POINT-TO-POINT SELECTION IN LINK SYSTEM'S

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ABSTRACT

This paper deals with link systems with different selection modes

A. point to point selection

B. point to point selection with repeated attempts (the maximum number of attempts can be prescribed)

C. point to group selection.

For these selection modes, the exact calculation of the probabilities of state and the characteristic traffic values, especially the point to point loss, are derived.

The considered link systems have $s \geq 2$ stages, the operation mode is groupselection and the special case of preselection is included.

The calculation method assumes Pure Chance Traffic of type 1 (PCT 1) as well as Pure Chance Traffic of type 2 (PCT 2).

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1. INTRODUCTION

The connecting network of modern telephone and /or data switching systems normally consists of space- or combined space and time division multiplex networks.

The space division multiplex networks are mostly multistage arrays with conjugated selection (link systems). The combined space and time division multiplex networks can be transformed into equivalent link systems, too; e.g. /1/.

For given traffic properties the main advantage of a link system, compared with a one stage connecting array, is the reduction of the number of crosspoints. However, this advantage requires on the other hand more complicated control. Therefore the selection to be made is of high interest.

The selection modes mostly used are:

- point to group selection:
  the marker controls a connection between a distinct free inlet and one out of all outgoing trunks of the considered group;
- point to point selection:
  both, inlet and outlet, are prescribed for the connection.

The second method seems to allow a simpler control, but considerably higher losses for the same traffic than the first method.

To combine the simplicity of the marker with the grade of service of point to group selection a third method can be realized,

- point to point selection with repeated attempts:
  for each attempt another free outgoing trunk is marked. The maximum number of attempts is normally limited to 3 or 4.

In traffic theory a large number of papers is known dealing with the first principle, e.g. /2/,/3/,/4/. (The two papers of K. Kümmerle /3/ and /4/ contain a presentation of a large number of calculation methods, e.g. the well-known publications of C. Jacobaeus, A.Elldin, P. Le Gall, A. Lotze, et.al.). In /5/ and /6/ models and calculation methods are described for the second principle.

In this paper the exact calculation of the probabilities of state and the characteristic traffic values for link systems are derived for these three operation principles basing on investigations for two stage link systems /7/, /8/, /9/.

For given structure of the link system, selection mode and offered traffic, the system of equations for the probabilities of state is derived. Then the characteristic traffic values (e.g. probability of loss, carried traffic) are determined by linear combination of the probabilities of state. Some results are presented in the last section.

ly link systems without gradings are considered. Investigations for link systems with gradings between the outlets of stage ν, $\nu \in [1,S)$ will be published /10/. Gradings between the outgoing trunks can be considered by means of /9/.

2. ASSUMPTIONS

2.1 STRUCTURE OF THE LINK SYSTEM

Link systems with S stages (fig.1) and the following parameters will be regarded:

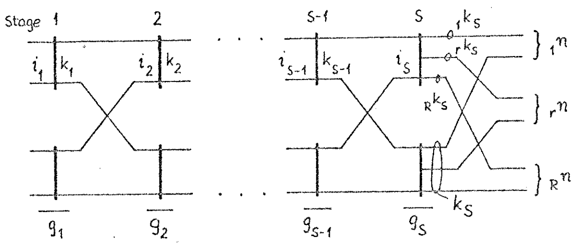

Fig. 1: Link system with S stages

S $\geq$ 2 stages

$i_\nu \gtrless k_\nu$; $\nu \in [1,S]$ expansion/concentration possible for each stage

$g_\nu k_\nu = g_{\nu+1} \cdot i_{\nu+1}$, $\nu \in [1,S)$ } without grading

$_rn = g_S \cdot {_rk_S}$ $\quad r \in [1,R]$

R = 1 preselection

$>$ 1 groupselection

2.2 TRAFFIC

Two types of offered traffic are distinguished:

a) Pure chance traffic of type 1 (PCT 1);

an infinite number of sources produces the offered traffic. The total arrival rate λ is constant and independent of the number of busy sources (Poisson Input).

b) Pure chance traffic of type 2 (PCT 2);

a finite number of sources produces the offered traffic. Each idle source has the constant arrival rate α (Bernoulli-Input).

The holding times are assumed to be negativ exponentially distributed with the mean value h; i.e. termination rate $\varepsilon = {}^1/h$.

