DDD NETWORK OPTIMIZATION IN FIELD ENGINEERING - FROM THEORY TO APPLICATION

A. Lotze
University of Stuttgart
Stuttgart, F.R. of Germany

ABSTRACT

This paper deals with teletraffic theory as an important tool to solve problems of tele-communication engineering. It is shown that -- by thorough simplifications -- theoretical results can be made applicable to a handy use in practice.

As an example, the calculation of arbitrary one-stage connecting arrays and link systems are considered in this paper.

Furthermore, special importance is attached to the problem how to dimension economically telephone networks with alternate routing.

As final result, a table book could be edited which enables, for the first time, this complicated task to be handled with high accuracy by means of a few tables only.

1. INTRODUCTION

1.1 General

During the Sixth ITC in Munich the President, Professor Dr. A. Jensen /27/, Mr. R.J. Chapuis /12/, Mr. I.A. Newstead and Mr. I. Tånge /43/, Mr. EPG Wright /60/ and many other prominent participants were focusing upon the necessity of a close cooperation between CCITT and ITC regarding the increasing need of handy tele-traffic outlines for field engineering. I should like to cite only one of these important voices, namely Dr. Jacobaeus /26/ mentioning in his invited paper:

"...we should also direct our attention to some extent to the technical applications. We need to be reminded that our task is, to-gether with the technicians and operations personnel of the manufacturers and administrations, to achieve better telecommunications plant, plant which is better matched to fulfil its functions for society and for the sub-scribers. To achieve this aim, theoretical, technical and purely practical advances must go hand in hand". So far Dr. Jacobaeus.

The aim of my paper is to present some typical examples considering these important objectives being of assistance and help to field engineer-ing problems. I should like to demonstrate the possibility to start from theoretical con-siderations achieving, nevertheless, handy solutions for practical engineering. However, the road to success, i.e. leading stepwise from a more interesting theory to methods of practical applicability and without losing significantly accuracy, can be long and hard. It desires also good contacts to manufacturers and administrations. Finally, one should achieve outlines for the considered problem which can be applied easily and correctly by field engi-neers without considering the entire theoreti-cal background, perhaps even without knowing much about it.

1.2 The Problems to be Solved

Outlines should be developed for loss systems regarding the following problems:

a) Probability of loss of one stage arrays and link systems with offered traffic PCT 1 or PCT 2, respectively.

b) Variance V of overflow traffic R behind groups with $k \leq n$.

c) Dimensioning of groups with offered overflow (A_{ov}, V).

d) Economic partitioning of the traffic to alternate routes.

e) Tabulated methods to b), c) and d) for field engineering.

f) Direct application of e) to link systems.

2. THE PROBABILITY OF LOSS IN ONE STAGE ARRAYS AND LINK SYSTEMS

Remark:

The alternate routing outlines described in Chapters 3 and 4 assume offered random traffic only from an infinite number of sources (Poissonian input). Nevertheless, for reasons of completeness, also loss calculation methods for a finite number of sources be concisely mentioned. Regarding the overflow problem for finite source traffic see /53,54/. As to the calculation of groups with offered smooth traffic see /2,3,28,42/.

2.1 Trunk Groups with Full Access

In many cases one can assume that the number of traffic generating sources q is so numerous that the arrival rate λ is practically independent of the number x of momentarily busy trunks, i.e. one assumes $q \to \infty$.

In public telephone networks this assumption is close to reality. Even though the actual number of inlets to any connecting array is finite it is mostly admissible to assume this "Poissonian input process" (also called "Pure Chance Traffic of Type 1; PCT 1"). In this case Erlang's Loss Formula is applied /11/.

$$B = E_n (A) = \frac{\frac{A^n}{n!}}{\sum_{i=0}^{n} \frac{A^i}{i!}} \qquad (1)$$

with

λ = arrival rate
h = mean holding time
$A = \lambda \cdot h$ = offered traffic
n = number of trunks in the group,

interarrival time and holding time being negative exponentially distributed. These assumptions are realistic enough for telephone traffic as checked by many extensive measurements, e.g. /14,24,25/.

Within PBX and for other connecting arrays with a comparatively small number of traffic sources ($q \lesssim 15 \cdot n$) one applies Erlang's Bernoulli Formula (EBF) /11/ for finite number of sources, tabulated e.g. in /48,63/. Here we have a constant call rate α per idle source; this type of traffic is also called "Pure Chance Traffic of Type 2; PCT 2".

$$B = \frac{\binom{q}{n} \alpha^n}{\sum_{i=0}^{n} \binom{q}{i} \alpha^i} \cdot \frac{q-n}{q-Y} \qquad (2)$$

with

q = number of traffic sources
α = call rate per idle source
n = number of trunks in the group
h = mean holding time (h=1)
Y = carried traffic

$$Y = \sum x \cdot p(x)$$

Interarrival time per idle source and holding time being negative exponentially distributed.

2.2 Trunk Groups behind Gradings with Limited Accessibility

2.2.1 Exact Calculation

The exact calculation of the probabilities of state in such groups, as well as of the loss probability, has been treated in many papers. Enormously large systems of linear equations generally to be evaluated numerically (except in case of Erlang's Ideal Gradings) do not permit the application of these exact methods in practice. (References see e.g. /9, 38/).

The number of interesting methods for an approximate loss calculation is therefore very high and still increasing. A survey can be found in /38/.

2.2.2 Grading Types

A large variety of different grading types is in use. Four typical grading types are shown in Fig. 1. Each one of these examples has g=9 selector groups (with e.g. 10..50 inlets each). The accessibility is k=10 and the number of outgoing trunks is n=45. The mean interconnection number ("grading ratio") is

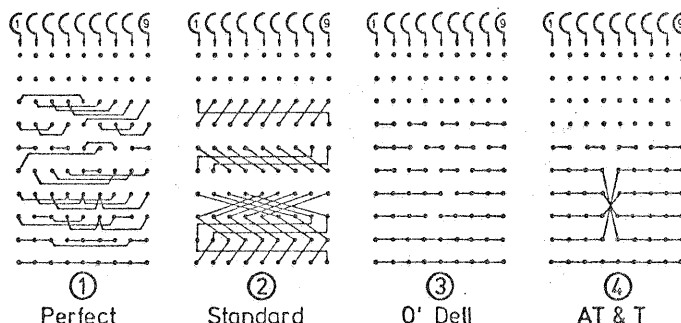$M = \frac{g \cdot k}{n} = 2$ for each of these grading examples.

Fig. 1: Typical Grading Types (n=45, k=10, g=9)

So-called "perfect" gradings with skipping (and sometimes also with slipping), e.g. Type 1 in Fig. 1, have comparatively small losses. Nevertheless, also simplified grading types yielding slightly higher losses /23/,e.g. Types 2, 3 and 4 in Fig. 1, can sometimes be economical because of the reduced manpower which is necessary for their performance and extension, respectively. It should be mentioned, however, that simplified gradings will be more loss sensitive in case of unbalanced offered traffic.

