

PPL — A Reliable Method for the Calculation of Point-To-Point Loss in Link Systems

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ABSTRACT

This paper belongs to a 3-paper-study presented at the 8th ITC; the other two papers deal with one sided link systems / 1 /, as well as with the comparison between the point-to-point selection mode versus the point-to-group selection mode / 2 /.

This paper here, being the basis for the other two papers, presents the new PPL-method for the calculation of the point-to-point loss in two-sided link systems. It uses quite a new way of solution basing on the derivation of an effective accessibility from a starting to a destination multiple. The calculated results for many various structures of link systems with $S=3,4,5$, and 6 stages are in good agreement with results obtained by simulation. Regardless of the easy programmability, a selection of design diagrams is included. They allow the direct design of a cross-point-saving link system with prescribed number of lines, carried traffic, and PP-loss by reading off one set of parameters from the diagram.

1. INTRODUCTION

During the last decade, a large number of interesting and valuable studies of the approximate calculation of the point-to-point loss (PPL) in link systems have been published. Most of the known approximate methods base on the fundamental graph method according to C.Y. Lee /36/. Further two studies became known basing on the idea of an average accessibility /20, 35/. Both yield a tool not yet sufficient. Therefore, as a rule, it was still impossible to obtain satisfactory results for the multistage link system structures without extensive simulation runs.

The authors apologize for not being able to quote the innumerable interesting publications on this topic. The list of references can therefore give only a selection of typical works.

In this paper another way of solution is developed. The PPL-method uses a similar idea as the method CLIGS (Calculation of Link Systems with Group Selection) published at the 7th ITC, Stockholm, 1973 /7,8/.

This new PPL-method yields a handy tool for field engineers to calculate the PP-loss with fairly good accuracy (see Chapter 4).

The following Chapter 5 outlines how to design PP-link systems simply and economically basing on the PPL-method. The analytical derivation of PPL is given in Chapter 6.

2. DEFINITIONS

2.1 Selection Modes

If an incoming call is to be switched through a link system to a trunk of a certain outgoing group, two strategies have to be distinguished regarding the selection procedure from the calling inlet to an idle outlet.

- Point-to-Group Selection (PGS-Mode)

Each call offered to an idle inlet in the first stage can hunt all accessible idle trunks of the desired group behind the last stage.

- Point-to-Point Selection (PPS-Mode)

If a call is offered to an idle inlet in the

first stage, an idle outlet of the desired outgoing group is determined. As a second step, the marker has to find a chain of idle links leading from the calling inlet to the a priori determined outlet of the desired trunk group. For economic reasons, many existing PP systems allow also second or more attempts, respectively, for a certain percentage of calls. The calculation method PPL considers the first attempt loss only.

2.2 Types of Traffic

PCT1 (Pure Chance Traffic of Type 1)

An infinite number of sources produces the offered traffic with the mean value A . The total call rate is constant and independent of the number of busy sources.

PCT2 (Pure Chance Traffic of Type 2)

A finite number of sources per multiple (being e.g. equal to the number of inlets per multiple) produces the offered traffic. Each idle source has the same constant call rate α . The idle times per source are negative exponentially distributed.

In both cases (PCT1, PCT2), the distribution of holding times is assumed to be negative exponential with the mean value of h_m .

2.3 Point-to-Point Loss

In publications, different definitions for the point-to-point loss can be found. Throughout this paper, point-to-point loss is defined as follows: Upon arrival at an idle inlet, a call suffers a point-to-point loss, if no chain of idle links through the link system can be found, provided at least one outlet to the desired outgoing group is still idle. Only those calls contribute to the arrival rate that find one inlet idle as well as at least one outlet of the desired outgoing trunk group idle. According to this definition, the point-to-point loss is defined as

$$B_{pp} = \text{lost calls/offered calls.}$$

3. SURVEY OF THE CONSIDERED STRUCTURES

In this chapter, a survey is given on the link system structures being investigated in the following sections.

For the time being the PPL-method is still restricted to symmetrical structures having the same average traffic load per multiple of the first and the last stage, respectively. Expansion between the inlets and outlets in the first stage multiples and analogously concentration between inlets and outlets of the last stage multiples are admissible.

The following notations are applied:

S	number of stages
i_j	inlets per multiple in stage j
k_j	outlets per multiple in stage j
g_{lj}	number of multiples belonging to a link block
g_{bj}	number of multiples belonging to a group of link blocks
$l_{j,j+1}$	average number of links from each multiple in stage j to each multiple within the link block or to each link block or to each group of link blocks in stage $j+1$, ($j=1..S-1$).

3.1 Structures with Three Stages

Figure 1 shows the notations applied to the considered 3-stage link systems. In all studied 3-stage link systems there exists one link between each multiple of the following stages.

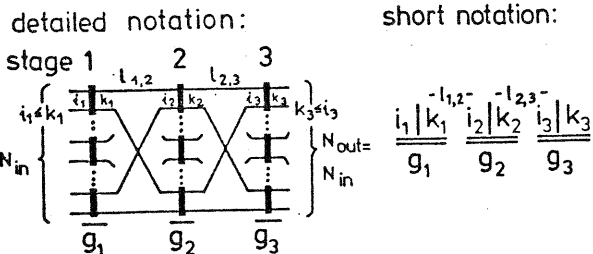


Fig. 1: Notations for 3-stage link systems

3.2 Structures with Four Stages

Figure 2 shows the notations applied to 4-stage link systems with single linkage (SL-systems). Single linkage implies $l_{j,j+1}=1$ ($j=1,2,3$).

All 4-stage link systems are wired with link blocks between stage 1-2 and 3-4.

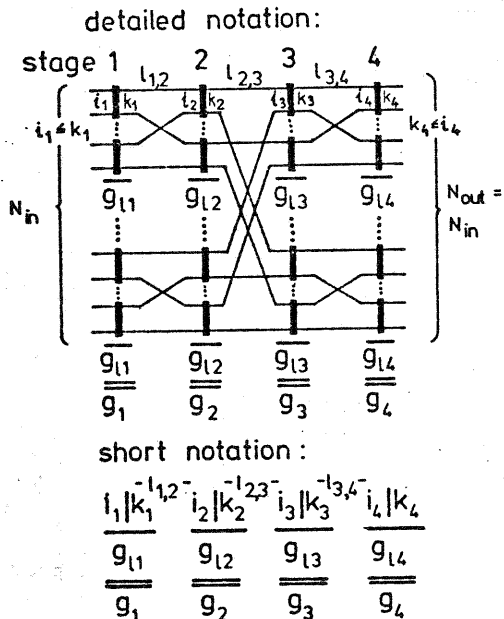


Fig. 2: Notations for 4-stage link systems

Besides SL-systems, also systems with $l_{j,j+1}=2,3$, etc. have been studied. Two types of multilinkage systems (ML) can be distinguished (cf. Fig. 3).

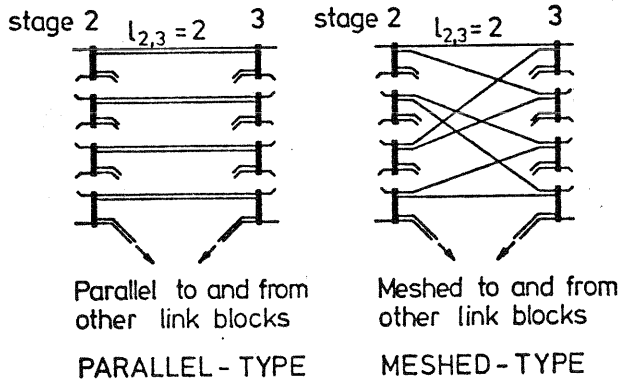


Fig. 3: Wiring in 4-stage link systems with $l_{j,j+1} > 1$

3.3 Structures with Five Stages

Figure 4 shows the notation applied to 5-stage link systems. Only SL-systems ($l_{j,j+1}=1$) with link blocks are considered.

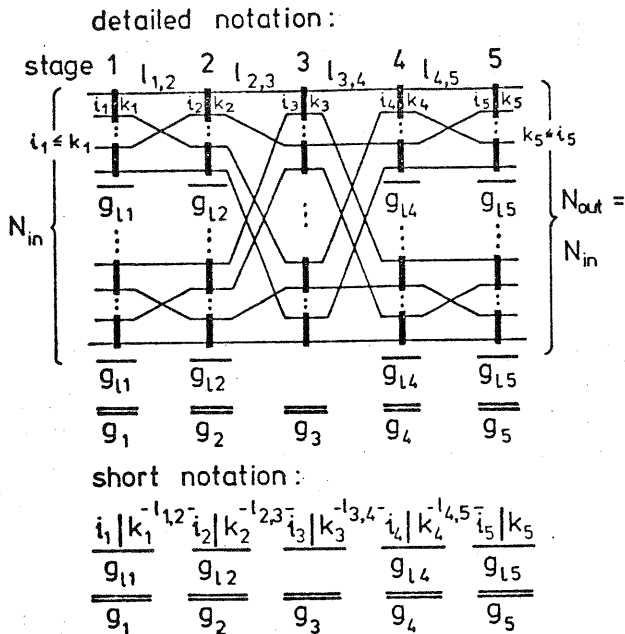


Fig. 4: Notations for 5-stage link systems

3.4 Structures with Six Stages

Figure 5 shows the notation applied to six stage link systems with single linkage ($l_{j,j+1}=1$).

These systems consist of link blocks (g_{lj} =number of multiples in a block in stage j) and of groups of link blocks (g_{bj} =number of multiples in a group of link blocks in stage j).

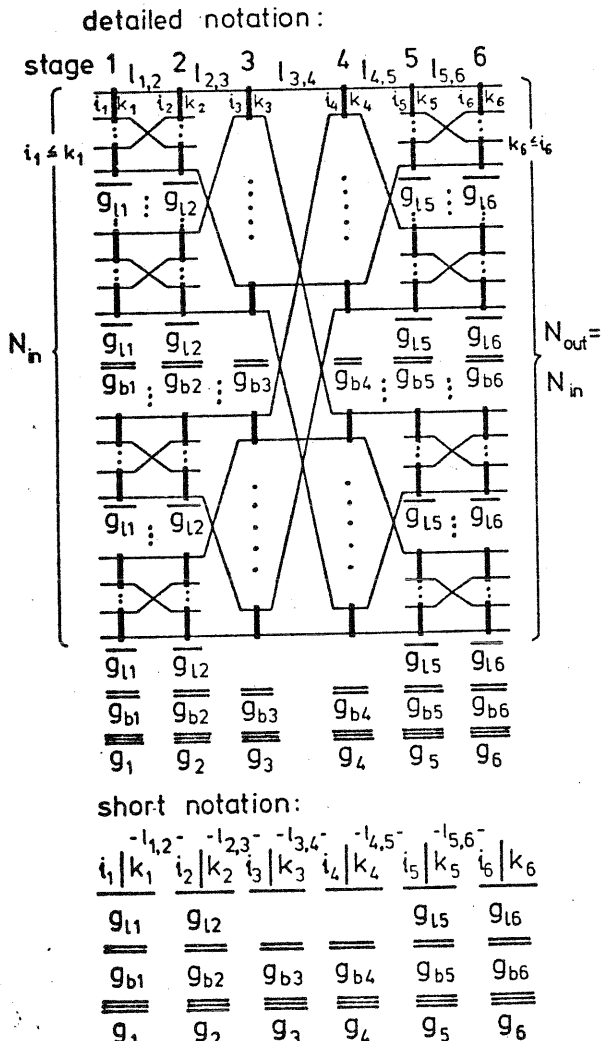


Fig. 5: Notations for 6-stage link systems

As to the wiring, one link from stage 1 to stage 2 must have access - via the connection graph - to as many multiples as possible in stage 5 which belong to the link block of the destination multiple in stage 6.

