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Session 8
GRADING AND LINK SYSTEMS
(Monday afternoon)

Chairman: H. Hochmuth

A TRAFFIC VARIANCE METHOD FOR GRADINGS OF ARBITRARY TYPE [80]

A. LOTZE

THE Equivalent Random Theory of R. J. Wilkinson and the Variance Coefficient Method of G. Bretschneider permit easy computation of variance and variance coefficient of an overflow traffic behind full available groups. The present paper deals with a computation method for gradings, that is to say primary groups as well as secondary groups with limited availability k, the number of lines being $n>k$.

To each out of g selector groups a pure chance traffic A_T may be offered. The probability of loss being B_k, a part-overflow traffic behind each selector group is

$$R_T = B_k \, . \, A_T.$$

This part-overflow traffic R_T will have a certain part-variance V_T or variance coefficient $D_T = V_T - R_T$. V_T and D_T shall be calculated.

Let us now replace one column of the grading by one gate, the "ON"—and "OFF"—states of which may have the same statistical properties as the busy and non-busy periods of the hunted k outlets of the grading.

The overflow traffic R_T and its variance V_T will remain unchanged by that substitute arrangement.

Next we look for a gate standing for a full available group having, too, k lines, to which another traffic A_0 is offered. This traffic A_0 may be chosen such that the blocking probability $E_k(A_0) = B_k$, being equal to the blocking probability of the grading. Then the overflow traffic amounts to

$$R_0 = A_0 \, . \, E_k(A_0) \neq R_T$$

In spite of this fact both gates obviously have the same average duration of blocking intervals and, too, of non-blocking intervals. Furthermore both gates have the same distribution function with regard to the duration of blocking intervals. Only the distribution function of non-blocking periods will not be exactly the same one for both gates, because the gate which stands for one column of the grading will be influenced by the type of the grading's interconnection method.

Admitting this only inaccuracy in calculating the variance V_T or variance coefficient D_T we are allowed to replace one single column of the grading by the following gate arrangement.

The wanted gate properties are considered to be generated by the offered traffic A_0. But during the ON-states of the gate, that is to say "all k outlets busy", the offered traffic may jump from the generating traffic A_0 to the actual traffic A_T.

Writing down the equations of state for that arrangement we find, by a way similar to that of Wilkinson and Riordan, the following formulae for variance V_T and variance coefficient D_T of the overflow traffic R_T behind one column of the grading:

$$V_T = R_T^2 \left[\frac{1}{E_k(A_0) \left\{ k+1-A_0(1-E_k(A_0)) \right\}} - 1 \right] + R_T$$

$$D_T = V_T - R_T$$
$$= p \, . \, R_T^2$$

$$p = \left[\frac{1}{E_k(A_0) \left\{ k+1-A_0(1-E_k(A_0)) \right\}} - 1 \right]$$

where p stands for the great bracket above and may be named the peakedness parameter.

Comparing this result with Bretschneider's variance coefficient D_0 of an overflow traffic behind a full available group having the same number of lines k and the same blocking probability $E_k(A_0) = B_k$ we find that in this case

$$D_0 = p \, . \, R_0^2.$$

Therefore the wanted part variance coefficient D_T of the grading is

$$D_T = \frac{D_0}{R_0^2} \, . \, R_T^2.$$

Taking into account the covariance between each pair out of the g part-overflow traffics R_T two approximate formulae can be derived for the inferior and superior limits of the total variance coefficient D which corresponds to the total overflow traffic R of the grading. The results are:

Inferior Limit $\quad D_I = V_I - R = p \, . \, R^2 . \frac{k}{n}$

Superior Limit $\quad D_{II} = V_{II} - R = D_I \cdot \left\{ 1 + \frac{n-k}{g.k} \right\}$

Average $\quad D_m = V_m - R = \frac{1}{2} \cdot \left\{ D_I + D_{II} \right\}$

g being the number of selector groups of the grading.

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The peakedness parameter $p=f(k, B_k)$ has been computed for the whole interesting range of k and B_k. It can be drawn directly from graphs, contained in the paper and furthermore in a new table book.

Finally we ask for the computation of a secondary grading to which several overflow traffics (R_i, D_i), as well as random traffics A_j are offered. The total offered traffic then amounts to:

$$R_{prim}=\overset{i}{\Sigma} R_i+\overset{j}{\Sigma} A_j$$

$$D_{prim}=\overset{i}{\Sigma} D_i$$

By means of these data and furthermore by the known availability k_{sec} of the secondary grading one can compute that ratio $\frac{n^*}{k^*}$ of one fitting Equivalent Substitute Primary Grading (SPG), which yields R_{prim} and D_{prim}. Using these data R_{prim}, D_{prim} and $\frac{n^*}{k^*}$ we can look up the RDA-Tables and read out directly the wanted data of the SPG, that is to say its number of lines n^*, availability k^* and its offered random traffic A^*.

Because of the properly chosen ratio $\frac{n^*}{k^*}$ this equivalent SPG is allowed to be considered as the first hunted part of a total grading, the second part of which consists of the secondary grading being calculated. Therefore the availability of this total grading amounts to:

$$k_{tot}=k^*+k_{sec}$$

and its total number of lines to

$$n_{tot}=n^*+n_{sec}$$

Being prescribed the secondary overflow, the loss of the total grading amounts to:

$$B_{tot}=\frac{R_{sec}}{A^*}$$

With (A^*, B_{tot}, k_{tot}) we can look up a loss table and find n_{tot}.

Hence, $n_{sec}=n_{tot}-n^*$

being the wanted number of lines of the secondary grading.

Vice versa one gets B_{tot} and $R_{sec}=A^*$. B_{tot} if the number of lines n_{sec} has been prescribed.

DISCUSSION

J. RUBAS: *Professor Lotze's paper complements his previous work on the dimensioning of gradings and now permits alternate routing computations to be performed in a network including graded routes.*

I would like to point out that the idea of an "on-off" gate in the derivation of the approximate formulae is essentially the same as the approach used by Mr. N. M. H. Smith in representing a loss system by a geometric group. It is interesting to note that professor Lotze's formulae give similar results to those obtained from the geometric group graphs (these are appended to Mr. J. N. Bridgford's paper No. 13, presented in session 1). Both methods, of course, give only the average efficiency of gradings designed in accordance with certain generally accepted standards.

Incidentally, what formula did Professor Lotze use to set the confidence limits for the estimates of the variance coefficients obtained by simulation?

A. LOTZE: *The valuable geometric group concept of Smith and Bridgford uses for each link between first and second stage of a two link group selector one individual gate, which is named a toggle in this paper and which has two state probabilities of "OFF/ON". This corresponds to the assumption of a binomial distribution on the link lines. By that method a geometric group 1, p, p^2, p^3 . . . arises from which one gets simplified values for the momentary blocking probabilities of the system. These p-values stand for the much more difficult exact combinatorial formulae, which would arise if one uses the idea of Erlang's Interconnection Formula.*

The idea of my paper uses another concept of gate-function. Here, one gate stands for a whole column of a grading and its statistical properties do not correspond to the average load per line. In spite of this, an auxiliary offered traffic A_o, which generates quite another load on a full available route, causes but the same statistical ON/OFF properties as the actual column of the grading has in case of actual traffic offered.

There is no doubt that in both cases the method of gate-substitute arrangements is successful. But the gate functions in the two theories are based on quite different substitute arrangements and have quite different meanings.

For the computation of confidence intervals Student's t-distribution has been applied, using ten up to thirty samples out of one trial, the number of calls being 100,000—300,000 for each point. A 95 per cent confidence interval has been computed.