B , arrival rate α, λ and termination rate ε, depend on the multiple v of stage 1 and on trunk group r. Therefore they will be denoted as $_r\alpha_v$, $_r\lambda_v$ and $_r\varepsilon_v$.

2.3 SELECTION MODES

Three selection modes are distinguished:

- point to point selection mode (PP-mode). The marker has to build up a connection from a distinct free inlet to a distinct free outgoing trunk of the considered group;

- point to point selection mode with repeated attempts (PR-mode). If the maximum number of attempts is limited e.g. to t attempts, then the marker has t attempts to build up a connection from the distinct free inlet to a distinct free outgoing trunk of the considered group;

- point to group selection mode (PG-mode). The marker has to build up a connection from a distinct free inlet to an outgoing trunk of the considered group.

I the number of attempts is equal to the number of free outgoing trunks of the considered group, the PR-mode is a realization of the PG-mode.

2.4 HUNTING MODES

Two hunting modes will be considered:

- sequential hunting with fixed home position. The outlets of the multiple will be hunted in sequential order and the first idle outlet will be occupied;

- random hunting. All idle outlets of the multiple will be occupied by the same probability.

3. DESCRIPTION OF THE LINK SYSTEM

3.1 ABBREVIATED NOTATIONS

In the following a number of abbreviated notations will be used:

$\{x\}$ occupation state with x occupied paths through the link system,
$(0 \leq x \leq \mathrm{Min}[g_\nu \cdot \mathrm{Min}[i_\nu, k_\nu], \nu \in [1,S]])$,
each state $\{x\}$ consists of a number of different occupation state patterns $\{\tilde{x}\}$.

$\{\tilde{x}\}$ occupation state pattern (state pattern) with x occupied paths

$\{\tilde{x}+1\}$ higher neighbouring state (HNS-) pattern with x+1 occupied paths, where x paths are identical to these of $\{\tilde{x}\}$

$\{\tilde{x}-1\}$ lower neighbouring state (LNS-) pattern with x-1 occupied paths which are identical to x-1 paths of $\{\tilde{x}\}$

$p(\tilde{x}), p(\tilde{x}+1), p(\tilde{x}-1)$ probabilities of state pattern (probabilities of state)

(w,v,ν) outlet w in multiple v of stage ν

(v,ν) multiple v of stage ν

$r|(m,S)$ trunk group r via multiple (m,S)

3.2 STRUCTURE DESCRIPTION

The wiring map of the link system with S stages is described uniquely by means of "wiring matrices $\Theta^{(\nu)}$", $\nu \in [1,S-1]$ with dimension $(g_\nu x k_\nu)$. Each element $\vartheta_{vw}^{(\nu)}$ denotes the number of the inlet of stage(ν+1) wired with the outlet (w,v,ν).

From this wiring matrix $\Theta^{(\nu)}$ a "multiple matrix $\phi^{(\nu)}$", $\nu \in [1,S-1]$ with the same dimension is derived. Each element $\varphi_{vw}^{(\nu)}$ denotes the number of the multiple of stage (ν+1) wired with the outlet (w,v,ν). If the inlets of stage ν are numbered: 1 (inlet no. 1 of multiple no. 1), ..., i_ν (inlet no. i_ν of multiple no. 1), $i_\nu+1$, ..., $g_\nu i_\nu$ (inlet no. i_ν of multiple no. g_ν), then $\varphi_{vw}^{(\nu-1)}$ can be determined according to (3.1):

$$\varphi_{vw}^{(\nu)} = \mathrm{ENTIER}\left(\frac{\vartheta_{vw}^{(\nu)} - 1}{i_{\nu+1}} + 1\right) \qquad \nu \in [1,S) \qquad (3.1)$$

3.3 DESCRIPTION OF THE OCCUPATION STATE PATTERN $\{\tilde{x}\}$

To denote the occupation state pattern $\{\tilde{x}\}$, with x occupied paths, $(g_\nu x k_\nu)$ dimensional "state matrices $S^{(\nu)}$", $\nu \in [1,S-1]$ are introduced. The element $s_{vw}^{(\nu)}$ characterizes the occupation state of the outlet (w,v,ν):

$$s_{vw}^{(\nu)} = 0 \text{ if the outlet is not occupied}$$
$$= \kappa_\nu \text{ if the outlet is occupied} \qquad (3.2)$$

where

$\nu = S-1$: $\kappa_\nu = r$ denotes the number of the outgoing trunk group r, $r \in [1,R]$ to which the path via the outlet (w,v,S-1) is connected.