3.5 System-to-Outgoing-Group Wiring

The outlets behind the last stage of a link system can be wired to the outgoing trunk groups in two different manners (cf. Fig.6):

a) SDM wiring (Space Division Multiplex)
In a SDM system, the trunks of a certain outgoing group are usually wired to the multiples of the last stage such that as many multiples as possible have access to a certain group.

b) TDM wiring (Time Division Multiplex)
One multiple of the last stage represents one trunk group, as it is the case in PCM systems, namely one multiple of the last stage corresponds to one PCM-highway.

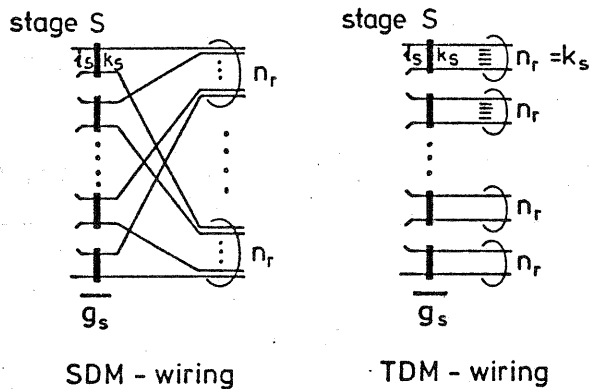


Fig.6: System-to-outgoing-group wiring

4. COMPARISON OF SIMULATION AND PPL-CALCULATION

4.1 Remarks on Simulation

Monte Carlo simulation was applied with a total of more than 500 million test calls. About 150 point-to-point selection systems of different structure types have been investigated. The simulations were carried out with sequential hunting from a home position in all stages (hunting mode H). Additionally, simulation runs were performed with sequential hunting from a randomly chosen starting point in the first stage only (hunting mode R). The outlets of the multiples in the last stage were wired in the SDM mode or TDM mode (cf. Chapter 3.5) with various trunk group sizes.

However, the influence of the trunk group size on the point-to-point loss (in contrast to the point-to-group loss) is negligibly small. On the other hand, there exists a difference in loss between SDM and TDM wiring. Therefore, the PPL-method disregards the trunk group size, but it takes into account the two different wiring modes.

All the following diagrams in this chapter show the point-to-point loss (B_{pp}) drawn vs. the average carried traffic per inlet Y/N . The authors decided to do so since plotting the loss vs. the average traffic offered would yield different curves depending on the individual traffic offered to the different trunk groups and on the offered traffic per inlet. The bold line denotes calculation results obtained by the PPL-method. Also, simulation results with 95% confidence intervals are given. For one loss value at least 100,000 test calls have been performed.

4.2 Structures with Three Stages

Figure 7 shows 3 structures with offered PCT 1. The wiring mode is of type SDM with groups of 10 trunks each. Here, the calculation tends to coincide with the tests performed with hunting mode R. As it can be seen, the losses in case of hunting mode H are remarkably smaller. This is due to the fact that sequential hunting from home position concentrates the traffic preferably in the first hunted links. Thus, traffic peaks have a better chance to come through via the less used hunting positions. This fictitious pushing up of the occupied links will be subsequently referred to as "push-up effect".

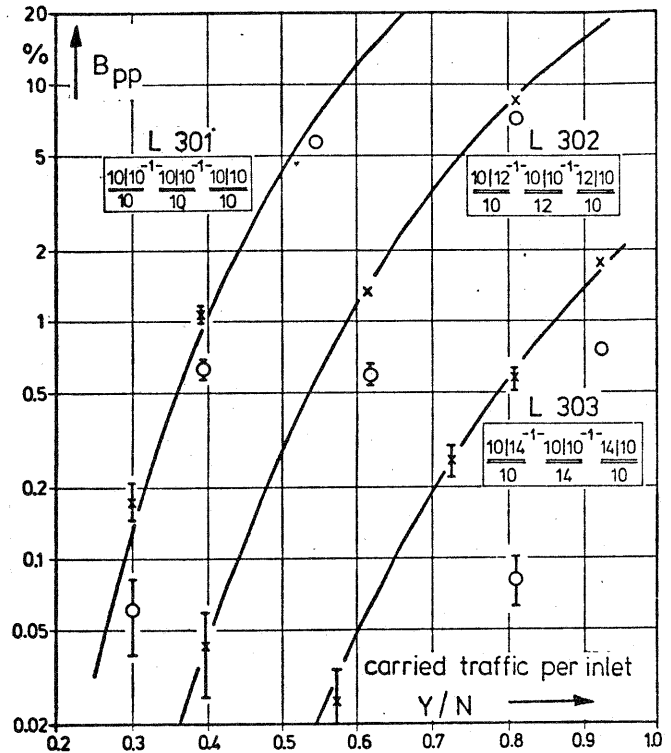


Fig.7: 3-stage link systems (SL) L301, L302, L303, offered PCT1, groups of 10 trunks each (SDM)
Simulation: \circ hunting mode H
 \times hunting mode R
PPL calculation: ———

4.3 Structures with Four Stages

4.3.1 Structures with Single Linkage (SL)

Figures 8 and 9 show the same two structures with offered PCT1 and PCT2, respectively. The wiring mode is of type SDM with groups of 50 trunks each. Additionally, further simulation runs were made with varied group sizes of outgoing trunk groups. The calculation curves lie in between the test results performed with hunting modes R and H. Here, the push-up effect is visible, too. However, the calculation considers only the mean value of the carried traffic between stage 2 and 3 which is an optimistic assumption with regard to the loss. Comparing the losses of PCT 1, and of PCT2 one can see that offered PCT2 causes smaller losses. The calculation takes this into account.

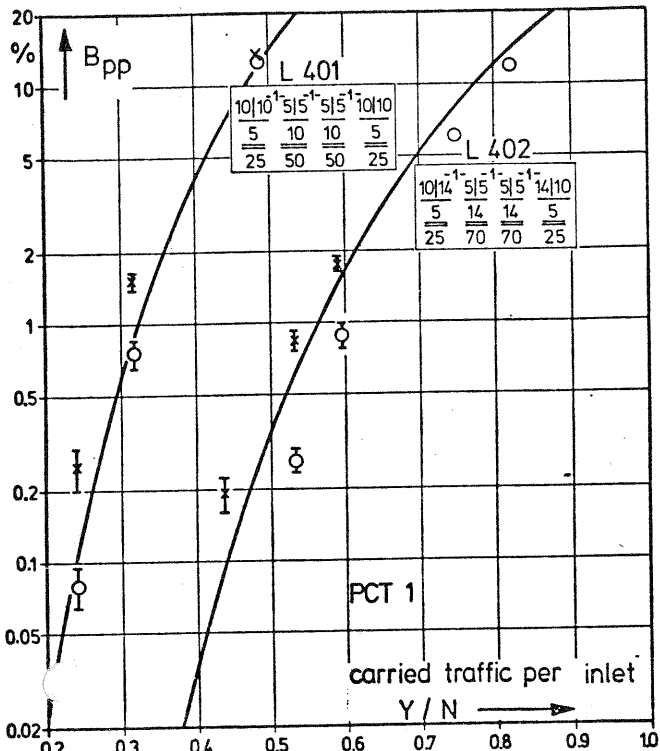


Fig. 8: 4-stage link systems (SL) L401, L402, offered PCT1, groups of 50 trunks each (SDM)
 Simulation: \circ hunting mode H
 \times hunting mode R
 PPL calculation: _____

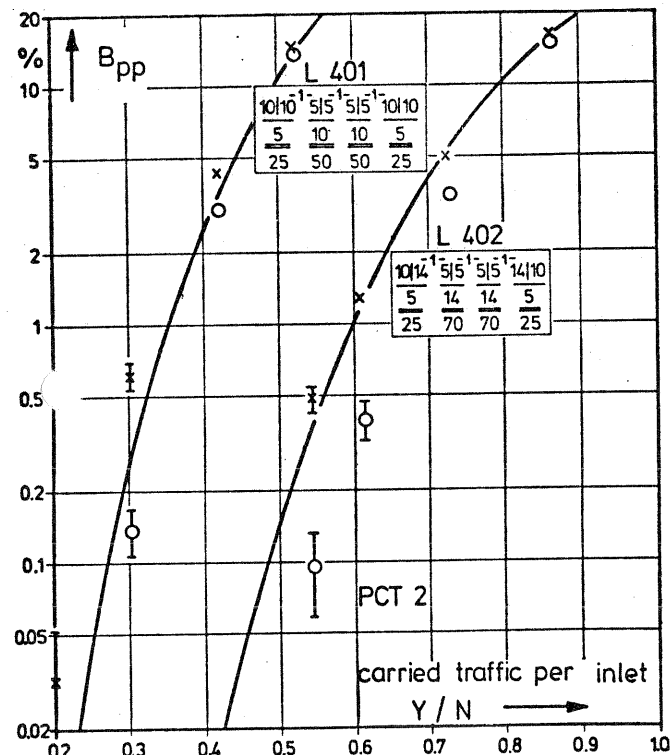


Fig. 9: 4-stage link systems (SL) L401, L402, offered PCT2, groups of 50 trunks each (SDM)
 Simulation: \circ hunting mode H
 \times hunting mode R
 PPL calculation: _____

Fig. 10 shows 3 structures with offered PCT1, hunting mode H, and SDM wiring with groups of 100 trunks each. Here, the number of links between the stages remains constant, but the number of inlets (and outlets, resp.) varies from 500 up to 1,000. Note the good accordance

between simulation and calculation. For reasons of clarity, simulations performed in hunting mode R have been omitted.