Here, the problem was to develop a loss formula for offered Poisson traffic (PCT 1) which was adaptable to all usual grading types and yielding losses close to the results of artificial traffic tests (and/or to measurements in real systems) from very low values of loss ($\lesssim 1^{o}/oo$) to very high ones ($\gtrsim 50$ %).

2.2.3 Approximate Loss Calculation for Offered PCT 1

One way to achieve a good approximate loss formula for a certain grading type is to apply the statistical equilibrium equations as used for "Erlang's Ideal Gradings". In this case one has to find out another (approximate) function c(x,k,n,A,grading type) for the state blocking probability in any state {x} which describes sufficiently accurate the special properties of the respective grading type /10,19,29,38/.

Investigations which lasted for many years were leading in 1961, however, also to another solution by means of the Modified Palm Jacobaeus Formula (MPJ) /23,34,38/. This loss formula yields reliable values of loss for "p e r f e c t" gradings (see Type 1 in Fig. 1) with skipping and sometimes also slipping, if a minimum grading ratio $M = 2$ is fulfilled. It holds for an accessibility k

$$B_{MPJ} = \frac{E_n(A_o)}{E_{n-k}(A_o)} \qquad (3)$$

with

$$Y = A_o \left[1 - E_n(A_o) \right] \quad \begin{array}{l} \text{being prescribed} \\ \text{for the calculation} \\ \text{of } B_{MPJ} \end{array}$$

and with

$$A_{actual} = \frac{Y}{1 - B_{MPJ}}$$

The admissible traffic A_{adm} offered to s i m p l i f i e d grading types (No. 2,3,4 in Fig. 1 and similar ones) has smaller values for the same parameters (n,k,B). For this purpose an adaptation formula was developed /21/. One obtains

$$A_{adm} = A_{MPJ} - \Delta A$$

with

$$\Delta A = F \cdot \left(\frac{n}{k} - 1 \right)^2 \cdot \frac{k-2}{60+4k} \cdot \frac{1-B}{1+k \cdot B^2} \qquad (4)$$

For a (standardized) minimum grading ratio (here $M=2$) the fitting parameter F depends on the grading t y p e only. It can be determined by one single series of informing traffic trials for the concerning type. This parameter is e.g. $F=0.3$ for Standard Gradings /15/ (Type 2 in Fig. 1). Furthermore, $F=1.1$ holds for O'Dell Gradings (Type 3 in Fig. 1) and $F=2.4$ for AT&T Gradings (Type 4 in Fig.1).

Formula (4) is valid for a boundary number of trunks n_b. For numbers $n_i > n_b$ the corresponding admissible traffic $A(n_i)$ is calculated -- regarding the prescribed loss B -- as

$$A(n_i) = A(n_b) \cdot \frac{n_i}{n_b} \dots \text{ where} \begin{cases} n_b = k^2, \text{ but at} \\ \text{most } n_b = 10 \cdot k \end{cases} \quad (5)$$

The reliability of this (heuristic) adaptation has been checked and is proved by extensive artificial traffic trials. Fig. 2 demonstrates this accuracy for the four grading types in Fig.1 as an example with $n=45$, $k=10$ and $g=9$.
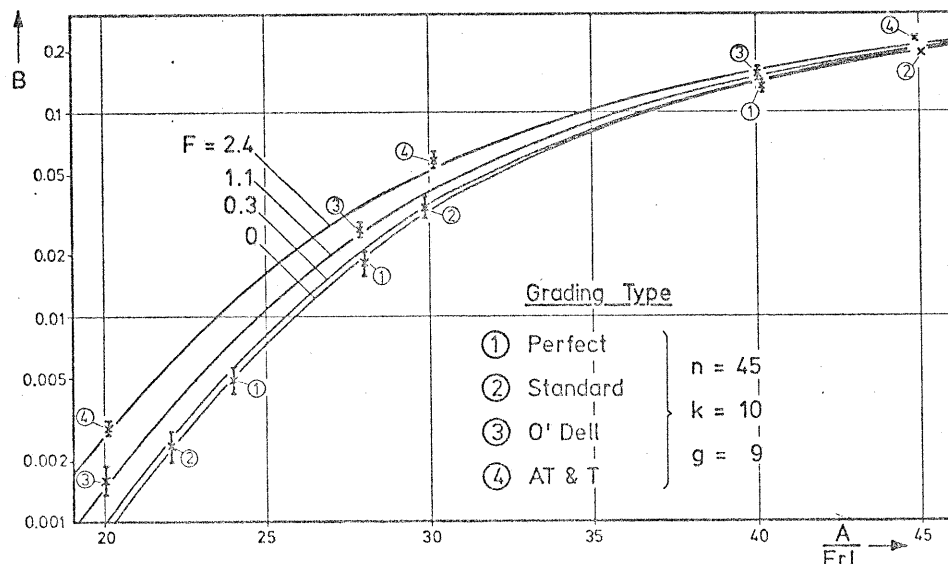
This MPJ Formula with $F=0.3$ is for instance part of the dimensioning outlines of the telephone administration in the FRG (Deutsche Bundespost). It is tabulated extensively for $F=0$ /40,61,62/ and $F=0.3$ /22,63/.

2.2.4 Approximate Loss Calculation for Offered PCT 2

Dimensioning formulae for field engineering have to be developed also for gradings with offered PCT 2, i.e. for random traffic from a finite number of traffic sources ($q \leq 5n$).

The following two approximate formulae yield results rather close to artificial traffic tests in case of "p e r f e c t" gradings (cf. Type 1 in Fig. 1):

a) The Bernoulli Interconnection Formula (BIF) /38/

$$B = \frac{\sum\limits_{x=k}^{n} (q-x) p(x) \frac{\binom{x}{k}}{\binom{n}{k}}}{q - Y} \qquad (6)$$

with

$$p(x) = \frac{\binom{q}{x} \alpha^x \prod\limits_{z=0}^{x-1} \left(1 - \frac{\binom{z}{k}}{\binom{n}{k}} \right)}{1 + \sum\limits_{i=1}^{n} \binom{q}{i} \alpha^i \prod\limits_{z=0}^{i-1} \left(1 - \frac{\binom{z}{k}}{\binom{n}{k}} \right)}$$

and

$$Y = \sum\limits_{x=0}^{n} x \cdot p(x)$$

$$A = \alpha \cdot (q - Y) \cdot h$$

q = number of sources
n = number of trunks
α = traffic intensity per idle source
k = accessibility
h = mean holding time (h=1)
Y = carried traffic
A = offered traffic

Fig. 2: Adaptation of the MPJ-Formula to Gradings of Various Type (Offered PCT1, Constant Call Intensity per Multiple)

———— Calculation

⫶ Simulation

b) The Bernoulli Quotient Formula (BQF) /4,8,38/
(Analogon to the MPJ-Formula for PCT1(Equ.(3))

$$B = \frac{B_n(\alpha_o, q)}{B_{n-k}(\alpha_o, q-k)} \qquad (7)$$

with

$$B_n(\alpha_o, q) = \frac{\binom{q}{n}\alpha_o^n}{\sum_{i=0}^{n}\binom{q}{i}\alpha_o^i} \cdot \frac{q-n}{q-Y}$$

The calculation prescribes: Y,n,k and q; hence

α_o by iteration.