$\nu \in [1, S-2] : \kappa_\nu = w$ denotes the number of the outlet $(w, \varphi_{\nu w}^{(\nu)}, \nu+1)$ connected with the outlet (w, v, ν).

The state pattern $\{\tilde{x}\}$ is described by

$$\{\| s_{ij}^{(1)} \|, \dots, \| s_{ij}^{(S-1)} \|\} \qquad (3.3)$$

It is useful to introduce a further quantity

$$s_{\nu w}^{(\nu)*} \begin{matrix} = 0 \\ = 1 \end{matrix} \Big\} \text{ if } s_{\nu w}^{(\nu)} \begin{cases} = 0 \\ > 0 \end{cases} \qquad (3.4)$$

4. EQUATION OF STATE

4.1 BASIC CONSIDERATIONS

Assuming stationarity, the following homogenous system of linear equations for the probabilities of state $p(\tilde{x})$ can be derived (statistical equilibrium) from the Chapman-Kolmogoroff-equation /10/:

$$\sum_i p_i(\tilde{x}+1) \cdot \varepsilon_i + \sum_j p_j(\tilde{x}-1) \cdot \lambda_j(\tilde{x}-1) \cdot \pi_j(\tilde{x}-1)$$
$$- p(\tilde{x}) \cdot \Big[\sum_i \lambda_i(\tilde{x}) \cdot \pi_i(\tilde{x}) + \sum_j \varepsilon_j \Big] = 0 \qquad (4.1a)$$

normalized by

$$\sum_{\tilde{x}} p(\tilde{x}) = 1 \qquad (4.1b)$$

where

$\sum_i, \sum_j$ contains all HNS-patterns $\{\tilde{x}+1\}_i$ or LNS-patterns $\{\tilde{x}-1\}_j$ respectively.

$\pi_i(\tilde{x}), \pi_j(\tilde{x}-1)$ characterizes the system influence that a call in $\{\tilde{x}\}$ or $\{\tilde{x}-1\}_j$, respectively generates $\{\tilde{x}+1\}_i$ or $\{\tilde{x}\}$ respectively.

$p_j(-1) = 0$ and $p_i(x = x_{max}+1) = 0$

$\sum_{\tilde{x}}$ contains all state patterns $\{\tilde{x}\}$

The main problem is to determine all HNS- and LNS-patterns and the individual coefficients π for a given state pattern $\{\tilde{x}\}$ in such a way that the equations for all probabilities of state $p(\tilde{x})$ can be determined per program by a digital computer.

Therefore a formal description is introduced. In the following four sections 4.2 to 4.5 the transitions from/to the neighbouring states will be considered for a certain ("considered") state pattern $\{\tilde{x}\}$.

The selection mode influences the two transitions

- from $\{\tilde{x}\}$ to the HNS-patterns $\{\tilde{x}+1\}$: $\{\tilde{x}\} \rightarrow \{\tilde{x}+1\}$
- from the LNS-patterns $\{\tilde{x}-1\}$ to $\{\tilde{x}\}$: $\{\tilde{x}-1\} \rightarrow \{\tilde{x}\}$

but it is without importance for the transitions

- from the HNS-patterns $\{\tilde{x}+1\}$ to $\{\tilde{x}\}$: $\{\tilde{x}+1\} \rightarrow \{\tilde{x}\}$
- from $\{\tilde{x}\}$ to the LNS-patterns $\{\tilde{x}-1\}$: $\{\tilde{x}\} \rightarrow \{\tilde{x}-1\}$

With these four transitions the system of equations (4.1) can be formulated in detail in section 4.6.

4.2 TRANSITION $\{\tilde{x}+1\} \rightarrow \{\tilde{x}\}$

For the considered state pattern $\{\tilde{x}\}$ all HNS-patterns $\{\tilde{x}+1\}$ and their individual transition coefficients must be determined.