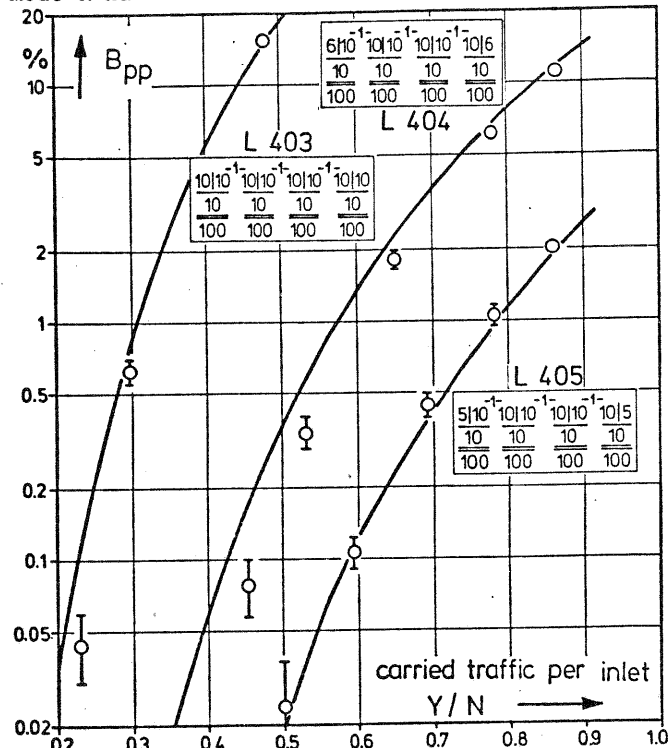


Fig. 10: 4-stage link systems (SL) L403, L404, L405, offered PCT1, groups of 100 trunks each (SDM)
 Simulation: \circ hunting mode H
 PPL calculation: _____

Fig. 11 shows 3 structures with offered PCT1 and hunting mode H. Here, TDM wiring was applied. All structures have the same switch size in the first and last stage, however, the number of link blocks increases from 2 up to 5. Also in this case, simulation and calculation results are very close.

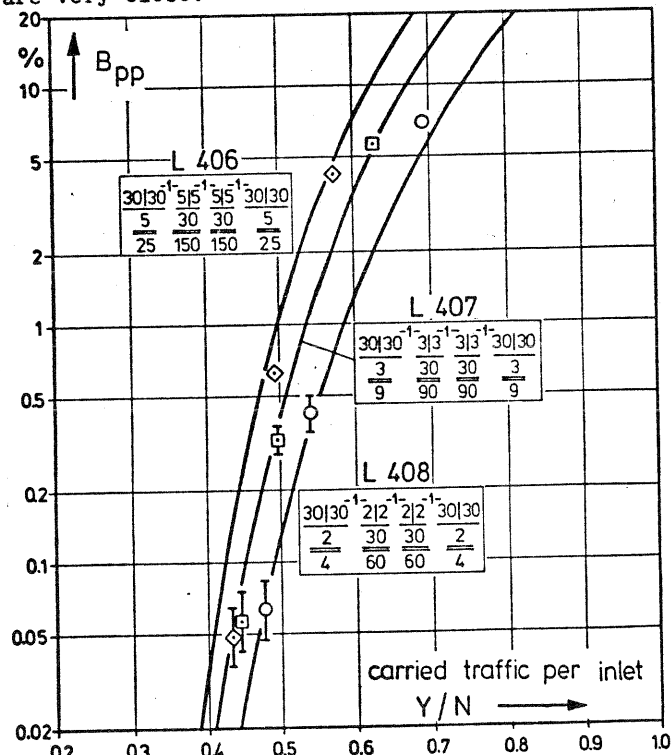


Fig. 11: 4-stage link systems (SL) L406, L407, L408, offered PCT1, groups of 30 trunks each (TDM)
 Simulation: \diamond \square \circ hunting mode H
 PPL calculation: _____

4.3.2 Multilinkage Systems (ML)

From Figures 12a and 12b, one can see the influence of parallel and meshed wiring (cf. Chapter 3.2) from stage 2 to stage 3 for the same structure. PCT1 was offered, and the wiring was SDM. The bold lines show the individually calculated curves of the loss. Simulation results are given for both hunting modes. As expected, meshed wiring yields lower losses.

Additionally, in both figures, a calculated loss curve of a single linkage structure is drawn (dashed line). This single linkage structure has about the same size ($N_{in} = N_{out} = 252$) and about the same number of crosspoints per line (CPL = 48.29 vs. 48 in the multilinkage system). The improvement of the grade of service is striking.

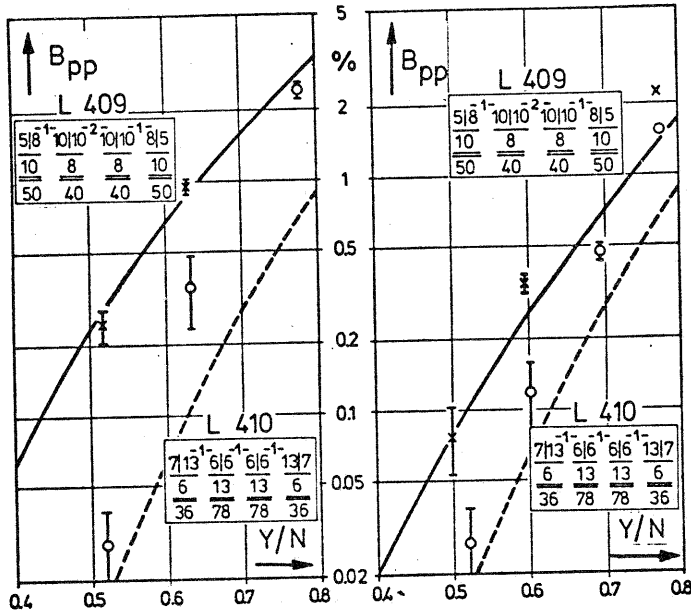


Fig. 12a: Parallel wiring

Fig. 12b: Meshed wiring

4-stage link system (ML) L409 with parallel (12a) and meshed (12b) wiring, offered PCT1, SDM wiring

Simulation: \circ hunting mode H
 \times hunting mode R

PPL calculation: — original system (ML) L409
 - - - crosspoint equivalent (SL) L410

Figures 13a and 13b show a further type of multilinkage structure. It has double linkage between all stages being either parallel or meshed (cf. Chapter 3.2, Fig. 3). PCT1 was offered, and the wiring of the outlets to the outgoing trunk group was performed in the SDM mode, too. The bold line represents the PPL calculation. Once again, the dashed line gives the much smaller loss of a corresponding single-linkage system.

A multilinkage structure with 2-4-2 links from multiple to multiple in Fig. 14 a,b has the same principal traffic behavior. This structure is comparatively expensive and crosspoint waisting. The corresponding single linkage system yields losses $< 0.02\%$, up to 0.95 erl/inlet.

Figures 12 through 14 emphasize the fact that has been proved by many other simulations and which also can be observed by the PPL calculation:

Using the same number of crosspoints per line, well designed single linkage structures are superior to multilinkage systems, in any case. Hence, in the following sections dealing with 5-stage and 6-stage structures, multilinkage is disregarded.

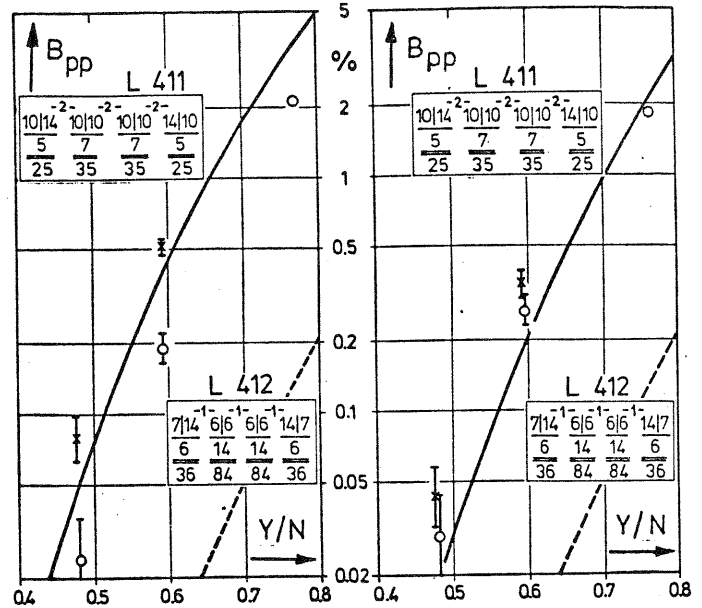


Fig. 13a: Parallel wiring

Fig. 13b: Meshed wiring

4-stage link system (ML) L411 with parallel (13a) and meshed (13b) wiring, offered PCT1, SDM wiring

Simulation: \circ hunting mode H

\times hunting mode R

PPL calculation: — original system (ML) L411
 - - - crosspoint equivalent (SL) L412

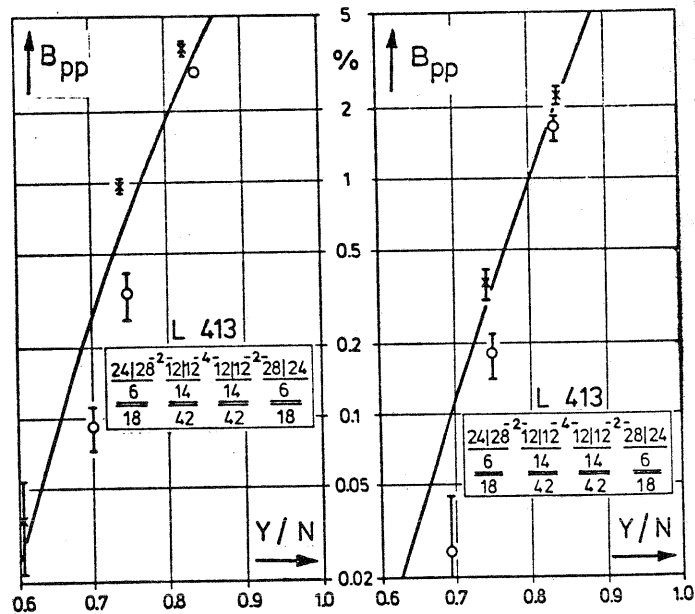


Fig. 14a: Parallel wiring

Fig. 14b: Meshed wiring

4-stage link system (ML) L413 with parallel (14a) and meshed (14b) wiring, offered PCT1, SDM wiring

Simulation: \circ hunting mode H

\times hunting mode R

PPL calculation: —

4.4 Structures with Five Stages

Figure 15 shows two 5-stage SL-structures. PCT1 was offered and SDM wiring applied. PPL and simulation are in good agreement.

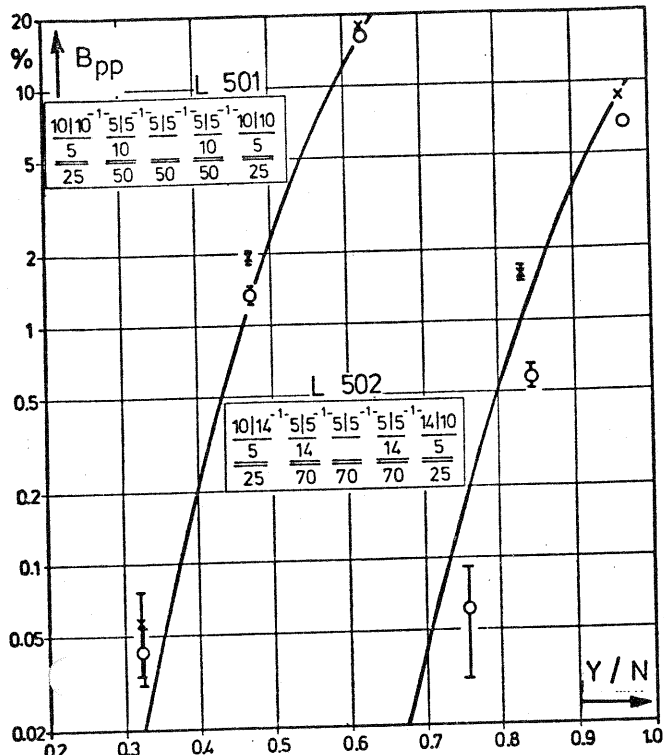


Fig. 15: 5-stage link systems (SL) L501, L502 offered PCT1, groups of 50 trunks each (SDM)

Simulation: \circ hunting mode H
 \times hunting mode R

PPL calculation: _____

4.5 Structures with Six Stages

Figure 16 shows three 6-stage SL-structures. PCT1 was offered, and the wiring mode was SDM. Simulation results are given for both hunting modes. As one can see, the accordance between calculation and simulation is very good.