The actual traffic offered is

$$A_{actual} = \frac{Y}{1-B}$$

Tables for all practically arising numbers
of sources q, numbers of trunks n, accessi-
bilities k and probabilities of loss B --
calculated by means of Equ. (6) or (7) --
would be very voluminous and unhandy.

This disadvantage can be avoided by a trans-
formation which uses only available PCT1 loss
tables, the FST Method; i.e. Finite Source
Transformation / 5/.

For any tabulated quadrupel (A_{PCT1},B,k,n) the
increase $\Delta A_{q/n}$ of the offered traffic which is
admissible for the actually concerned source
to trunk ratio q/n can be read from the cor-
responding loss tables of this grading type
and for offered PCT1, with

$$\Delta A_{q/n} = 0.3 \cdot \frac{n-0.77 \cdot A_{PCT1} \cdot (1-B)}{\frac{q}{n} + 1} \qquad (8)$$

$$A_{adm}\left(\frac{q}{n}\right) = A_{PCT1} + \Delta A_{q/n}$$

A_{PCT1} for considered grading type and tables.

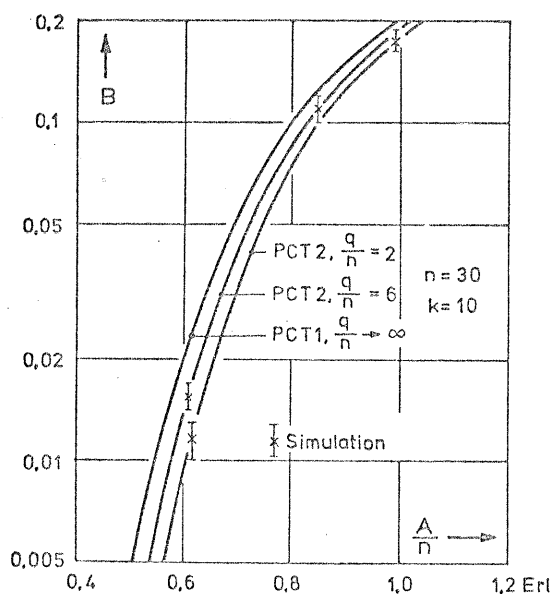Fig.3:
Probability of loss B vs. the offered traffic
per trunk A/n for n=30, k=10 and varying
source to trunk ratio q/n for a standard
grading: (Simulation with 95% confidence
interval)

For instance one considers a standard grading
(cf. Type 2 in Fig.1) with a fitting parameter
F=0.3. Fig. 3 shows the accuracy of the FST
Method as well as the influence of q/n on the
probability of loss.

2.3 Loss Calculation and Structures of Two-Sided Link Systems

2.3.1 Exact Calculation

Although one stage arrays with full or limited
access will be of practical interest still for
many years the use of link systems is making
progress.

To a large extent, the exact calculation of
state distributions and loss probabilities of
link systems has been theoretically solved
for two-sided link systems with preselection,
group selection, point-to-point selection /32/.

The rank of the concerning linear equation
systems to be evaluated is, however, in case
of usual sizes of link systems beyond any
computer capacity at the present time.
Nevertheless, the check of approximate
solutions by exact solutions is now in sight,
for at least small systems.
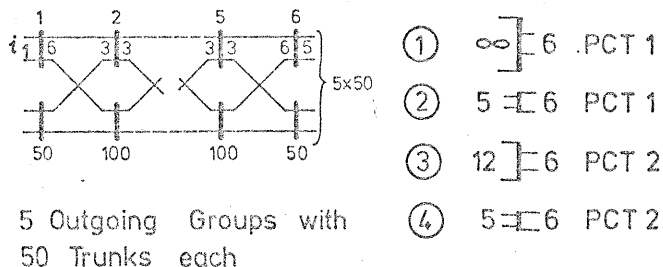
2.3.2 Approximate Loss Calculation

Field engineers need reliable and handy ap-
proximate loss calculation methods. Further-
more, outlines for crosspoint economic
structures are desired. These problems have
been investigated among others by the Stuttgart
University Teletraffic Team since more than
ten years, e.g. /30,31,37/. An analysis of
calculation methods can be found in /30,31/
containing also a large number of references.

Reliable approximate loss formulae should be
applicable for losses between $\lesssim 1^o/oo$ and
$\gtrsim 50$ %, in particular for point-to-group
hunting with regard to systems with high
usage groups and alternate routing. As con-
tinued development of former methods /35,37/
two closely related new methods for loss
calculation are therefore presented -- together
with further results -- in another paper of
this ITC /7/. These methods meet the above
mentioned demands.

Besides the case of group selection and
offered PCT1 dealt with in / 7 /, this loss
calculation method can analogously be applied
also to systems with traffic concentration in
the first and/or further stages, furthermore
to link systems with offered PCT2 (finite
number of sources). An example for the demon-
stration of accuracy is given in Fig. 4 .

The special influence to the loss of a so-
called "internal traffic" running twice through
a link system has also been investigated in
detail and is published at this ITC / 6 /.

a) The System

1   2   5   6
i_1 6 3|3 3|3 6|5   ① ∞⊏6 .PCT 1
                           } 5×50
                           ② 5⊏6 PCT 1
50 100 100 50   ③ 12⊏6 PCT 2
                           ④ 5⊏6 PCT 2

5 Outgoing Groups with
50 Trunks each

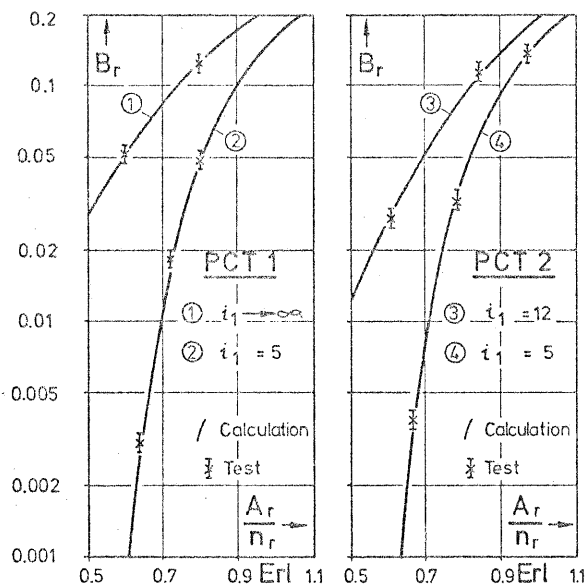## b) Probability of loss B versus offered traffic per trunk A_r/n_r

Fig. 4:
Probability of loss in case of PCT1 and PCT2

2.3.3 Suitable Structures

As to favourable structures with regard to
small losses and small crosspoint require-
ments the detailed outlines and results
in the above mentioned paper /7/ show clearly
the advantage of so-called w i d e link
systems, at least in case of group selection
and point-to-group hunting (see also Fig.4).