4.2.1 HNS-PATTERN $\{\tilde{x}+1\}$

According to the definition of the HNS-pattern $\{\tilde{x}+1\}$ in section 3.1 all state patterns must be determined having one additional occupied path compared with $\{\tilde{x}\}$. Therefore within each matrix $\| s_{ij}^{(\nu)} \|$ only the value of one element $s_{\nu w}^{(\nu)}$ changes from 0 to κ_ν, all other elements remain unchanged. With the Kronecker-Symbol $\delta_{i\nu}$

$$\delta_{i\nu} \begin{matrix} = 1 \\ = 0 \end{matrix} \Big\} \text{ if } i \begin{cases} = \nu \\ \neq \nu \end{cases} \qquad (4.2)$$

the matrix becomes

$$\| s_{ij}^{(\nu)} + \kappa_\nu \cdot \delta_{i\nu}^{(\nu)} \cdot \delta_{jw}^{(\nu)} \|$$

The condition "all elements $s_{\nu w}^{(\nu)}$, $\nu \in [1, S)$ which changed their value from 0 to κ_ν establish one additional connected path" can be regarded by

$$\prod_{\nu=1}^{S-1} (1 - s_{\nu w}^{(\nu)*})$$

All patterns with one additional connection are obtained by

$$\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{w_1=1}^{k_1} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} \{\| s_{ij}^{(1)} + \kappa_1 \delta_{i\nu}^{(1)} \delta_{jw}^{(1)} \|, \dots, \| s_{ij}^{(S-1)} + \kappa_{S-1} \delta_{i\nu}^{(S-1)} \delta_{jw}^{(S-1)} \|\} \prod_{\nu=1}^{S-1} (1 - s_{\nu w}^{(\nu)*}) \qquad (4.3)$$

To get all HNS-patterns out of these patterns two restrictions must be regarded.
There must be at least in $\{\tilde{x}\}$

1) one idle source in multiple $(v,1)$; i.e. $\psi_v = 1$

$$\psi_v \begin{matrix} = 1 \\ = 0 \end{matrix} \Big\} \text{ if } s_{v.}^{(1)*} \begin{cases} < i_1 \\ = i_1 \end{cases} \quad \text{for PCT 2} \qquad (4.4.a)$$

$$\psi_v = 1 \quad \text{for PCT 1} \qquad (4.4.b)$$

where

$$s_{v.}^{(\nu)*} = \sum_{w=1}^{k_\nu} s_{\nu w}^{(\nu)*} \qquad (4.5)$$

2) one idle outlet of multiple $(\varphi_{\nu w}^{(S-1)}, S)$ to trunk group r; i.e. $(1 - {}_r\tau_{\nu w}^{(S-1)}) = 1$

$${}_r\tau_{\nu w}^{(S-1)} \begin{matrix} = 1 \\ = 0 \end{matrix} \Big\} \text{ if } \sum_{y=1}^{g_{S-1}} \sum_{z=1}^{k_{S-1}} \delta_{s_{yz}^{(S-1)}, r} \cdot \delta_{\varphi_{yz}^{(S-1)}, \varphi_{\nu w}^{(S-1)}} \begin{cases} = {}_r k_S \\ < {}_r k_S \end{cases} \qquad (4.6)$$

With these two conditions the HNS-patterns $\{\tilde{x}+1\}$ are given from (4.3) as

$$\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{w_1=1}^{k_1} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} \{\| s_{ij}^{(1)} + \kappa_1 \delta_{i\nu}^{(1)} \delta_{jw}^{(1)} \|, \dots, \| s_{ij}^{(S-1)} + \kappa_{S-1} \delta_{i\nu}^{(S-1)} \delta_{jw}^{(S-1)} \|\} \cdot$$
$$\cdot \prod_{\nu=1}^{S-1} (1 - s_{\nu w}^{(\nu)*}) \cdot (1 - {}_r\tau_{\nu w}^{(S-1)}) \cdot \psi_v \qquad (4.7)$$

4.2.2 TRANSITION

A HNS-pattern $\{\tilde{x}+1\}$ has one additional connection from multiple $(v,1)$ to group r. The transition to $\{\tilde{x}\}$ is therefore given by the termination rate ${}_r\varepsilon_v$ of this connection. The transition rate from all HNS-patterns $\{\tilde{x}+1\}$ into $\{\tilde{x}\}$ results from (4.7)

$$\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{w_1=1}^{k_1} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} p(\| s_{ij}^{(1)} + \kappa_1 \delta_{i\nu}^{(1)} \delta_{jw}^{(1)} \|, \dots, \| s_{ij}^{(S-1)} + \kappa_{S-1} \delta_{i\nu}^{(S-1)} \delta_{jw}^{(S-1)} \|) \cdot$$
$$\cdot \prod_{\nu=1}^{S-1} (1 - s_{\nu w}^{(\nu)*})(1 - {}_r\tau_{\nu w}^{(S-1)}) \cdot \psi_v \cdot {}_r\varepsilon_v \qquad (4.8)$$