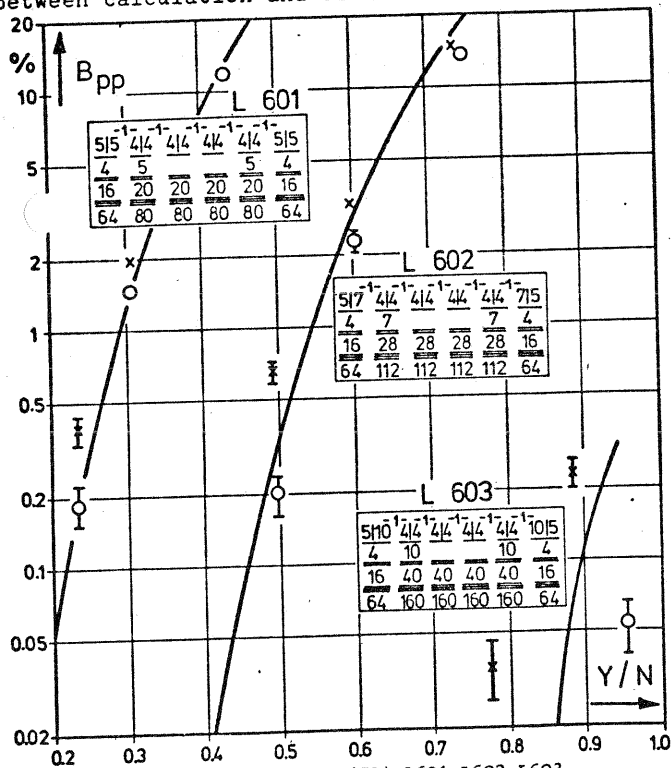


Fig. 16: 6-stage link systems (SL) L601, L602, L603 offered PCT1, groups of 64 trunks each (SDM)

Simulation: \circ hunting mode H
 \times hunting mode R

PPL calculation: _____

5. PRACTICAL APPLICATION

(Nik-Charts for quick manual design of SL-link systems with 4 and 6 stages)

Although the PPL-method is relatively easy to program on a computer, in the following a quick and handy method is outlined how to design SL-systems with 4 or 6 stages, respectively. This can be done by using the Nik-Charts (Fig. 17-25).

The design is performed in 4 steps as follows:

- Step 1 The number of stages is chosen (cf. remarks on the number of stages at the end of Ex.2).
- Step 2 One calculates the number of inlets (i_1) in the first stage. A most crosspoint saving SL-structure can be achieved only if $i_1, i_2, \dots, i_{s-1}; k_2, k_3, \dots, k_s$ all have the same value. Thus one gets the well known equation $i_1 = \sqrt[S/2+1]{N}$; (eventually to be rounded) where $[S/2+1]$ denotes the integer part of $S/2+1$.
- Step 3 Using the Nik-Chart with i_1 as chart parameter and the given total number of $N_{in} = N_{out} = N$, furthermore the prescribed carried traffic per inlet as well as the planned point-to-point loss, one obtains the required number of outlets k_1 in the first stage multiple.
- Step 4 These 3 parameters N, i_1 and k_1 allow the design of a SL-link system. The following examples illustrate this way.

5.1 Design of Group Selection Units (GSU) with Fixed Size

Example 1

Be given $N=4096$ inlets and outlets, respectively. For a traffic carried of 2867.2 Erlang, i.e. 0.70 Erl. per inlet, a loss of $B_{pp}=5\%$ is prescribed.

- Step 1 Be chosen the number of stages $S=4$
- Step 2 $i_1 = \sqrt[S/2+1]{N} = \sqrt[3]{4096} = 16$
- Step 3 With $N=4096$ and $i_1=16$ use Nik-Chart for PCT1 with parameter $i_1=16$ (Fig. 21). For 0.70 Erl. per inlet and $B_{pp}=5\%$ one reads off an expansion $k_1/i_1 \approx 1.35$ from the bold line curve drawn for 4096 lines. Hence, one gets $k_1 = 1.35 \cdot 16 = 21.6$, be chosen $k_1=22$.
- Step 4 With these 3 parameters, N, i_1 , and k_1 , the following 4-stage SL-link system is obtained:

$$N_{in} = \begin{matrix} 16 & 22 & 16 & 16 & 22 & 16 \\ 16 & 22 & 22 & 16 & 16 & 16 \\ 256 & 352 & 352 & 256 & & \end{matrix} \begin{matrix} N_{out} = \\ N_{in} = 4096 \end{matrix}$$

The total number of crosspoints required amounts to 360,448 which is 88 crosspoints per line (trunk) ($CPL=88$) and $88/0.7=125.71$ crosspoints per Erl ($CPE=125.71$).

Example 2

Be given again $N=4096$, $B_{pp}=5\%$ and $Y/N=0.70$ Erl. (cf. Example 1).

- Step 1 This time be chosen the number of stages $S=6$.
- Step 2 $i_1 = \sqrt[S/2+1]{N} = \sqrt[4]{4096} = 8$
- Step 3 With $N=4096$ and $i_1=8$ use Nik-Chart with parameter $i_1=8$ (Fig. 24). For $Y/N=0.70$ Erl. per inlet and $B_{pp}=5\%$ one reads off an expansion $k_1/i_1 \approx 1.31$ from the bold line curve drawn for 4096 lines. Hence, one gets $k_1 = 1.31 \cdot 8 = 10.48$, be chosen $k_1=10$.
- Step 4 With these parameters, N, i_1 , and k_1 , the following 6-stage link system is obtained:

$$N_{in} = \left\{ \begin{array}{cccccc} 8|10 & 8|8 & 8|8 & 8|8 & 8|8 & 10|8 \\ \hline 8 & 10 & & & 10 & 8 \\ \hline 64 & 80 & 80 & 80 & 80 & 64 \\ \hline 512 & 640 & 640 & 640 & 640 & 512 \end{array} \right\} \begin{array}{l} N_{out} = \\ \\ \\ N_{in} = \\ \\ \\ 4096 \end{array}$$

The total number of crosspoints required amounts to 245,760 which is 60 crosspoints per line (CPL=60) and 85.7 crosspoints per Erlang (CPE=85.7).

Comment on Example 1 and Example 2

For an equal number of lines (N) and an equal amount of traffic carried (Y/N), two systems were designed for S=4 and S=6 stages, resp. The system with 6 stages requires remarkably less crosspoints, namely about one third less than the one with 4 stages. However, the 6-stage system is more sensitive to overload. This can approximately be read off from Fig.24. With the planning value of $k_1=1.31$ the loss increases from 5% to 10% as the load carried increases from 0.70 to 0.735 Erl. per inlet. This corresponds to an overload of 5% only.

Similarly, for the solution with 4 stages, one obtains PP-loss of 10% as the traffic carried per line is increased to 0.76 Erl. per inlet (cf. Fig.21). This, however, corresponds to an overload of about 8.6%. The decision, whether S=4 or S=6 should be realized, therefore depends, in practice, on the crosspoint requirements mentioned above, and on the overload sensitivity demanded. Furthermore, the software and/or hardware costs of the common control as well as available switch sizes and wishes regarding the modular expansion of the system will have an influence on the decision.

5.2 Design of Group Selection Units (GSU) Providing Future Extensions

Example 1 and Example 2 described a so-called master size where exactly holds $N=i_1^3$ or $N=i_1^4$, respectively. Also, master size systems guarantee the minimum number of crosspoints per line for a given traffic carried and for a prescribed loss Bpp. In practice, also GSU's have to be designed which grow stepwise from an initial size to a final extension.

There are 2 different strategies that can be applied for this purpose.

5.2.1 SYG - the Concept of Symmetrical Growth

The concept of symmetrical growth will be outlined in Example 3.

Example 3

Smallest size $N \approx 300$
Main size $N \approx 500$
Largest size $N \approx 2000$

Furthermore, a loss Bpp=1% and a traffic carried per inlet $Y/N=0.8$ are prescribed.

Step 1 Be chosen S=4 stages

Step 2 Starting with the economically most interesting main size $N \approx 500$, one obtains

$$i_1 = \frac{[S/2+1]}{\sqrt{N}} = \frac{3}{\sqrt{500}} = 7.94,$$

hence, $i_1=8$ is chosen.

Step 3 With $N \approx 500$ and $i_1=8$ use Nik chart with parameter $i_1=8$ (Fig.18). For $Y/N=0.8$ Erl. per inlet and Bpp=1% one reads off an expansion $k_1/i_1 \approx 1.83$ from the bold line curve drawn for 512 lines. Therefrom, $k_1=1.83 \cdot 8 = 14.64$. Regarding the maximum size of $N \approx 2000$, one gets $k_1/i_1 \approx 1.87$ which means $k_1=14.96$, so $k_1=15$ will be chosen. This guarantees a loss Bpp=1% also for the largest size. The loss will be somewhat lower than 1% for the smallest size (cf. analytical derivation Chapter 6).

Step 4 With these parameters, N, i_1 , and k_1 , the structure of the main size is as follows:

$$N_{in} = \left\{ \begin{array}{cccc} 8|15 & 8|8 & 8|8 & 15|8 \\ \hline 8 & 15 & 15 & 8 \\ \hline 64 & 120 & 120 & 64 \end{array} \right\} \begin{array}{l} N_{out} = \\ \\ \\ N_{in} = 512 \end{array}$$

The total number of crosspoints required amounts to 30,720, which is 60 crosspoints per line (CPL=60) and 75 crosspoints per Erl. (CPE=75).

Smallest size: ($N \approx 300$)

In a 4-stage SL-link system the following equation holds: $N=i_1 \cdot i_2 \cdot i_3$

with $i_2=i_3$ for symmetrical reasons, one obtains

$$i_2 = \sqrt{N/i_1}, \text{ hence, } i_2 = \sqrt{300/8} = 6.12$$

which is rounded off to $i_2=6$. Hence, the minimum size becomes $N=288$.

$$N_{in} = \left\{ \begin{array}{cccc} 8|15 & 6|6 & 6|6 & 15|8 \\ \hline 6 & 15 & 15 & 6 \\ \hline 36 & 90 & 90 & 36 \end{array} \right\} \begin{array}{l} N_{out} = \\ \\ \\ N_{in} = 288 \end{array}$$

The total number of crosspoints is 15,120 which is 52.5 crosspoints per line (CPL=52.5) and 65.625 crosspoints per Erl. (CPE=65.625). Analogously, we get for the largest size $N \approx 2000$

$$i_2 = \sqrt{N/i_1}, \text{ hence, } i_2 = \sqrt{2000/8} = 15.81$$

which is rounded up to $i_2=16$.