2.3.4 Point-to-Point Selection

Point-to-Point selection is not further re-
garded in this paper. However, one should
remember that for equal crosspoint require-
ment the point-to-point loss (without repeated
attempts) is much higher than the loss for
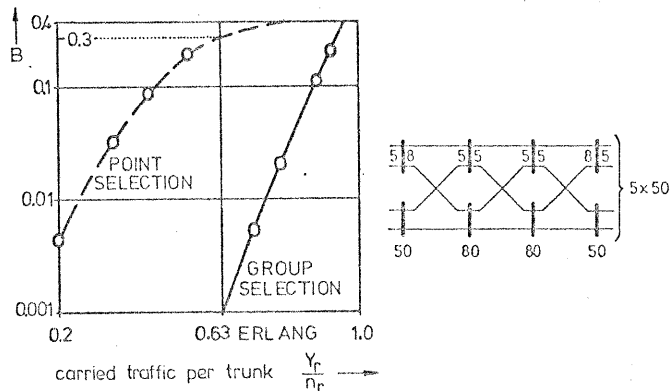point-to-group hunting. Fig. 5 gives an
example.

carried traffic per trunk $\frac{Y_r}{n_r}$ ——➤

Fig. 5:
Group selection and point selection

3. TELEPHONE NETWORKS WITH ALTERNATE ROUTING

3.1 General Problems Regarding Alternate Routing

Each telephone administration needs suffi-
ciently accurate but easily applicable out-
lines for the economic dimensioning of trunk
groups in dialling networks with alternate
routing (local, nationwide or international
networks, respectively).

The problems to be solved can be seen from
Fig. 6A and 6B.
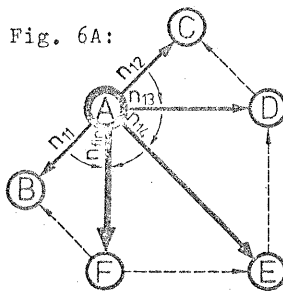
Fig. 6A:

Fig. 6B:

DIRECT OFFERED
TRAFFICS FROM

A TO F: A_{10}
A TO B: A_{11}
A TO C: A_{12} .
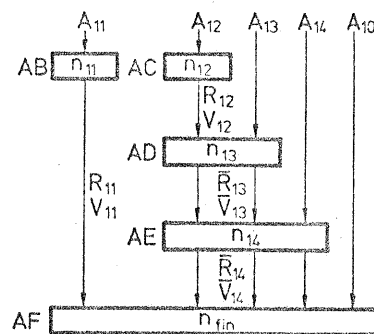A TO D: A_{13}
A TO E: A_{14}

Fig. 6:
Tandem exchange with alternate routing

The exchange of Fig. 6A is drawn more sche-
matically in Fig. 6B with regard to the various
single and multiple overflows as well as to
the directly offered traffics.

Fig. 6 is demonstrating that the outlines
should have the following features:

- Dimensioning of trunk groups for prescribed
  probability of loss in case of offered
  random traffic and with full or limited
  accessibility (see Chapter 2).

- Outgoing trunk groups of a tandem exchange
  with alternate routing to which overflowing
  peaked traffic rests are offered have also
  to be dimensioned correctly for prescribed
  probability of loss or overflow, respectively.
  Suitable calculation methods are dealt with
  in Section 3.3.

- The traffics flowing through a tandem ex-
  change with alternate routing have to be
  partitioned economically to the various
  alternatively hunted outgoing groups. This
  problem is treated in Section 3.4. The ex-
  pression "economic" means that simple and
  fast dimensioning methods should yield a
  reasonably close approach to a theoretical
  cost minimum for all outgoing groups as a
  whole.

The developed methods as a whole will be
summarized in Chapter 4.

3.2 Conditions for Practical Application

Any alternate routing dimensioning outline should take into account that in many countries the use of public DDD networks (besides eventually local alternate routing) has a very wide extent; e.g. in the FRG the administrative personnel has to perform traffic measurements and a new dimensioning of DDD trunk groups once or twice a year for about 8000 trunk groups outgoing from tandem toll exchanges.

The following conditions for applicable outlines result from these facts:

1. Variance to mean $Z=\frac{V}{R}$ of overflowing traffic R
   - only from tables
   - if at all !

2. Only from tables:
   a) group size in case of offered overflow (A_{ov}, V)
   b) standardized cost ratios final/ high usage trunks
   c) economic partitioning of the traffic to high usage groups and final group

3. No iteration procedures to be performed by planning engineers.

These conditions were strictly observed for the dimensioning outlines to be developed.

3.3 Dimensioning of Trunk Groups with Offered Overflow Traffic

As it is known, the ERT Method by Wilkinson and Riordan is used for groups with offered overflow traffic and full accessibility /59/.

For trunk groups with limited access the exact variance calculation is very complicated /17, 50,51,52/. A good and reliable approximate variance formula for limited accessibility whose structure is very similar to the exact one of J. Riordan (for full access) has, however, been found /20,36/.

Equations (9), (10) and (11) show the concerning formulae for the variance V of an overflowing traffic rest R.

Full Access (k=n) ERT Method	(9)	Limited Access (k<n) RDA Method	(10)
$V=R\cdot(1+p\cdot R)$		$V=R\cdot(1+p\cdot R\cdot\frac{k}{n})$	

where in any case

$$p=\frac{1}{B\cdot(k+1-A_o\cdot(1-B))}-1 \qquad (11)$$

with

$A_o = A$ for full accessibility

and with

$A_o = f(k,B)$ and $B = f(A,n,k,grading\ type)$ in case of limited access.

As one can see, the function p in Equ. (11) is only dependent on k and B, respectively. Therefore, the above variance formula (10) is true for the overflow behind any grading type with offered PCT1 because the pair (B,k) implies already the relevant data of a certain grading type.
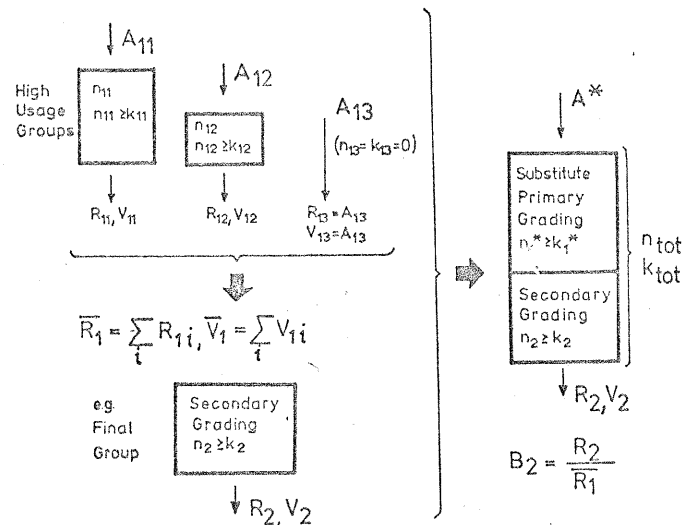

Fig. 7:
Principle of the ERT Method and RDA Method

Fig. 7 shows the wellknown idea of R.I. Wilkinson, i.e. how to calculate the loss of trunk groups to which overflow traffic is o f f e r e d :

The actual overflowing traffic rest $\overline{R}_1 = \sum R_{1i}$ and its corresponding variance $\overline{V}_1 = \sum V_{1i}$ offered to a secondary group (left hand side) is generated as overflow of one fictitious substitute primary group (right hand side of Fig.7) by means of an offered Equivalent Random Traffic A^* and n_1^* trunks. The quantities A^* and n_1^* have to be determined iteratively or by means of Rapp's approximate formulae /45/.