4.3 TRANSITION $\{\tilde{x}\} \rightarrow \{\tilde{x}+1\}$

Here the HNS-patterns do not have to be described in detail, it is only necessary to determine whether a transition from $\{\tilde{x}\}$ to a HNS-pattern is possible.

A call occurs in multiple $(v,1)$ to group r with arrival rate ${}_r\lambda_v$

$${}_r\lambda_v = {}_r\alpha_v (i_1 - s_{v.}^{(1)*}) \quad \text{for PCT 2} \qquad (4.9.a)$$

$$= {}_r\lambda_v = \text{const.} \quad \text{for PCT 1} \qquad (4.9.b)$$

Now, it must be determined according to the selection modes if this call is successful or not.

4.3.1 PP-MODE

With probability ${}^{m}_{r}\Gamma$ an outlet of multiple (m,s) to trunk group r will be marked.

$$
{}^{m}_{r}\Gamma = \left.\begin{matrix} \dfrac{{}^{m}_{r}\gamma}{{}_{r}\gamma} \\ = 0 \end{matrix}\right\} \text{if } {}_{r}\gamma \begin{cases} > 0 \\ = 0 \end{cases} \qquad (4.10)
$$

where

$$
{}^{m}_{r}\gamma = {}_{r}k_S - \sum_{v=1}^{g_{S-1}} \sum_{w=1}^{k_{S-1}} \delta_{s_{vw}^{(S-1)},r} \cdot \delta_{\varphi_{vw}^{(S-1)},m} \qquad (4.11)
$$

number of idle outlets in multiple (m,S) to trunk group r

$$
{}_{r}\gamma = \sum_{m=1}^{g_S} {}^{m}_{r}\gamma \qquad (4.12)
$$

number of idle outlets to trunk group r

If the marked outlet of multiple (m,S) to group r is available via a free path from multiple (v,1) then the new connection will be built up. It is $(1 - {}^{m}_{r}\chi_v^{(1)}) = 1$ if at least one of such a free path exists.

Determination of ${}^{m}_{r}\chi_v^{(1)}$

To get ${}^{m}_{r}\chi_v^{(1)}$ it must be determined for all multiples $(v,\nu+1)$, $v = \varphi_{vw}^{(\nu)}$, $\nu \epsilon [1,S-2]$ if the number of free outlets to multiple (m,S) is greater than 0 and for multiple (m,S) if all outlets to trunk group r are occupied.

$\nu = S-1$: An outlet (w,v,S-1) is blocked to r|(m,S), i.e. ${}^{m}_{r}\omega_{vw}^{(S-1)} = 1$ if it is

- not occupied, i.e. $s_{vw}^{(S-1)*} = 0$
- wired to multiple (m,S) and blocked to trunk group r, i.e. $\delta_{\varphi_{vw}^{(S-1)},m} \cdot {}_{r}\tau_{vw}^{(S-1)} = 1$

  or wired to another multiple of stage S, i.e. $(1 - \delta_{\varphi_{vw}^{(S-1)},m}) = 1$

$$
{}^{m}_{r}\omega_{vw}^{(S-1)} = (1 - s_{vw}^{(S-1)*}) \cdot [1 - \delta_{\varphi_{vw}^{(S-1)},m} \cdot (1 - {}_{r}\tau_{vw}^{(S-1)})] \qquad (4.13.a)
$$

The multiple (v,S-1) is blocked to r|(m,S) if all outlets are occupied or blocked to r|(m,S), i.e.

$$
{}^{m}_{r}\chi_v^{(\nu)} = \left.\begin{matrix} 1 \\ 0 \end{matrix}\right\} \text{if } s_{v.}^{(\nu)*} + {}^{m}_{r}\omega_{v.}^{(\nu)} \begin{cases} = k_\nu \\ < k_\nu \end{cases} \qquad (4.14)
$$

where

$$
{}^{m}_{r}\omega_{v.}^{(\nu)} = \sum_{w=1}^{k_\nu} {}^{m}_{r}\omega_{vw}^{(\nu)} \qquad (4.15)
$$