Thus, the maximum size becomes $N=2048$.

$$N_{in} = \left\{ \begin{array}{cccc} 8|15 & 16|16 & 16|16 & 15|8 \\ \hline 16 & 15 & 15 & 16 \\ \hline 256 & 240 & 240 & 256 \end{array} \right\} \begin{array}{l} N_{out} = \\ \\ \\ N_{in} = 2048 \end{array}$$

The total number of crosspoints required amounts to 184,320 which is 90 crosspoints per line (CPL=90) and 112.5 crosspoints per Erlang (CPE=112.5).

Comment on Example 3

This method of designing (SYG) allows the use of a fixed switch size only in the first and in the last stage. The switches in stage 2 and 3 have to be enlarged from 6|6 to 16|16. Also the link blocks have to be enlarged. Analogously, one proceeds with SYG for a 6-stage system. However, here a fixed multiple size for stage 1 and 2 as well as for stage 5 and 6 should be provided to reduce the changes in case of extensions.

Extending link systems in the above described manner, saves crosspoints in the initial sizes (CPL=52.5 vs. CPL=90 for the final size in Example 3). But whenever the link system has to be altered in size, one has to replace the middle stage switches and to form new link blocks, too. This, however, can outweigh the initial crosspoint savings. Therefore, another strategy is outlined below.

5.2.2 CBS - the Concept of Constant Block Size

The concept of constant block size leaves the switch sizes and link block sizes unchanged. It requires a constant number of crosspoints per line from the smallest up to the largest planned extension size of the link system. This means, that one tolerates a certain pre-investment of crosspoints. The structure of the link blocks is determined by the planned maximum size. Basing on Example 2 for a group selection unit with 6 stages, the following example outlines the design according to CBS. Once again, Nik-Charts are used for the design.

Example 4

be chosen the number of stages $S=6$.

Smallest size (initial size) $N_{in}=N_{out}=512$

Largest size (final size) $N_{in}=N_{out}=4096$

Dimensioning starts with the final size:

With $N_{max}=4096$ given, one gets $i_1 = \sqrt[4]{4096} = 8$.

For a prescribed loss $B_{pp}=5\%$ at an average traffic carried per inlet $Y/N=0.7$, for the middle stages one obtains $i=k=8$ (see Nik-Chart Fig.24). Thus, one gets the following structure:

$$N_{in} = \left\{ \begin{array}{cccccc} 8 & 10 & 8 & 8 & 8 & 8 & 10 & 8 \\ \hline 8 & 10 & 8 & 8 & 8 & 8 & 10 & 8 \\ \hline 64 & 80 & 80 & 80 & 80 & 80 & 64 & \\ \hline 512 & 640 & 640 & 640 & 640 & 640 & 512 & \end{array} \right\} \begin{array}{l} N_{out} = \\ N_{in} = \\ 4096 \end{array}$$

This above structure is of type SL, i.e. single linkage. Stages 1,2, and 3 form 8 groups of link blocks with 512 lines each. The same holds true for stages 4,5, and 6.

Now, intermediate sizes are considered. The smallest size is $N=512$. The one left hand side group of 8 link blocks has to be wired to the one right-hand side group of 8 link blocks.

$$N_{in} = \left\{ \begin{array}{cccccc} 8 & 10 & 8 & 8 & 8 & 8 & 10 & 8 \\ \hline 8 & 10 & 8 & 8 & 8 & 8 & 10 & 8 \\ \hline 64 & 80 & 80 & 80 & 80 & 80 & 64 & \\ \hline 512 & & & & & & & 512 \end{array} \right\} \begin{array}{l} N_{out} = \\ N_{in} = \\ 512 \end{array}$$

The $80 \cdot 8 = 640$ links from stage 3 to 4 can be wired either in the parallel or in the meshed mode (cf. Fig.3). In both cases, the loss will be significantly below the B_{pp} planned for the maximum size. It could also be calculated by the PPL-method described in Chapter 6.

Next, $N=1024$ is considered. Here in this case, one has two groups of link blocks in each half of the link system.

$$N_{in} = \left\{ \begin{array}{cccccc} 8 & 10 & 8 & 8 & 8 & 8 & 10 & 8 \\ \hline 8 & 10 & 8 & 8 & 8 & 8 & 10 & 8 \\ \hline 64 & 80 & 80 & 80 & 80 & 80 & 64 & \\ \hline 128 & 160 & 160 & 160 & 160 & 160 & 128 & \end{array} \right\} \begin{array}{l} N_{out} = \\ N_{in} = \\ 1024 \end{array}$$

Analogously, one gets an extension size of 2048, if using 4 groups of link blocks on the left-hand side and also 4 groups of link blocks on the right-hand side. Again, the loss B_{pp} will be significantly lower than in the planned maximum size for $N_{max}=4096$ lines.

As to the other intermediate sizes, i.e. for $N_{in}=N_{out}=1536, 2560, 3072$ and 3584 , the outlets in the multiples in stage 3 cannot be partitioned evenly among the groups of link blocks. In other words, $l_{3,4}$ is not an integer number. As a rule, the wiring can be performed in such a manner, that the number of links leading into a certain group of link blocks does not differ significantly.

A further extension beyond the planned final size (4096 trunks) is also possible. In that case, the sizes of the switches in the middle stages should be increased, as it was suggested in example 3 (concept of symmetrical growth).

5.3 Nik-Charts for 4-Stage Systems

Fig. 17 through 22 are Nik-Charts for 4-stage link systems. Whereas Fig. 17,18,20,21, and 22 were calculated for offered PCT1, Fig.19 was calculated for PCT2.

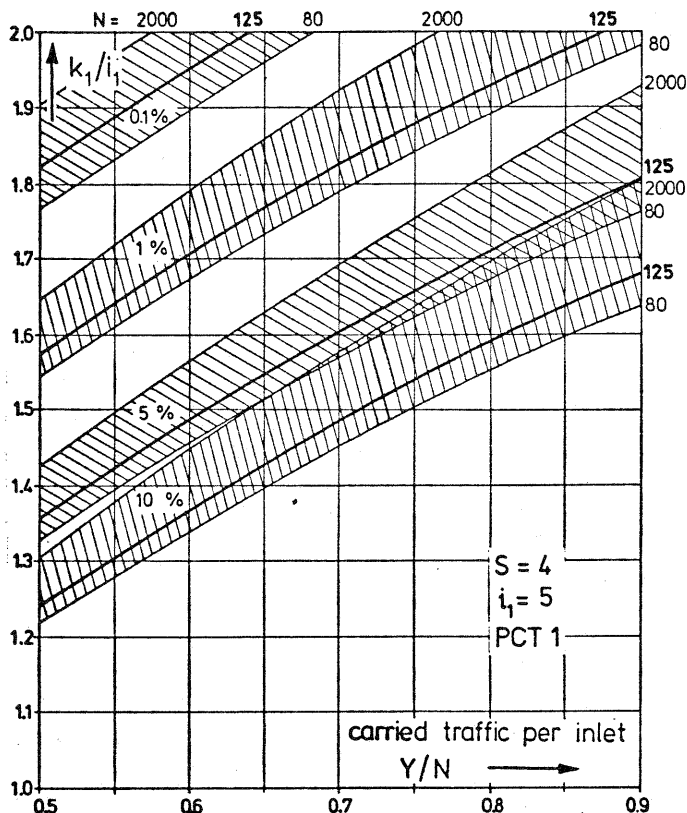


Fig.17: Nik-Chart for $S=4$ stages, with $i_1=5$ inlets per first stage multiple and with offered PCT1

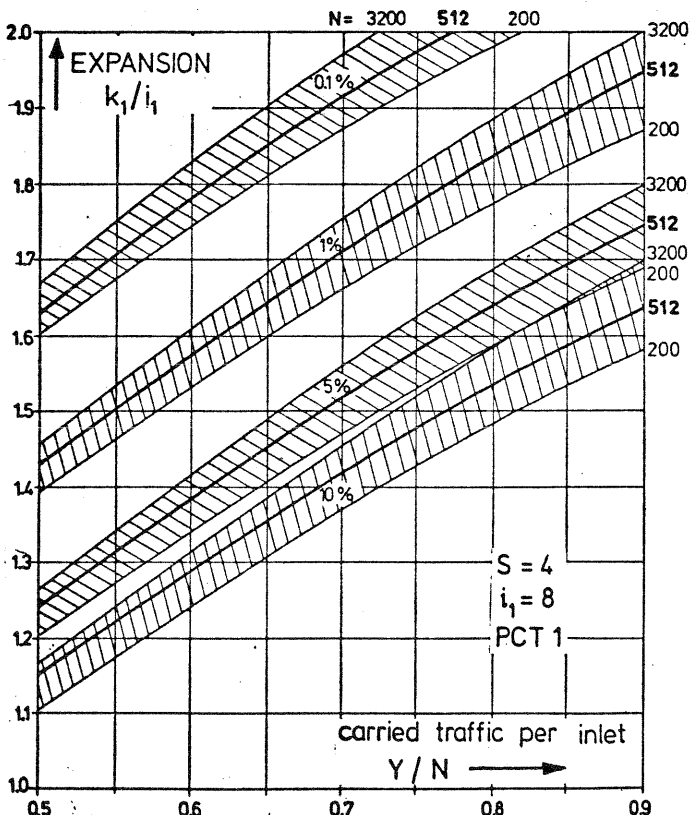


Fig.18: Nik-Chart for $S=4$ stages, with $i_1=8$ inlets per first stage multiple and with offered PCT1

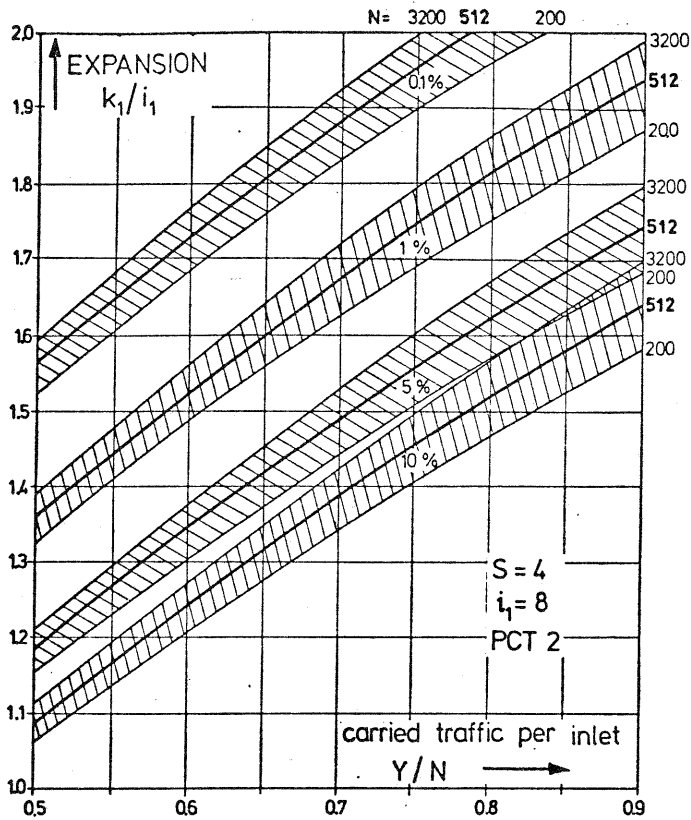


Fig.19: Nik-Chart for $S=4$ stages, with $i_1=8$ inlets per first stage multiple and with offered PCT2