In case of a secondary g r a d i n g one needs also a substitute primary grading, whereas a further paramter k^* has to be determined suitably /20,36,62/.

The loss probability of the total group with $n_1^* + n_2$; $k_1^* + k_2$ (on the right hand side) can now be calculated for any type of grading with high accuracy (cf. Section 2.23 and 2.1) as well as for full access.

Thus, one finds the overflowing traffic R_2 of the t o t a l substitute group and from R_2 one gets the probability of loss of the actual secondary group as

$$B_2 = \frac{R_2}{\overline{R}_1}.$$

The traffic $\overline{R}_1$ is -- as offered traffic -- also called A_{ov}.

This method is both elegant and easy to understand, however, rather time consuming as soon as a large number of groups has to be dimensioned.

In order to save calculation time, it has been investigated whether an average, uniform "standardized" value $Z = \frac{V}{A_{ov}}$ could be applied. As a result, the Central Telecommunications Department (FTZ) of the Deutsche Bundespost found that for all national DDD tandem exchanges in the public telephone network a uniform value Z=1.5 ... 2.0 would be accurate enough. Using only one parameter (e.g. Z=2.0) the number of necessary tables is significantly reduced. Furthermore, it saves any individual variance calculation by planning engineers !

Thus one obtains the simplest dimensioning procedure for groups with offered overflow traffic.

3.4 The Economic Partitioning of the Traffic Offered to Alternatively Hunted Routes

The problem how to economically partition the traffic to the various high usage groups and at least to a final group regarding the different trunk costs per route is well known. I would like to refer only to the brilliant and extensive investigations of Y. Rapp /45,46,47/, C.W. Pratt /44/, R. Schehrer /49/ and many other scientists, e.g./4155,57,58/.

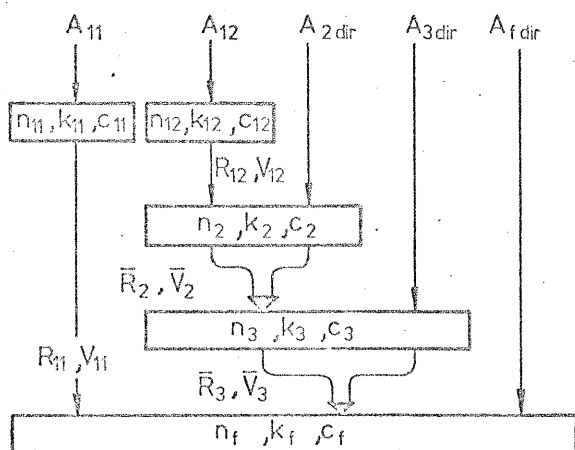In each case a cost function has to be set up. Fig. 8 shows an example:

Fig. 8:
Traffic flow in a tandem exchange

The cost function holds

$$c_{total} = \qquad\qquad\qquad\qquad\qquad (12)$$

$$c_{11} \cdot n_{11}(A_{11}, k_{11}, c_f, \Delta Y_f)$$

$$+c_{12} \cdot n_{12}(A_{12}, k_{12}, k_2, k_3, n_2, n_3, c_2, c_3,$$

$$c_f, \Delta Y_2, \Delta Y_3, \Delta Y_f)$$

$$+c_2 \cdot n_2(R_{12}, V_{12}, A_{2dir}, k_2, k_3, n_3, c_3, \Delta Y_3, \Delta Y_f)$$

$$+c_3 \cdot n_3(\bar{R}_2, \bar{V}_2, A_{3dir}, k_3, \Delta Y_f)$$

$$+c_f \cdot n_f(R_{11}, V_{11}, \bar{R}_3, \bar{V}_3, A_{fdir}, \Delta Y_f)$$

where $\Delta Y_2, \Delta Y_3$ are the marginal occupancies of the groups No. 2 and 3 and

$$\Delta Y_f = f(R_{11}, V_{11}, \bar{R}_3, \bar{V}_3, A_{fdir}, n_f, k_f, B_f)$$

is the marginal capacity of the final route.

One recognizes the mutual dependencies between all trunk groups being sequentially hunted.

Only the partial differentiation with respect to all free parameters, i.e. number n_{ij} of trunks per group, can yield the strictly exact economical optimum, that is to say the minimum costs for all outgoing routes as a whole.

However, because of the mutual influence of each group to the other ones which are hunted in series, this could only be done by means of much too complicated iteration procedures. This leads (in particular for the case of multiple overflow) to further unavoidable simplifications for practical use:

1) One dimensions all high usage groups of second, third etc. choice uniformly for an overflow of B=20 % (in any case high occupancy and economy!).

2) The marginal capacity ΔY_{fin} of the final route is assumed to be only $\Delta Y_{fin} = f(k_{fin}, B_{fin}) \neq f(n_{fin})$.

3) Basing on 1) and 2) one obtains a simple formula for the cost ratio P in case of single, double, triple overflow by partial derivation of the cost function; see Fig. 9 (cf. Annex in /22/).
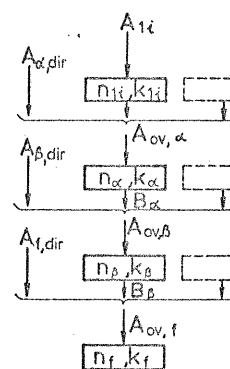
Fig. 9:
Network with triple overflow

Partial derivation of the cost function yields for triple overflow a cost ratio

$$P_{III} = P_{f/1i} \cdot \underbrace{\left[\frac{1-B_\alpha}{P_{f/\alpha}} + \frac{B_\alpha \cdot (1-B_\beta)}{P_{f/\beta}} + B_\alpha \cdot B_\beta \right]}_{\text{from a table}} \qquad (13)$$

(overflow $B_\alpha = B_\beta = 20\% = const.$).

For double overflow $B_\beta = 1.0$ $\qquad P_{II} = P_{f/1i} \cdot \dfrac{0.8}{P_{f/\alpha}}$ (14)

For single overflow $B_\alpha = 1.0$ $\qquad P_I = P_{f/1i}$. (15)

As it is known, the marginal occupancy of the considered first choice group has to be

$$\Delta Y_1 = \frac{\Delta Y_{fin}}{P} \qquad \text{where}$$

$P = P_I$ or P_{II} or P_{III}, respectively. Therewith n_{1i} can be determined. Details of the derivation can be drawn from the Annex in /22/.