$\nu \epsilon [1,S-2]$: An outlet (w,v,ν) is blocked to r|(m,S), i.e. ${}^{m}_{r}\omega_{vw}^{(\nu)} = 1$, if

- it is not occupied, i.e. $s_{vw}^{(\nu)*} = 0$
- the multiple $(v = \varphi_{vw}^{(\nu)}, \nu+1)$ is blocked to r|(m,S), i.e. ${}^{m}_{r}\chi_v^{(\nu+1)} = 1$

$$
{}^{m}_{r}\omega_{vw}^{(\nu)} = (1 - s_{vw}^{(\nu)*}) \cdot {}^{m}_{r}\chi_v^{(\nu+1)} \qquad (4.13.b)
$$

The multiple (v,ν) is blocked to r|(m,S) if ${}^{m}_{r}\chi_v^{(\nu)} = 1$, acc. to (4.14).

With probability ${}_{r}E_{v,1}$ the incoming call to multiple (v,1) can be connected to the marked outlet of trunk group r :

$$
{}_{r}E_v = {}_{r}E_{v,1} = \sum_{m=1}^{g_S} (1 - {}^{m}_{r}\chi_v^{(1)}) \cdot {}^{m}_{r}\Gamma \qquad (4.16)
$$

The transition rate from $\{\tilde{x}\}$ to all HNS-patterns $\{\tilde{x}+1\}$ is given by

$$
p(\|s_{ij}^{(1)}\|, \ldots, \|s_{ij}^{(S-1)}\|) \sum_{r=1}^{R} \sum_{v_1=1}^{g_1} {}_{r}E_v \cdot {}_{r}\lambda_v \qquad (4.17)
$$

4.3.2 PR-MODE

The first attempt was sucessful with probability ${}_{r}E_{v,1}$. A second attempt is made if

- there are two or more idle outlets to trunk group r and
- the first marked outlet was not available from multiple (v,1).

Therefore the number of available outlets remains constant $\sum_m (1 - {}^{m}_{r}\chi_v^{(1)}) \cdot {}^{m}_{r}\gamma$ and the number of markable outlets is reduced to ${}_{r}\gamma - 1$.

Now, with probability ${}_{r}E'_v$ an available outlet will be marked

$$
{}_{r}E'_v = \frac{\sum_m (1 - {}^{m}_{r}\chi_v^{(1)}) \cdot {}^{m}_{r}\gamma}{{}_{r}\gamma - 1} = {}_{r}E_{v,1} \cdot \frac{{}_{r}\gamma}{{}_{r}\gamma - 1} \qquad (4.18)
$$

The probability for a sucessful second attempt is given by

$$
{}_{r}E_{v,2} = \left.\begin{matrix} (1 - {}_{r}E_{v,1}) \cdot {}_{r}E_{v,1} \cdot \dfrac{{}_{r}\gamma}{{}_{r}\gamma - 1} \\ = 0 \end{matrix}\right\} \text{if } {}_{r}\gamma \left\{\begin{matrix} \geq 2 \\ < 2 \end{matrix}\right\} \text{and } {}_{r}\gamma - {}^{m}_{r}\gamma \begin{cases} > 0 \\ = 0 \end{cases} \qquad (4.19)
$$

The probability for success in one of the first two attempts is given by

$$
{}_{r}E_v = {}_{r}E_{v,1} + {}_{r}E_{v,2} \qquad (4.20)
$$

The probability for a successful t-th attempt is given by the recursion formula

$$
{}_{r}E_{v,t} = \left.\begin{matrix} (1 - {}_{r}E_{v,t-1}) \cdot {}_{r}E_{v,1} \cdot \dfrac{{}_{r}\gamma}{{}_{r}\gamma - t + 1} \\ = 0 \end{matrix}\right\} \text{if } {}_{r}\gamma \left\{\begin{matrix} \geq t \\ < t \end{matrix}\right\} \text{and } {}_{r}\gamma - {}^{m}_{r}\gamma \begin{cases} \geq t-1 \\ < t-1 \end{cases} \qquad (4.21)
$$