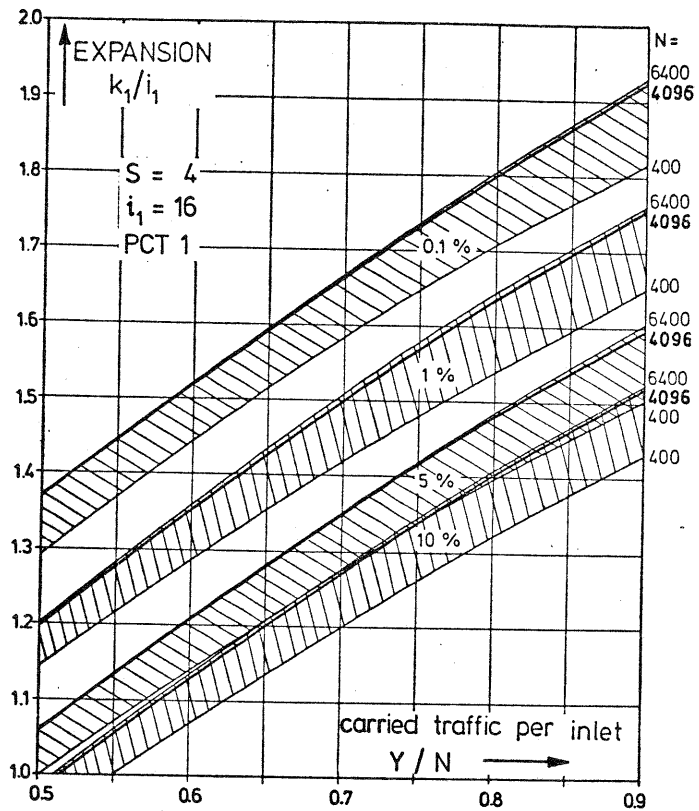


Fig.21: Nik-Chart for $S=4$ stages, with $i_1=16$ inlets per first stage multiple and with offered PCT1

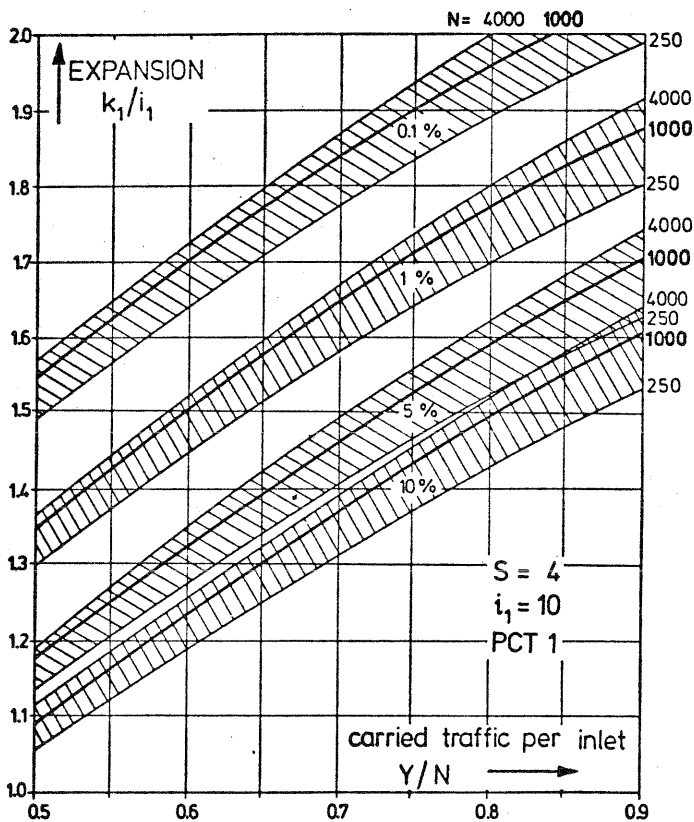


Fig.20: Nik-Chart for $S=4$ stages, with $i_1=10$ inlets per first stage multiple and with offered PCT1

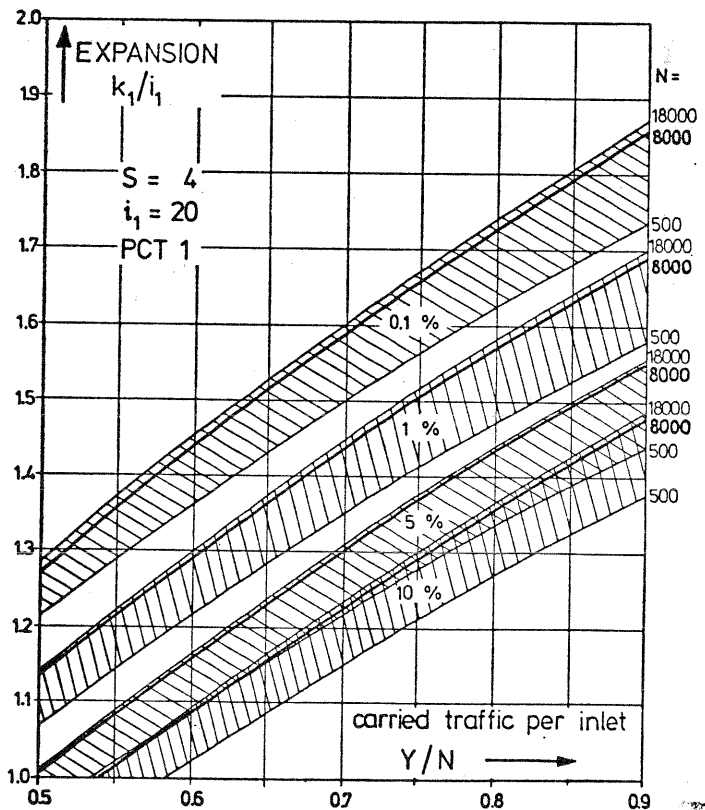


Fig.22: Nik-Chart for $S=4$ stages, with $i_1=20$ inlets per first stage multiple and with offered PCT1

5.4 Nik-Charts for 6-Stage Systems

Fig. 23, 24 and 25 are Nik-Charts for 6-stage link systems, where PCT1 was applied in the calculation.

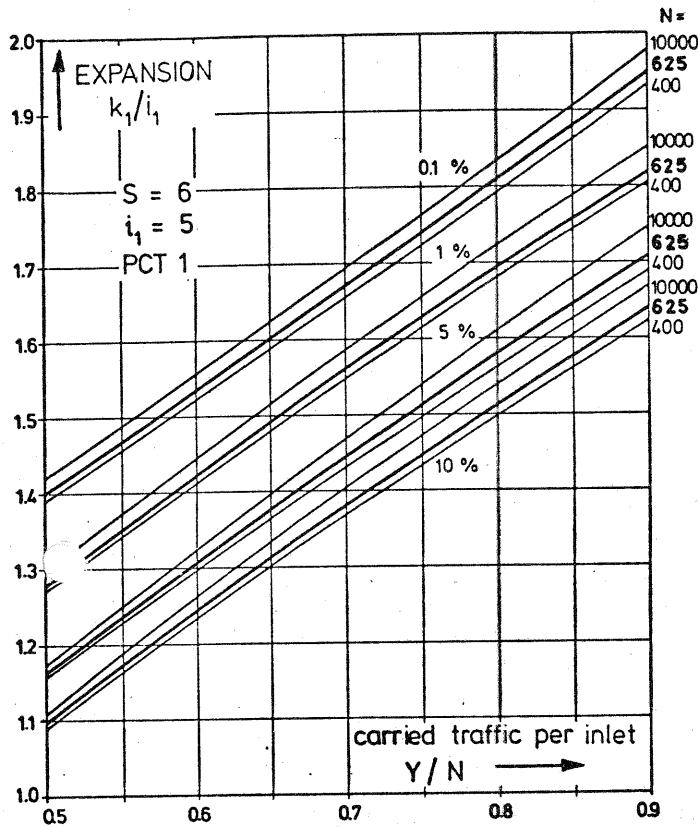


Fig. 23: Nik-Chart for S=6 stages, with $i_1=5$ inlets per first stage multiple and with offered PCT1

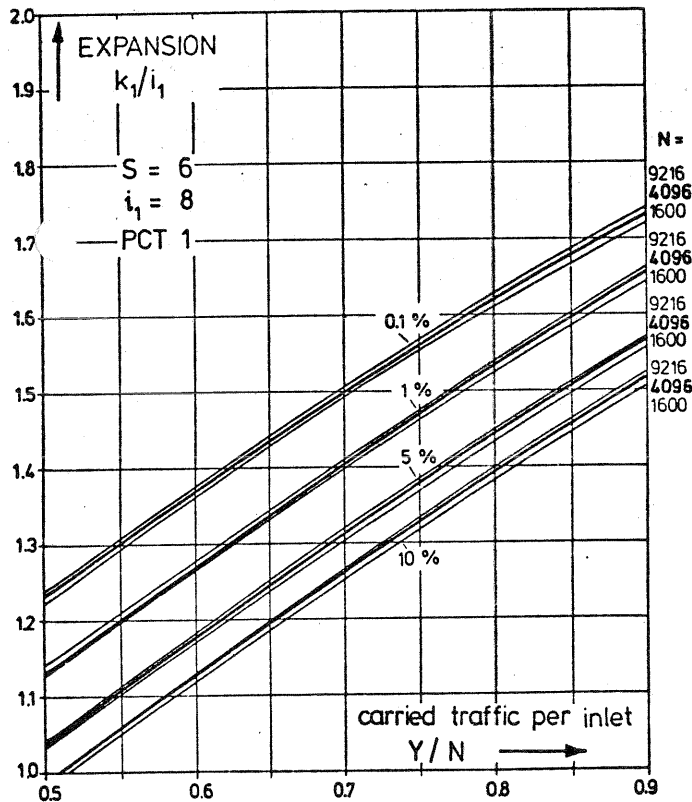


Fig. 24: Nik-Chart for S=6 stages, with $i_1=8$ inlets per first stage multiple and with offered PCT1

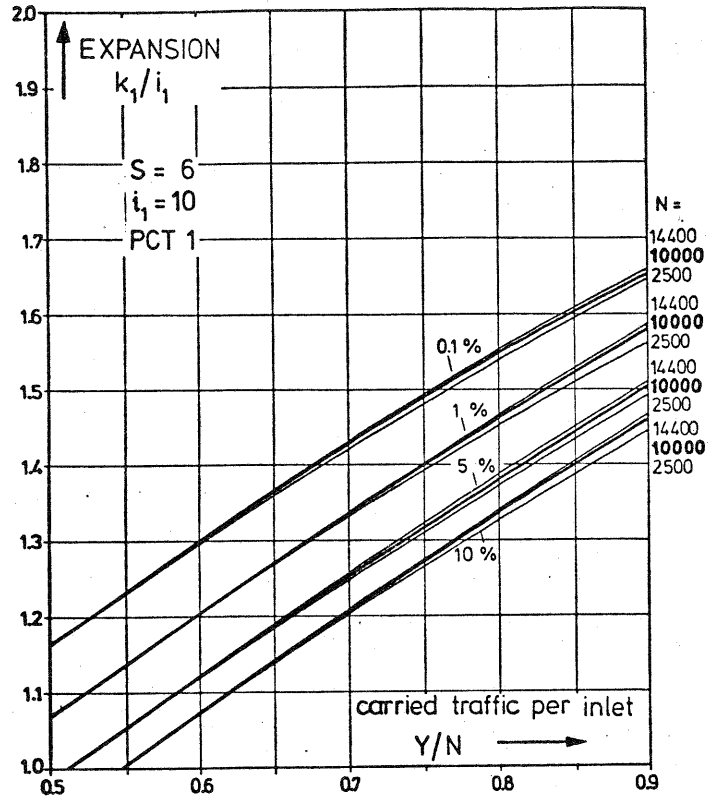


Fig. 25: Nik-Chart for S=6 stages, with $i_1=10$ inlets per first stage multiple and with offered PCT1

6. ANALYTICAL DERIVATION OF THE PPL-FORMULA

6.1 Basic Idea

If point-to-point selection in a link system with S stages is applied, the i_s links from the last but one stage to the destination multiple may be considered as a "trunk group" (cf. Fig. 26).