4. THE DEVELOPED PLANNING METHOD

4.1 General

Basing on the considerations of Sections 3.2, 3.3 and 3.4 a Table Book /22/ was calculated which allows a very fast and economic dimensioning of the outgoing trunk groups in any tandem exchange with alternate routing. An extremely high number of test calculations has proved that the achieved network costs are very close (approx. ± 5%) to the theoretical minimum. Hereby the unavoidable inaccuracies of forecasted traffic values, standardized cost ratios etc. are not yet regarded.

4.2 The Procedure of Dimensioning

In any case the high usage groups of first order are dimensioned first, then the ones of second, third order etc. and at least the final group, respectively.

The accessibility of high usage groups of first, second ... order and the final group is set up before dimensioning the number of trunks and selectors (to be possibly changed again if the sum of available outlets of the respective switching stage is too small).

In this regard one has to notice that the subsequently hunted high usage groups of first order, furthermore possible high usage groups of second, third ... order and the final group as a whole have the characteristic features of one grading.

Therefore, the smallest necessary total number of outgoing trunks for a given overall grade of service is obtained (similar to an ordinary grading which is sequentially hunted from fixed home position), if the number of interconnected outlets increases from the first to the last hunting position.

Consequently, $2 \leq M_1 \leq M_2 \leq M_3 \leq M_f \leq g$ has to be realized for the grading ratios of the individual groups (whereby g is the number of grading (partial access) groups).

The use of the three main types of tables is explained by the example given in Fig.10 .
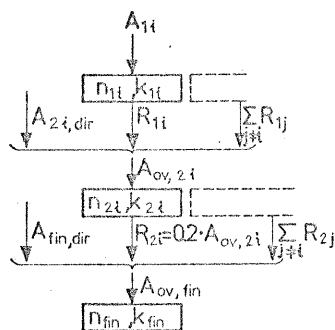

Fig. 10: Network with double overflow

a) Dimensioning of High Usage Groups of First Order (First Choice Groups):
The number of trunks n_1 and the corresponding overflowing traffic rest R_1 of the respective high usage group of first order can be read from the Table Fig. 11.

By means of the following data

A_{1i} offered random traffic to the respective group No. 1i

k_{1i} accessibility of this group No. 1i

k_{fin} accessibility of the final group

P cost ratio regarding the costs per trunk in the group No. 1i, as well as the trunk costs in the subsequently hunted groups (cf. Section 3.4 and the Annex in /22/)

one obtains from this Table in Fig. 11

n_{1i} the number of trunks of the group No. 1i

R_{1i} the overflowing traffic rest behind this group No. 1i

k of the final group	Cost Ratio P		$k_1 = 10$		
≤ 15	.. .. 1.6		1.8	2.0	..
k_{fin}	P		2.2	2.5	..
≥ 26	1.1 ..		2.5	3.0	..
	n_1 R_1		n_1 R_1	n_1 R_1	
$A_1 = 1$	.. ..		..	..	..
..	.. ..		..	..	..
A_1	n_1 R_1 38 2.75		40 2.15		..
31	.. .. 38 3.25		40 2.59	41 2.29	..
32	.. .. 39 3.42		41 2.75	43 2.16	..
33	.. .. 41 3.23		42 2.90	44 2.31	..
34	.. .. 42 3.40		43 3.06	45 2.45	..
..	.. ..		..	..	..

Fig. 11:
Table for the determination of the number of trunks n_1 and the overflowing traffic R_1 for high usage groups of first order as a function of the offered random traffic A_1, the cost ratio P and the accessibilities k_1 of the high usage group of first order and k_{fin} of the final group.

b) Dimensioning of High Usage Groups of Second, Third ... Order:
The offered traffic $A_{ov,2i}$ (Fig.10) includes all offered overflowing traffic rests R_{1i} and the directly offered random traffic $A_{2i,dir}$. These groups are dimensioned for a uniform probability of overflow B=20% and for the average (constant) variance-to-mean ratio $Z = \frac{V}{A_{ov}} = 2.0$ of the overflow.

The number of trunks n_{2i} for $A_{ov,2i}$ and the given accessibility k_{2i} can be obtained by the Table Fig. 12.

Fig. 12:
Table for the determination of the number of trunks n for high usage groups of second, third order etc. as a function of the offered overflow traffic A, the accessibility k and the probability of overflow B

c) Dimensioning of the Final Group:
In order to dimension final groups for a prescribed probability of loss, the Table in Fig. 13 is used.

Fig. 13:
Table for the determination of the number of trunks n for final groups as a function of the offered overflow traffic A, the accessibility k and the probability of loss B

4.3 Example

Let us regard a system with the offered traffics, accessibilities and cost ratios, respectively, as in Fig. 14.
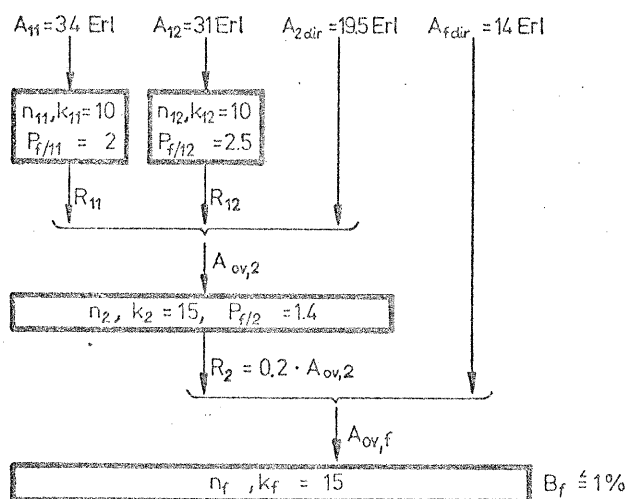

Fig. 14: System with double overflow

From Equation (14) one gets the cost ratios P_{II}:

$$P_{II,11} = 2 \cdot 0.771 = 1.54 \approx 1.6$$

$$P_{II,12} = 2.5 \cdot 0.771 = 1.93 \approx 2.0$$

From the table in Fig. 11 one obtains

$$n_{11} = 42 \qquad n_{12} = 41$$

$$R_{11} = 3.40 \text{ Erl} \qquad R_{12} = 2.29 \text{ Erl}$$

Therewith

$$A_{ov,2} = R_{11} + R_{12} + A_{2dir} = 25.2 \text{ Erl}$$

is the traffic offered to the second choice group.

From the table in Fig. 12 : $n_2 = 27$.

The traffic offered to the final group is

$$A_{ov,fin} = R_2 + A_{fin,dir} = (5.0 + 14.0) \text{Erl} = 19 \text{Erl}$$

From this follows by means of the table in Fig. 13

$$n_{fin} = 38 .$$

4.4 Two Auxiliary Tables

One table allows the determination of the r a n d o m traffic which is offered to a first high usage group ($A_{random} = A_{1i} = f(Y_{1i,measured}, n_{1i}, k_{1i})$).

Another table allows the same procedure for high usage groups of second, third, etc. order to which overflowing traffic (Z=2.0 assumed) is offered.

