDF_{F_r}, the f_r are generated as:

$$f_r = \mathrm{CDF}_{F_r}^{-1}(u) = a/2\pi \sqrt{\frac{1}{(1-u)^2} - 1} \quad (4)$$

where u is uniformly distributed over the range $[0, 1)$. The amplitudes $\{c_n\}_{n=1}^{N}$ of (1) are all equal to $\sqrt{2/N}$.

IV. Power Sampling Method

In the SoS model, it is up to the user to select the number of sinusoids N, which influences both, the accuracy and the computational cost of the implementation. Since the c_n values are constant, the performance of the generated channel mainly depends on the selection of the spatial frequencies f_x and f_y, respectively f_r. In MCM, the spatial frequencies f_r are chosen by random. For a large number of N, the empirical CDF constructed from the set $\{f_r\}_{n=1}^{N}$ converges to the CDF_{F_r} in (3). As stated by [1], [7], the number of sinusoid N required to achieve satisfying correlation properties is more than 1000. This leads to a significant computational effort.

We will show that if the parameters of the sinusoids are carefully selected, N can be reduced. We denote this new selection method as the Power Sampling Method (PSM). The key idea of PSM is that, in contrast to the MCM, the spatial frequencies f_r are not chosen by random but derived from the power spectrum of the theoretical stochastic process $s(x, y)$. The details of the PSM are described in the following sections.

A. Representation of $\hat{s}(x, y)$ in terms of power

The average power of a given periodic signal $x(t)$ with period T_0 can be expressed as:

$$P_x = 1/T_0 \int_{-T_0/2}^{+T_0/2} x^2(t)\mathrm{dt} \quad (5)$$

If $x(t)$ is represented as a Fourier series expansion, then, according to Parseval's theorem, the power P_x can be calculated by the summation of the powers of all sinusoids [10]. The average normalized power of each sinusoid $s(t) = A\cos(2\pi ft)$ with period T_s and amplitude A is:

$$P_s = 1/T_s \int_{-T_s/2}^{+T_s/2} A^2 \cos^2(2\pi ft)\mathrm{dt} = A^2/2 \quad (6)$$

Therefore, by just knowing the amplitudes of the sinusoids, the power of a signal that is represented by a Fourier series expansion can be calculated. It can also be shown that the power of $x(t)$ at a given frequency value f can be found by calculating the power of the sinusoids with frequency f.

If equation (1) for $\hat{s}(x, y)$ is rewritten with the help of (2) in terms of $f_{r,n}$, a similar approach can be developed:

$$\hat{s}(x,y) = \sum_{n=1}^{N} c_n \cos(2\pi f_{r,n}(\cos(\xi_n)x + \sin(\xi_n)y) + \theta_n) \quad (7)$$

The term $(\cos(\xi_n)x + \sin(\xi_n)y)$ translates into a distance, which is independent of ξ_n and equation (7) can be interpreted as the Fourier series expansion of $\hat{s}(x, y)$ with the $f_{r,n}$ as the frequencies and the c_n as the amplitude values.

Since the amplitudes c_n are constant, each sinusoid has a power of $c_n^2/2$. Hence, it is possible to plot the cumulative power of $\hat{s}(x, y)$ as a function of $f_{r,n}$. Keeping this idea in mind, it can be argued that the CDF_{F_r} of (3) is the same as the function of the normalized cumulative power of $s(x, y)$ as a function of f_r. As a result, the power of $\hat{s}(x, y)$ with respect to f_r can be given to:

$$G_s(f_r) = 1 - \frac{a}{\sqrt{a^2 + 4\pi^2 f_r^2}} \quad (8)$$

where $G_s(f_r)$ is the theoretical CDF of the average power of $s(x, y)$ from 0 to f_r. A plot of this function for a decorrelation distance of 20m is given in Fig. 2.

This argument is supported by the analysis of a channel generated according to the MCM of [1]. As an example, if a channel is generated from 1000 sinusoids with a decorrelation distance of 20m, according to (3), half of the $f_{r,n}$ values should be smaller than 0.0096. The c_n values of all sinusoids are equal to $\sqrt{2/1000}$ and every sinusoid thus has the same power of $1/1000$. According to (8), half of the total power of $s(x, y)$ is then located in the frequency range below 0.0096.

In order to get a good approximation of the stochastic process $s(x, y)$, it can be concluded that the normalized cumulative power that is calculated from the selected $f_{r,n}$ values, denoted as $G_{\hat{s}}(f_r)$, should approximate $G_s(f_r)$.