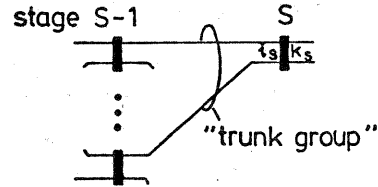


Fig. 26: The "trunk group" from stage (S-1) to the destination multiple in stage S

Therefore, it is sufficient for the calling inlet to have access to one idle link out of i_s "trunks" leading to the destination multiple. First, the effective accessibility (k_{eff}) to such a "trunk group" has to be calculated as a function of the link system structure and of the traffic carried. Chapters 6.3 and 6.4 deal with the calculation of this accessibility for single linkage (SL) and multilinkage (ML) systems. By means of the accessibility, one can determine the blocking probability $c(x_s, k_{eff}(x_1))$ of the hunted destination group consisting of i_s "trunks", if x_1 outlets of the starting multiple and x_s inlets of the destination multiple are occupied.

$$c(x_s, k_{eff}(x_1)) = \frac{\binom{x_s}{k_{eff}(x_1)}}{\binom{i_s}{k_{eff}(x_1)}} \quad (1)$$

The probability that x_1 out of i_1 inlets in a first stage multiple are occupied is denoted $p(x_1)$.

In the case of offered PCT1, this probability $p(x_1)$ is calculated as follows:

The carried traffic y_1 per multiple in the first stage is prescribed. Hence,

$$p(x_1) = \frac{\frac{A_0 x_1}{x_1!}}{\sum_{j=0}^{i_1-1} \frac{A_0^j}{j!}} \quad (2)$$

where

$$A_0 = \frac{y_1}{1 - E_{1,i_1}(A_0)} \quad (3)$$

to be determined by iteration.

This assumption of the Erlang formula has been proved as a suitably good approximation.

Furthermore, be $p(x_s)$ the probability that x_s out of k_s outlets of the destination multiple in the last stage are occupied. Here, one has to distinguish the 2 wiring modes behind the last stage of the link system (cf. Chapter 3.5). In case of SDM (space division multiplex) wiring, the probability that one certain outlet out of k_s outlets is occupied, is assumed to be independent of the idle/busy state of the other ones. Therefore, the use of a Bernoulli distribution yielded good results in all cases.

$$p(x_s) = \binom{k_s}{x_s} \cdot (y_s/k_s)^{x_s} \cdot (1 - y_s/k_s)^{k_s - x_s} \quad (4)$$

where y_s is the carried traffic of the considered destination multiple.

In case of TDM (time division multiplex) wiring, the k_s outlets form a outgoing trunk group. Consequently, an Erlang distribution is applied.

$$p(x_s) = \frac{\frac{A_0 x_s}{x_s!}}{\sum_{j=0}^{k_s} \frac{A_0^j}{j!}} \quad (5)$$

where

$$A_0 = \frac{y_s}{1 - E_{1,k_s}(A_0)} \quad (6)$$

to be determined by iteration.

Therewith, one obtains

$$B_{PP} = \sum_{x_1=0}^{i_1-1} \frac{p(x_1)}{1-p(i_1)} \cdot \sum_{x_s=0}^{k_s-1} \frac{p(x_s) \cdot c(x_s, k_{eff}(x_1))}{1-p(k_s)} \quad (7)$$

The summation only includes values up to (i_1-1) and (k_s-1) since B_{PP} is defined as the loss (cf. Chapter 2.3) which is caused by internal blocking during those time intervals, when at least one idle inlet of a considered multiple in stage 1, as well as one idle outlet to the desired outgoing trunk group exists. The condition "at least one idle outlet in the desired outgoing trunk group" implies that also a destination multiple exists where this above mentioned idle trunk is connected to.

In the case of PCT2, the probability $p(x_1)$ is calculated as follows:

As the number of traffic sources per multiple is assumed to be equal to the number of inlets i_1 , one gets

$$p(x_1) = \binom{i_1}{x_1} \cdot (y_1/i_1)^{x_1} \cdot (1 - y_1/i_1)^{i_1 - x_1} \quad (8)$$

The probability distribution $p(x_s)$ in the case of SDM remains unchanged (see Eq. 4).

In case of TDM wiring, the number $q = N_{in}$ of all traffic sources in the first stage, contributing as a whole to the traffic carried of one certain outgoing PCM group (one destination

multiple), is

$$q \gg k_s$$

Therefore, again $p(x_s)$ as in Eq.5 is applied.

Therewith, one obtains an analogous formula for PCT2:

$$B_{PP} = \sum_{x_1=0}^{i_1-1} p(x_1) \frac{1 - x_1}{1 - y_1} \cdot \sum_{x_s=0}^{k_s-1} \frac{p(x_s) \cdot c(x_s, k_{eff}(x_1))}{1 - p(k_s)} \quad (9)$$

The total loss of a certain considered outgoing group with n_r trunks and the traffic carried y_r is composed of B_{PP} and the probability that all n_r trunks of this group No. r are busy. This probability can be fairly well approximated by Erlang's loss formula. Therewith, the total loss becomes

$$B_{tot} = E_{1,n_r}(A_{O,r}) + (1 - E_{1,n_r}(A_{O,r})) \cdot B_{PP} \quad (10)$$

where

$$A_{O,r} = \frac{y_r}{1 - E_{1,n_r}(A_{O,r})} \quad (11)$$

to be determined by iteration.

It should be pointed out that B_{PP} depends only on the total traffic carried of the link system, whereas E_{1,n_r} depends only on the traffic carried of the considered outgoing trunk group.

6.2 Effective Accessibility

Strictly spoken, the effective accessibility should consider the momentary states of all sections of the graph which connects starting and destination multiple. But to achieve a tolerable amount of computation work, the probabilities of state were only regarded between stage 1 and 2, and furthermore between stage $S-1$ and stage S . For the intermediate stages, only the mean value of the traffics carried (y_j) in a multiple of stage j is regarded. The authors define the expression "accessibility" as follows:

- all accessible idle trunks of the above mentioned "trunk group" leading to the destination multiple, and additionally
- all those occupied links connected to this destination multiple which can be accessed and therefore be re-occupied as soon as they become idle.

This definition is obviously in full agreement with the definition of accessibility regarding a one-stage grading.

This effective accessibility is calculated in different ways depending on the number of stages, and whether single linkage or multilinkage is applied

6.3 Effective Accessibility k_{eff} in Single Linkage Systems

The effective accessibility k_{eff} is composed of several terms, as pointed out above. These terms will be explained in full detail using a 4-stage system as an example.

6.3.1 $S=4$ stages:

$$\text{Term A} = (k_1 - x_1) \cdot (1 - y_2/k_2) \cdot (1 - y_3/k_3) \quad (12)$$

Term A denotes the average number of idle inlets to the destination multiple that can be accessed from the starting multiple via one or more chains of idle links.

Furthermore, equation (12) considers that x_1 out of k_1 outlets in the starting multiple are momentarily occupied.

$$\text{Term B} = x_1 / g_4 \quad (13)$$

Term B represents that average fractional part of the x_1 occupied outlets in the first stage multiple which are connected with the considered destination multiple. As soon as they become idle

they are ready for an immediate re-occupancy for a call to the destination multiple.

$$\text{Term C} = (k_1 - x_1) \cdot y_2 / g_4 \quad (14)$$

Term C represents analogously that fractional part of the number of occupied outlets in the second stage multiples, that are

- a) accessible via idle outlets from the first stage, and
- b) connected via a chain of occupied links from stage 2 to the destination multiple.

$$\text{Term D} = (k_1 - x_1) \cdot (1 - y_2 / k_2) \cdot y_3 / g_{l4} \quad (15)$$

Term D represents that fractional part of the number of occupied outlets in the third stage multiples, that are

- a) accessible via idle outlets from the first stage and
- b) connected with the destination multiple.

However, using term D in the above described form causes an underestimation of loss in the calculation for systems with $i_1 < k_1$, and here preferably in the range where the carried traffic per inlet is small. The reason for this underestimation arises from the fact that the calculation of the accessibility considers only the average values of the traffic carried in the multiples, except for the starting multiple. These average values would cause too optimistic values of k_{eff} .

Hence, for more accurate results, the term D has been split up in term D_1 and D_2 . The term D_2 has been multiplied with a heuristic factor y_3 / k_3 that reduces the effective accessibility the more the traffic carried is smaller.

The terms D_1 and D_2 read as follows:

$$\text{Term } D_1 = (i_1 - x_1) \cdot (1 - y_2 / k_2) \cdot y_3 / g_{l4} \quad (16)$$

$$\text{Term } D_2 = (k_1 - i_1) \cdot (1 - y_2 / k_2) \cdot y_3 / g_{l4} \cdot y_3 / k_3 \quad (17)$$

As one can see, the term D_2 disappears for $k_1 = i_1$, and it holds $D_1 = D$ in this case of a so-called "narrow" system.

Finally, the sum of the terms A, B, C, D_1 and D_2 yields the effective accessibility for 4-stage systems.

$$\text{It holds: } k_{\text{eff}}(x_1) = A + B + C + D_1 + D_2 \quad (18)$$

This equation (18) is valid for SL-systems with 4 stages.

6.3.3 S=3 stages

The effective accessibility for systems with 3 stages is calculated similarly.

$$\text{Term A} = (k_1 - x_1) \cdot (1 - y_2 / k_2) \quad (19)$$

$$\text{Term B} = x_1 / g_3 \quad (20)$$

$$\text{Term C} = (k_1 - x_1) \cdot y_2 / g_3 \quad (21)$$

Term D does not exist here.

$$\text{It holds: } k_{\text{eff}}(x_1) = A + B + C \quad (22)$$

The equation is valid for SL-systems with 3 stages.