5. ON THE APPLICATION TO LINK SYSTEMS WITH ALTERNATE ROUTING (POINT TO GROUP SELECTION MODE)

5.1 Extension of the Method to Link Systems

Up to now, the alternate routing problem has been discussed only with respect to one stage group selection arrays having constant accessibilities per outgoing group.

The same methods can also be applied to link systems by means of the same tables.

5.2 Link Systems with Full Accessibility

Link systems with exact full access according to Clos /13/ et al, furthermore "wide" group selection link systems with "practically" full access (cf. /7/) are not problematic.

The outgoing groups are calculated for offered random traffic as well as for offered overflow traffic like behind a one stage array with full accessibility.

5.3 Link Systems with Limited Accessibilities

Alternate routing with full accessible groups only, is not in any case the most favourable way. Full access to all trunk groups saves trunks but it leads to a comparatively small capacity reserve of the network as a whole. In case of a cable breakdown or other reasons of overload a highly used network with full access groups only has the smallest capacity reserves and could yield a high increase of loss / 1,39 /.

Therefore, link systems with limited access and a smaller trunk occupancy, but with a lower increase of loss in case of overload, could also be considered.

The difficulty to dimension those link systems
is due to the well known fact that the
effective (limited) accessibility / 7 / per
outgoing group depends on the total traffic
carried by the link system and the group size
itself /17,18/.Consequently two ways of di-
mensioning are possible:

a) Iteration procedures regarding -- for pre-
   scribed traffic and loss -- the mutual
   dependency between accessibility and group
   size.

b) The design of smaller link system modules
   having practically full access to its N_m
   outlets. These N_m outlets are divided up to
   the outgoing groups and facilitate limited
   but constant accessibilities k_i to each
   group No. i, where $\sum_i k_i = N_m$.

The outlets k_i per group No. i of such LCA-
modules have now to be suitably graded to
the respective number n_i of outgoing trunks
(interconnection number $M \geq 2$). Such trunk
groups could be dimensioned like one stage
gradings. The tables according to Chapter 4
can be applied.

6. CONCLUSION

The methods described above prove that the
results of teletraffic theory can, in many
cases, be simplified so far that useful and
practical methods are achieved for the daily
work of telephone engineers without signi-
ficant loss of accuracy.

7. ACKNOWLEDGEMENT

LITERATURE

/1/ ASGERSEN, C.:
    A new Danish traffic routing plan with
    single stage alternate routing and con-
    sistent use of service protection finals
    for first routed traffic to tandem offices.
    5th ITC New York 1967, Prebook 407-417.

/2/ BÄCHLE, A.:
    On the calculation of full available
    groups with offered smoothed traffic.
    17th Report on Studies in Congestion
    Theory, Institute of Switching and Data
    Technics, University of Stuttgart, 1973.

/3/ BÄCHLE, A.:
    On the calculation of full available
    groups with offered smoothed traffic.
    7th ITC Stockholm 1973, Congressbook
    223/1-6.

/4/ BÄCHLE, A., HERZOG,U.:
    Die Berechnung einstufiger Koppelanord-
    nungen mit unvollkommener Erreichbarkeit
    bei angebotenem Zufallsverkehr 2. Art.
    Institute of Switching and Data Technics,
    University of Stuttgart, Monograph, 1966.

/5/ BAZLEN, D.:
    The dimensioning of trunk groups for
    standard gradings of the German GPO in
    case of finite number of traffic sources.
    Nachrichtentechn. Z. 25 (1972) 1, 50-52.

/6/ BAZLEN, D.:
    Link systems with both-way connections
    and outgoing finite-source traffic.
    7th ITC Stockholm 1973,Congressbook 316/1-8.

/7/ BAZLEN, D., KAMPE, G., LOTZE, A.:
    On the influence of hunting mode and link
    wiring on the loss of link systems.
    7th ITC Stockholm 1973, Congressbook
    232/1-12.

/8/ BOTSCH, D.:
    International standardizing of loss
    formulae ?
    5th ITC New York 1967, Prebook 90-95.

/9/ BRETSCHNEIDER, G.:
    Exact loss calculations of gradings.
    5th ITC New York 1967, Prebook 162-169.

/10/ BRIDGFORD, J.N.:
     The geometric group concept and its
     application to the dimensioning of link
     access systems.
     4th ITC London 1964, Doc. 13.

/11/ BROCKMEYER, E., HALSTRØM,H.L., JENSEN, A.:
     The life and works of A.K. Erlang.
     Transact. Danish Acad. Techn. Sci.No.2
     (1948), Copenhagen.

/12/ CHAPUIS, R.:
     Cooperation CCITT - ITC.
     6th ITC Munich 1970, Congressbook 51/1.

/13/ CLOS, C.:
     A study of non-blocking switching networks.
     Bell Syst.Tech.J. 32 (1953) 2, 406-424.

/14/ HAYWARD JR., W.S., WILKINSON, R.I.:
     Human factors in telephone systems and
     their influence on traffic theory,
     specially with regard to future facilities.
     6th ITC Munich 1970, Congressbook 431/1-10.

/15/ HEINRICH, G., TRAUTMANN, K.:
     Simplified standard gradings behind single-
     stage switching arrangements.
     Nachrichtentechn. Z. 21 (1968)7,412-417.

/16/ HERZOG, U.:
Die exakte Berechnung des Streuwerts von
Überlaufverkehr hinter Koppelanordnungen
beliebiger Stufenzahl mit vollkommener bzw.
unvollkommener Erreichbarkeit.
A.E.Ü. 20 (1966) 3, 180-184.

/17/ HERZOG, U.:
A general variance theory applied to link
systems with alternate routing.
5th ITC New York 1967, Prebook 398-406.

/18/ HERZOG, U.:
Die Bemessung von Linksystemen mit alter-
nativer Leitweglenkung nach dem verallge-
meinerten RDA-Streuwertverfahren.
Nachrichtentechn. Z. 23 (1970) 3, 143-150.

/19/ HERZOG, U.:
Calculation of fully available groups and
gradings for mixed pure chance traffic.
Nachrichtentechn. Z. 24(1971) 12,627-629.

/20/ HERZOG, U., LOTZE, A.:
The RDA-method, a method regarding the
variance coefficient for limited access
trunk groups.
Nachrichtentechn. Z. (Commun. J.) 7
(1968) 2/3, 47-52.

/21/ HERZOG, U., LOTZE, A., SCHEHRER, R.:
Calculation of trunk groups for simplified
gradings.
Nachrichtentechn. Z. 22(1969)12, 684-689.

/22/ HERZOG,U.,LÖRCHER,W.,LOTZE,A.,SCHEHRER,R.:
Alternate routing tables for the economic
dimensioning of telephone networks.
Institute of Switching and Data Technics,
University of Stuttgart, 1973.

/23/ HOFSTETTER, H., TRAUTMANN, K.:
Der Einfluß der Mischung auf die Verkehrs-
leistung der Abnehmerschaltglieder hinter
einstufigen Vermittlungsanordnungen.
Nachrichtentechn.Z. 16(1963) 12, 635-642.