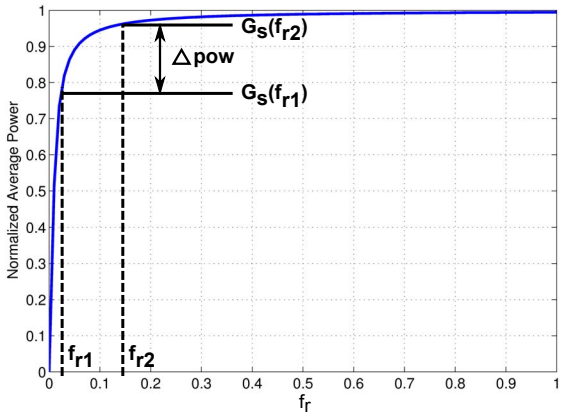

Fig. 2. Normalized cumulative power distribution $G_s(f_r)$

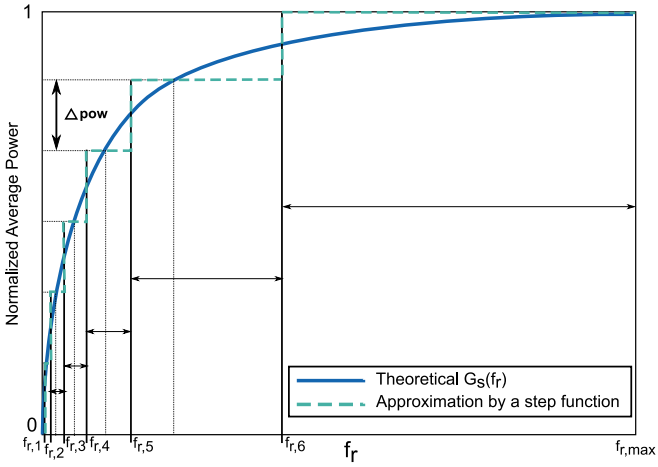

Fig. 3. Approximation of $G_s(f_r)$ in the PSM

B. Relation between $f_{r,n}$ and c_n

Figure 2 shows the function $G_s(f_r)$ for a decorrelation distance of 20m. $G_s(f_{r1})$ is the value of the power of $s(x, y)$ obtained by the total power of sinusoids whose $f_{r,n}$ values are smaller than f_{r1}. Accordingly, the power of $s(x, y)$ over the region $[f_{r1}, f_{r2})$ is:

$$\Delta\text{pow} = G_s(f_{r2}) - G_s(f_{r1}) \tag{9}$$

If the $f_{r,n}$ values of the sinusoids that constitute $\hat{s}(x, y)$ are generated in a way that there is only one f_r value in the region $[f_{r1}, f_{r2})$, then its amplitude c_n should be chosen to match the overall power in this region. Using equations (6) and (9), the amplitude c_n of the corresponding sinusoid can thus be calculated to

$$c_n = \sqrt{2 \cdot \Delta\text{pow}} \tag{10}$$

In our previous example, if the $f_{r,n}$ variables are generated in a way that there is only one f_r value in the region [0,0.0096), then the power of the sinusoid will be:

$$\Delta\text{pow} = G_s(0.0096) - G_s(0) = 0.5 \tag{11}$$

Therefore, the c_n of the corresponding sinusoid should be adapted in a way that its power equals 0.5. Consequently, according to (10), its amplitude has to be chosen to $c_n = 1$.

C. New approach to the selection of $f_{r,n}$ and c_n

In order to obtain better performance, the $f_{r,n}$ and c_n values of the sinusoids should be calculated in a way that the error between the $G_{\hat{s}}(f_r)$ of the generated sinusoids and the thoretical $G_s(f_r)$ becomes minimal. First, the codomain of $G_s(f_r)$ is divided into N equidistant intervals, where N is the number of sinusoids that shall be used to approximate $s(x, y)$. The size of the interval determines the power Δpow of the corresponding sinusoids. The amplitudes c_n can then be calculated from Δpow_i using (10). Second, the $f_{r,n}$ values of the sinusoids are chosen to be at the center of each interval. Figure 3 illustrates the selection of the $f_{r,n}$ and c_n values according to our PSM.

As an example, we conducted simulations with 200 sinusoids, with $f_{r,n}$ and c_n values calculated as follows: The codomain of $G_s(f_r)$ is divided into 200 equal parts, that means each sinusoid has a power value of $\Delta\text{pow}_i = 1/200$. c_n is calculated using (10). As we know the starting and end values of each region that are $G_s(f_{r,LowerBound})$ and $G_s(f_{r,UpperBound})$ respectively, we can find the corresponding $f_{r,n}$ of the region using the inverse of the equation (8) as $G_s^{-1}((G_s(f_{r,LowerBound}) + G_s(f_{r,UpperBound}))/2)$.

V. Numerical Results

In this section, the computational effort and the accuracy of MCM and PSM are compared.