6.3.3 S=5 stages

Analogously, one obtains for

$$\text{Term A} = (k_1 - x_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot (1 - y_4 / k_4) \quad (23)$$

Here, it should be noted that the product $(k_1 - x_1)(k_2 - y_2)(1 - y_3 / k_3)$ must not exceed the value of g_{l4} being that number of multiples in stage 4 from which access to the destination multiple is possible (cf. Fig.4).

$$\text{Term B} = x_1 / g_5 \quad (24)$$

$$\text{Term C} = (k_1 - x_1) \cdot y_2 / g_5 \quad (25)$$

$$\text{Term D} = (k_1 - x_1) \cdot (k_2 - y_2) \cdot y_3 / g_5 \quad (26)$$

$$\text{Term } E_1 = (i_1 - x_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot y_4 / g_{l5} = P_1 \cdot y_4 / g_{l5} \quad (27)$$

$$\text{Term } E_2 = (k_1 - i_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot y_4 / g_{l5} \cdot y_4 / k_4 = P_2 \cdot y_4 / g_{l5} \cdot y_4 / k_4 \quad (28)$$

Again the last term, E, has been split up for reasons outlined above. Similarly to term A, the sum of products P_1 and P_2 must not exceed the value of g_{l4} . Because of splitting, it holds:

$$P_1 \leq (i_1 / k_1) \cdot g_{l4} \quad \text{and} \quad P_2 \leq (1 - i_1 / k_1) \cdot g_{l4} \quad (29)$$

For SL-systems with 3 or 4 stages, this limitation has no effect, and therefore it is omitted.

The sum $B + C + D + E_1 + E_2$ must not exceed the average traffic carried y_5 .

The reason is that the sum represents the number of occupied links of the "trunk group" leading to the destination multiple (cf. Chapter 6.2, definition of accessibility).

It holds:

$$k_{\text{eff}} = A + B + C + D + E_1 + E_2 \quad (30)$$

$$\text{where} \quad B + C + D + E_1 + E_2 \leq y_5 \quad (31)$$

The equation is valid for SL-systems with 5 stages.

6.3.4 S=6 stages

$$\text{Term A} = (k_1 - x_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot (1 - y_4 / k_4) \cdot (1 - y_5 / k_5) \quad (32)$$

Analogously to systems with 5 stages, here, the product $(k_1 - x_1)(k_2 - y_2)(1 - y_3 / k_3)(1 - y_4 / k_4)$ must not exceed the value of g_{l5} (cf. Fig.5).

$$\text{Term B} = x_1 / g_6 \quad (33)$$

$$\text{Term C} = (k_1 - x_1) \cdot y_2 / g_6 \quad (34)$$

$$\text{Term D} = (k_1 - x_1) \cdot (k_2 - y_2) \cdot y_3 / g_6 \quad (35)$$

$$\text{Term E} = (k_1 - x_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot y_4 / g_{l6} \quad (36)$$

$$\text{Term } F_1 = (i_1 - x_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot (1 - y_4 / k_4) \cdot y_5 / g_{l6} = P_1 \cdot y_5 / g_{l6} \quad (37)$$

$$\text{Term } F_2 = (k_1 - i_1) \cdot (k_2 - y_2) \cdot (1 - y_3 / k_3) \cdot (1 - y_4 / k_4) \cdot y_5 / g_{l6} \cdot y_5 / k_5 = P_2 \cdot y_5 / g_{l6} \cdot y_5 / k_5 \quad (38)$$

Again, the last term, F, is split up. In the same manner as above, the sum $(P_1 + P_2)$ must not exceed the value of g_{l5} . Because of splitting it holds:

$$P_1 \leq (i_1 / k_1) \cdot g_{l5} \quad \text{and} \quad P_2 \leq (1 - i_1 / k_1) \cdot g_{l5} \quad (39)$$

Similarly, it holds $(B + C + D + E + F_1 + F_2) \leq y_6$ for reasons mentioned above.

$$k_{\text{eff}} = A + B + C + D + E + F_1 + F_2 \quad (40)$$

This equation is valid for SL-systems with 6 stages.

6.4 Effective Accessibility in Multilinkage Systems

6.4.1 Multilinkage between Stage 2 and 3 in 4-Stage Systems

Two types of multilinkage are possible.

- Parallel linkage (cf. Fig.3)

The effective accessibility is calculated in the following manner:

$$\text{Term A} = (k_1 - x_1) \cdot (1 - (y_2 / k_2)^{l_{2,3}}) \cdot (1 - y_3 / k_3) \quad (41)$$

$$\text{Term B} = x_1 / g_4 \quad (42)$$

$$\text{Term C} = (k_1 - x_1) y_2 / g_4 \quad (43)$$

$$\text{Term D}_1 = (i_1 - x_1) \cdot (1 - (y_2/k_2))^{l_{2,3}} \cdot y_3 / g_{l_4} \quad (44)$$

$$\text{Term D}_2 = (k_1 - i_1) \cdot (1 - (y_2/k_2))^{l_{2,3}} \cdot y_3 / g_{l_4} \cdot y_3 / k_3 \quad (45)$$

Comparing equations (41) through (45) to those in Chapter 6.3.1 (12)-(17) one notices that the second factor in the terms A, D₁ and D₂ has been changed, regarding the blocking probability of l_{2,3} parallel links. Again, it holds:

$$k_{\text{eff}}(x_1) = A + B + C + D_1 + D_2 \quad \text{where } l_{2,3} \geq 2 \quad (46)$$

For $l_{2,3} = 1$, one gets the equations (12)-(17) in Chapter 6.3.1.

- Meshed linkage (cf. Fig.3)

Under the optimistic assumption that meshing between stage 2 and 3 increases the accessibility as much as possible, one obtains the following terms A, D₁, and D₂. Terms B and C remain unchanged.

$$\text{Term A} = (k_1 - x_1) \cdot l_{2,3} \cdot (1 - y_2/k_2) \cdot (1 - y_3/k_3) \quad (47)$$

The product $(k_1 - x_1) \cdot l_{2,3} \cdot (1 - y_2/k_2)$ must be limited to g_{l_3} which is the maximum number of multiples in stage 3 having links to the destination multiple.

$$\text{Term D}_1 = (i_1 - x_1) \cdot l_{2,3} \cdot (1 - y_2/k_2) \cdot y_3 / g_{l_4} = P_1 \cdot y_3 / g_{l_4} \quad (48)$$

$$\begin{aligned} \text{Term D}_2 &= (k_1 - i_1) \cdot l_{2,3} \cdot (1 - y_2/k_2) \cdot y_3 / g_{l_4} \cdot y_3 / k_3 \quad (49) \\ &= P_2 \cdot y_3 / g_{l_4} \cdot y_3 / k_3 \end{aligned}$$

Analogously to Chapters 6.3.3 and 6.3.4, the products P₁ and P₂ are limited to

$$P_1 \leq (i_1/k_1) \cdot g_{l_3} \quad \text{and} \quad P_2 \leq (1 - i_1/k_1) \cdot g_{l_3} \quad (50)$$

The sum B+C+D₁+D₂ must not exceed the average traffic carried \bar{y}_4 . The reason is the same one as outlined in Chapters 6.3.3 and 6.3.4. Hence,

$$k_{\text{eff}}(x_1) = A + B + C + D_1 + D_2 \quad (51)$$

$$\text{where } B + C + D_1 + D_2 \leq y_4 \quad (52)$$

However, numerous simulation results have proved that the calculation by means of equations (47)-(52) yields too optimistic losses. The reason lies in the fact that the patterns of idle links between stages 2 and 3, may often give access to certain third stage multiples twice or more, whereas other multiples cannot be reached. Therefore, the authors suggest the following approximation which has been successful in all investigated systems that have meshed linkage between stage 2 and 3:

The loss B_{pp} is calculated twice, i.e. separately with k_{eff} according to equ. (41)-(46) for parallel linkage as well as with k_{eff} according to equ. (47)-(52). Then, the final B_{pp} for meshed systems is obtained by an interpolation as follows:

$$\begin{aligned} B_{\text{pp}}(\text{meshed}) &= \frac{1}{3} B_{\text{pp}} \quad (\text{with } k_{\text{eff}} \text{ acc. eq. 41-46}) \\ &+ \frac{2}{3} B_{\text{pp}} \quad (\text{with } k_{\text{eff}} \text{ acc. eq. 47-52}) \end{aligned} \quad (53)$$

An example is given in Figure 12.

6.4.2 Multilinkage between All Stages in 4-Stage Systems

Parallel or meshed linkage from stage 1 to stage 2 as well as from stage 3 to stage 4 does not have any significant effect on the loss. As to parallel or meshed linkage between the middle stages, the same considerations hold as outlined in 6.4.1.

It holds:

$$\text{Term A} = \frac{1}{1,2} (k_1 - x_1) \cdot (\dots) \cdot l_{3,4} \cdot (1 - y_3/k_3) \quad (54)$$

$$\text{Term B} = x_1 / g_4 \quad (55)$$

$$\text{Term C} = \frac{1}{1,2} (k_1 - x_1) y_2 / g_4 \quad (56)$$

$$\text{Term D}_1 = \frac{1}{1,2} (i_1 - x_1) \cdot (\dots) \cdot y_3 / g_{l_4} \quad (57)$$

$$\text{Term D}_2 = \frac{1}{1,2} (k_1 - i_1) \cdot (\dots) \cdot y_3 / g_{l_4} \cdot y_3 / k_3 \quad (58)$$

Once again, in the case of single linkage, these equations simplify to the set of equations (12) through (17). An example is given in Fig.13.

The factor $1/1,2$ in Equations 54,56,57,58 expresses the above mentioned fact that multi-linkage between stage 1 and 2 does not contribute to the effective accessibility k_{eff} . This has been confirmed by many simulations.

A special modification of the term C is necessary if $l_{1,2}, l_{3,4} \neq l_{2,3}$. Here, it holds:

$$\text{Term C} = \frac{1}{1,2} (k_1 - x_1) \cdot y_2 / g_4 \cdot V \quad (59)$$

$$\text{The factor V is defined as } V = \frac{l_{2,3}}{1,2} \quad (60)$$

with the limitation $V \leq 1,2$.

This heuristic factor may be explained by the fact that the number of relevant occupied links which contribute to term C is greater. An example is shown in Fig. 14.

6.4.3 Multilinkage in a 6-Stage System between the Middle 2 Stages

Here, the calculation is performed analogously to Chapter 6.4.1, for parallel as well as for meshed linkage. Therewith one can also calculate the loss of intermediate extension sizes of link systems if the CBS concept is applied (cf. Chapter 5.2.2).

7. OUTLOOK

Nik-Charts for further sets of system parameters including also 3, 5, and 8 stages, are under work. The extension of PPL to the case of repeated attempts was also solved but is beyond the topic of this paper. A publication will follow.

A further publication will deal with the PPL-calculation of subscriber link systems with concentration and expansion, respectively.

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