/24/ IVERSEN, V.B.:
Analyses of real teletraffic processes
based on computerized measurements.
To be published in Ericsson Technics 29(1973)

/25/ IVERSEN, V.B., NYGAARD, J.:
Investigations of traffic arrival pro-
cesses based upon extensive traffic
measurements.
6th ITC Munich 1970, Congressbook
534/1-7.

/26/ JACOBAEUS, C.:
Application of traffic theory in industry
and administration.
6th ITC Munich 1970, Congressbook 131/1-9.

/27/ JENSEN, A.:
What does ITC expect from CCITT ?
6th ITC Munich 1970, Congressbook 511/1-2.

/28/ KATZ, S.S.:
Statistical performance analysis of a
switched communications network.
5th ITC New York 1967, Prebook 566-575.

/29/ KÜHN, P.:
Waiting time distributions in multi-queue
delay systems with gradings.
7th ITC Stockholm 1973, Congressbook
242/1-9.

/30/ KÜMMERLE, K.:
An analysis of loss approximations for
link systems.
a) 5th ITC New York 1967, Prebook 327-336
b) A.E.Ü. 25 (1971) 9/10, 466-471.

/31/ KÜMMERLE , K.:
Berechnungsverfahren für mehrstufige
Koppelanordnungen mit konjugierter Durch-
schaltung.
9th Report on Studies in Congestion
Theory, Institute of Switching and Data
Technics, University of Stuttgart, 1969.

/32/ LÖRCHER, W.:
Point-to-point selection in link systems.
7th ITC Stockholm 1973, Congressbook
317/1-8.

/33/ LOTZE, A.:
Loss formula, artificial traffic checks
and quality standards for characterizing
one stage gradings.
3rd ITC Paris 1961, Doc. 28.

/34/ LOTZE, A.:
Verluste und Gütemerkmale einstufiger
Mischungen.
Nachrichtentechn. Z. 14 (1961), 449-453.

/35/ LOTZE, A.:
3. Bericht über verkehrstheoretische
Untersuchungen (CIRB).
Institute of Switching and Data Technics,
University of Stuttgart. 1963.

/36/ LOTZE, A.:
A traffic variance method for gradings of
arbitrary type.
a) 4th ITC London 1964, Doc. 80
b) Post Off. Telecommun. J. Special Issue:
Report of the Proceedings of the Fourth
International Teletraffic Congress,
London 1964, p. 50.

/37/ LOTZE, A.:
Optimum link systems.
5th ITC New York 1967, Prebook 242-251.

/38/ LOTZE, A.:
History and development of grading theory.
a) 5th ITC New York 1967, Prebook 148-161
b) A.E.Ü. 25 (1971) 9/10, 402-410.

/39/ LOTZE, A.:
Problems of traffic theory in the design
of international direct distance dialling
networks.
Nachrichtentechn. Z. (Commun. J.) 7
(1968) 2/3, 41-46.

/40/ LOTZE, A., WAGNER, W.:
Tables of the Modified Palm-Jacobaeus-
Loss-Formula.
Institute of Switching and Data Technics,
University of Stuttgart, 1962.

/41/ LOTZE, A., SCHEHRER, R.:
The design of alternate routing systems
with regard to the variance coefficient.
Nachrichtentechn. Z. (Commun. J.) 7
(1968) 2/3,52-56.

/42/ MINA, R.:
Modification of the input process used
in classical traffic theory.
6th ITC Munich 1970, Congressbook 428-434.

/43/ NEWSTEAD, I. A., TÅNGE, I.:
What the CCITT expects of traffic theory.
6th ITC Munich 1970, Congressbook 512/1-6.

/44/ PRATT, C.W.:
The concept of marginal overflow in
alternate routing.
5th ITC New York 1967, Prebook 51-58.

/45/ RAPP, Y.:
Planning of junction network in a multi-
exchange area.
I. General Principles.
Ericsson Technics 20 (1964) 1, 77-130.

/46/ RAPP, Y.:
Planning of junction network in a multi-exchange area.
II. Extensions of the principles and applications.
Ericsson Technics 21 (1965) 2, 187-240.

/47/ RAPP, Y.:
Planning of junction network in a multi-exchange area.
III. Optimum types of physical and carrier circuits.
5th ITC New York 1967, Prebook 581-586.

/48/ RUBAS, J.:
Table of the Engset loss formula.
Fundamental Planning Section, Traffic Engineering Report No. 11, Postmaster-General's Department Headquarters, Melbourne, 1969.

/49/ SCHEHRER, R.:
Optimal design of alternate routing systems.
5th ITC New York 1967, Prebook 378-389.

/50/ SCHEHRER, R.:
On the exact calculation of overflow systems in switching networks.
10th Report on Studies in Congestion Theory, Institute of Switching and Data Technics, University of Stuttgart, 1969.

/51/ SCHEHRER, R.:
On the exact calculation of overflow systems.
a) 6th ITC Munich 1970, Congressbook 147/1-9
b) A.E.Ü. 25 (1971) 9/10, 426-430.

/52/ SCHEHRER, R.:
Exact calculation of overflow systems with one non-ideal grading and one ideal grading or full available group.
Nachrichtentechn. Z. 24 (1971)10, 532-534.

/53/ SCHEHRER, R.:
On the calculation of overflow systems with a finite number of sources and full available groups.
Proceedings of the Symposium on Computer-Communications Networks and Teletraffic, Polytechnic Institute of Brooklyn, April 4-6,1972, pp. 227-245.

/54/ SCHEHRER, R.:
On the calculation of overflow systems with full available groups and finite source traffic.
16th Report on Studies in Congestion Theory, Institute of Switching and Data Technics, University of Stuttgart, 1973.

/55/ SEGAL, M.:
Traffic engineering of communication networks with a general class of routing schemes.
4th ITC London 1964, Doc. 24.

/56/ WALLSTRÖM, B.:
Congestion studies in telephone systems with overflow facilities.
Ericsson Technics 22 (1966) 3, 187-351.

/57/ WALLSTRÖM, B.:
Methods for optimizing alternative routing networks.
Ericsson Technics 25 (1969) 1, 3-28.

/58/ WEBER, J.H.:
Some aspects of routing and control in communications networks.
4th ITC London 1964, Doc. 11.

/59/ WILKINSON, R.I., RIORDAN, J.:
Theories for toll traffic engineering in the USA.
a) 1st ITC Copenhagen 1955
b) Bell Syst.Techn.J. 35 (1956) 421-514.

/60/ WRIGHT, E.P.G.:
Cooperation: CCITT-ITC.
6th ITC Munich 1970, Congressbook 1/1.

/62/ Tables for overflow variance coefficient and loss of gradings and full available groups.
Institute of Switching and Data Technics, University of Stuttgart, 2nd Edition,1966.

/63/ Telephone traffic theory - tables and charts. Part 1
Siemens AG, Berlin-München, 1970.

/61/ Dimensioning data for planning of communication systems.
Telefonbau und Normalzeit, Frankfurt 1966.