The transition rate is given according to (4.17) where

$$
{}_{r}E_v = \sum_{l=1}^{t} {}_{r}E_{v,l} \qquad (4.22)
$$

4.3.3 PG-MODE

Now the connection must be built up between the multiple (v,1) and one (out of all) idle outlets to trunk group r. Therefore the conditions (4.10),...,(4.15) can be combined. The coefficient ${}^{m}_{r}\chi_v^{(\nu)}$ can be replaced by ${}_{r}\chi_v^{(\nu)}$ where $(1 - {}_{r}\chi_v^{(\nu)}) = 1$ denotes that an idle outlet to trunk group r is available from an idle outlet (w,v,ν). ${}_{r}\chi_v^{(\nu)}$ is given according to (4.14) with ${}^{m}_{r}\omega_{v.}^{(\nu)}$ replaced by ${}_{r}\omega_{v.}^{(\nu)}$. In the same way ${}_{r}\omega_{v.}^{(\nu)}$ can be calculated according to (4.15) with ${}^{m}_{r}\omega_{vw}^{(\nu)}$ replaced by ${}_{r}\omega_{vw}^{(\nu)}$ according to (4.23).

$$
{}_{r}\omega_{vw}^{(\nu)} = \left.\begin{matrix} (1 - s_{vw}^{(S-1)*}) \cdot {}_{r}\tau_{vw}^{(S-1)} \\ = (1 - s_{vw}^{(\nu)*}) \cdot {}_{r}\chi_v^{(\nu+1)} \end{matrix}\right\} \text{if } \nu \begin{cases} = S-1 \\ \epsilon\, [1,S-2] \end{cases} \qquad (4.23)
$$

where $(v,\nu+1) = (\varphi_{vw}^{(\nu)}, \nu+1)$

The probability for success is given by

$$
{}_{r}E_v = (1 - {}_{r}\chi_v^{(1)}) \qquad (4.24)
$$

and the transition rate according to (4.17) with ${}_{r}E_v$ according to (4.24).

4.4 TRANSITION $\{\tilde{x}\} \rightarrow \{\tilde{x}-1\}$

This transition rate depends on the termination rate ${}_{r}\varepsilon_v$ of the established connections. A connection between the outlet (w,v,1) and trunk group r exists if

$$
\sum_{w_2=1}^{k_2} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)},\kappa_\nu} = 1 \qquad (4.25)
$$

Therefore the transition rate from $\{\tilde{x}\}$ to all LNS-patterns $\{\tilde{x}-1\}$ is given by

$$
p(\|s_{ij}^{(1)}\|, \ldots, \|s_{ij}^{(S-1)}\|) \sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{w_1=1}^{k_1} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)},\kappa_\nu} \cdot {}_{r}\varepsilon_v \qquad (4.26)
$$

4.5 TRANSITION $\{\tilde{x}-1\} \rightarrow \{\tilde{x}\}$

Firstly, all out of the neighbouring lower patterns must be determined which change to $\{\tilde{x}\}$ if a call occurs in multiple (v,1) to group r. Only these patterns are considered to be LNS-patterns. They depend on the selection- and on the hunting mode.

Secondly the transition rate can be determined.

A call occurs in multiple (v,1) to group r with arrival rate ${}_r\lambda_v^*$

$$
\begin{aligned}
{}_r\lambda_v^* &= {}_r\alpha_v \cdot (i_1 - s_{v.}^{(1)*} + 1) \quad \text{for PCT 2} \\
&= {}_r\lambda_v \quad \text{for PCT 1}
\end{aligned}
\tag{4.27}
$$

4.5.1 SELECTION MODES

4.5.1.1 PP-MODE

Similar to (4.3) together with (4.25) all interesting patterns with x-1 identical connections as $\{\tilde{x}\}$ are given by

$$
\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{m=1}^{g_S} \sum_{w_1=1}^{{}^m_r\mu_v^{(1)}} \cdots \sum_{w_{S-1}=1}^{{}^m_r\mu_v^{(S-1)}} \{\| s_{ij}^{(1)} - \kappa_1 \delta_{iv}^{(1)} \delta_{jw}^{(1)} \|, \ldots, \| s_{ij}^{(S-1)} - \kappa_{S-1} \delta_{iv}^{(S-1)} \delta_{jw}^{(S-1)} \| \} \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)}, \kappa_\nu}
\tag{4.28}
$$