The comparison of the computational effort is straightforward - it directly depends on the number of sinusoids that need to be calculated to determine the shadowing value at a given position. In a simple simulation the signal-to-interference ratio of a mobile moving for one hour with 3km/h through an area defined by 19 hexagonally shaped cells was calculated every 200ms. 20 shadowing values were calculated at each position update of the mobile - one per cell and a common value to correlate the shadowing of all cells. We measured an execution time of 799s for the MCM with 1058 sinusoids and 141s for PSM with 200 sinusoids on an Intel E5420 processor with optimized g++ compiler settings (-g0 -O3). These values are in line with our expectations.

The accuracy of the implementation is judged by comparison to the performance of the defining stochastic process' ACF, i.e. the theoretical ACF. Here, the correlation model of [5] will serve as a reference.

There are two cases to distinguish: short- and long distances. For long distances, a good compliance within about two times the decorrelation distance is considered sufficient for system level simulations. Towards the other end of the scale, the minimum required distance can be calculated from the user's speed and the system's minimum time interval. Typical values

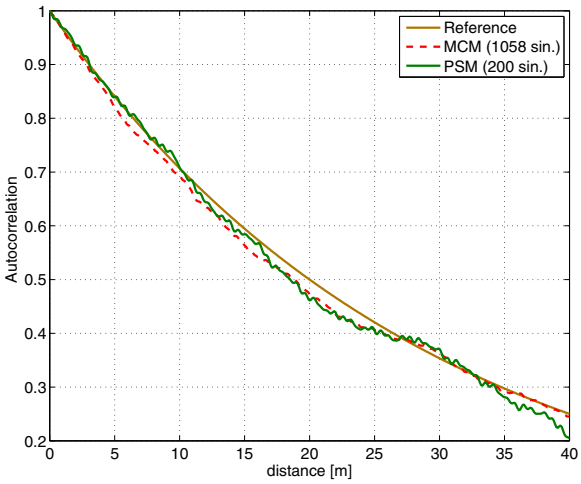

Fig. 4. Reference and resulting ACF

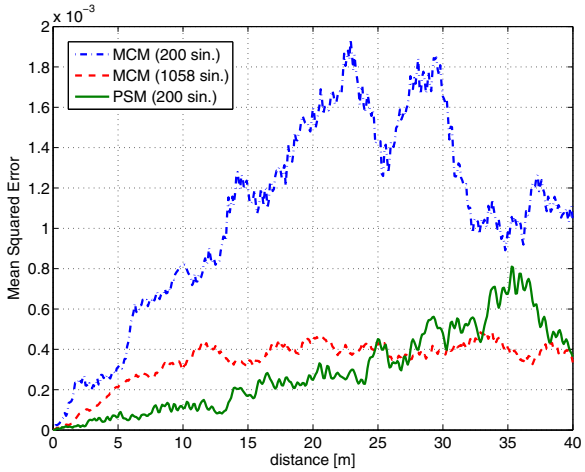

Fig. 5. MSE of both methods for short distances

for LTE system level simulations are a user speed of 3km/h and a system time step of 1ms, i.e. the minimum distance is 8.3e-4 m. Of course, these values have to be adapted according to the given simulation environment.

All examples given in this section assume a decorrelation distance d_c of 20m. The examples using the MCM are generated using Matlab as described in [1]. The parameters of PSM are selected as described in subsection IV-C.

Figure 4 shows the one-dimensional ACF of both, the PSM and the reference ACF. Over the entire range shown in the diagram, a good correspondence between both can be observed.

In order to assess the deviation of the implementations from the reference process more systematically, the mean squared error between the generated process' ACF and the reference ACF is used. Figure 5 compares the mean squared error for the different implementation methods and number of sinusoids. The diagram shows that the error of MCM with 1058 sinusoids and PSM with 200 sinusoids is comparable, whereas the error of the MCM with only 200 sinusoids is much higher.

Concluding, it can be stated that the implementation of PSM achieves a similar accuracy as MCM, but allows for a reduction of the number of sinusoids by a factor of five. Due to the lower number of sinusoids, this directly results in a five times lower computation time for the shadowing values. As an example, in our system level simulations for handover performance studies, by using PSM we achieved a speed up of the overall simulation time by a factor of about two.

VI. Conclusion

We have presented the Power Sampling Method (PSM) for the parametrization of the well established sum-of-sinusoids model for the implementation of a spatially correlated shadow fading channel. In comparison to the parameter selection using the Monte-Carlo Method (MCM) that was proposed in [1], this new method significantly reduces the required number of sinusoids and hence also the computational complexity. This is achieved by a selection of the spatial frequencies and amplitudes of the sinusoids according to the power profile of the stochastic process.

In the example presented in the numerical results section, a clear performance gain is shown by achieving the same level of accuracy with the use of less number of sinosuids in PSM method as compared to MCM method.

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