The boundary values ${}^m_r\mu_v^{(\nu)} \le k_\nu$, $\nu \in [1,S)$ depend on the hunting mode:

Random hunting: ${}^m_r\mu_v^{(\nu)} = k_\nu \qquad \nu \in [1,S)$ (4.29.a)

Sequential hunting: i.e. the first hunted outlet of multiple (v,ν) which is idle to $r|(m,s)$ will be connected. Therefore by means of ${}^m_r\mu_v^{(\nu)}$ all outlets are excluded which follow after an outlet being idle to $r|(m,s)$ in $\{\tilde{x}\}$.

$$
{}^m_r\mu_v^{(\nu)} = \inf\left(\mu^* \,\middle|\, \sum_{z=1}^{\mu^*+1} {}^m_r u_{vz}^{(\nu)} < \mu^*+1 \;,\; \mu^* \in [0, k_\nu]\right)
\tag{4.29.b}
$$

with

$$
\begin{aligned}
{}^m_r u_{vz}^{(\nu)} &= s_{vz}^{(\nu)*} + {}^m_r\omega_{vz}^{(\nu)} \quad &\text{if } z \in [1, k_\nu] \\
&= 0 &\text{if } z = k_\nu + 1
\end{aligned}
$$

To get the LNS-patterns out of the patterns according to (4.28) further conditions must be introduced:

1) the terminated connection characterized by $\prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)}, \kappa_\nu}$ must be a connection from outlet (w,v,1) to (m,S) in $\{\tilde{x}\}$, i.e.

$$
\delta_{\varphi_{vw}^{(S-1)}, m} = 1
\tag{4.30}
$$

2) the marked outlet to trunk group r must be of multiple (m,S). This occurs by probability ${}^m_r\Gamma^*$

$$
{}^m_r\Gamma^* = \frac{{}^m_r\gamma + 1}{{}_r\gamma + 1}
\tag{4.31}
$$

3) the marked outlet of multiple (m,S) to trunk group r must be available from multiple (v,1) via the above mentioned path. This condition depends on the hunting mode.

A. Random Hunting

Here the number of free outlets of multiple (v,ν) to $r|(m,s)$ must be determined. In each stage ν, $\nu \in [2,S)$ the multiple $v = \varphi_{vw}^{(\nu-1)}$ connected with the outlet $(w,v,\nu-1)$ and the multiples $e = \varphi_{v\varsigma}^{(\nu-1)}$ connected with outlets $(\varsigma \neq w, v, \nu-1)$ or $(\varsigma, e, \nu-1)$ must be regarded. In the first stage only multiple (v,1) is of importance.

Multiple $(v, \nu = S-1)$: The number of idle outlets to $r|(m,S)$ is given by

$$
{}^m_r\pi_{vw}^{(\nu)} = k_\nu - s_{v.}^{(\nu)*} - {}^m_r\omega_{v.}^{(\nu)} + 1 - {}^m_r\beta_{vw}^{(\nu)}
\tag{4.32}
$$

where ${}^m_r\beta_{vw}^{(S-1)}$ denotes the outlets of multiple (v,S-1) which are blocked to $r|(m,S)$ in $\{\tilde{x}\}$ but free to $r|(m,S)$ if the connection via outlet (w,v,S-1) has been terminated

$$
{}^m_r\beta_{vw}^{(S-1)} = \sum_{\varsigma=1}^{k_{S-1}} (1 - s_{v\varsigma}^{(S-1)*}) \cdot \delta_{\varphi_{v\varsigma}^{(S-1)}, \varphi_{vw}^{(S-1)}} \cdot {}_r\tau_{vw}^{(S-1)}
\tag{4.33.a}
$$

with $m = \varphi_{vw}^{(S-1)}$

Multiple $(e \neq v, \nu = S-1)$: It is blocked to $r|(m,S)$ in $\{\tilde{x}\}$ and has at least one idle outlet to $r|(m,S)$ in the lower pattern i.e. ${}^m_r\chi_e^{(S-1)*} = 1$ if

- all outlets are occupied or blocked to $r|(m,S)$; i.e. ${}^m_r\chi_e^{(S-1)} = 1$ according to (4.14)
- if at least one outlet ς (which is not occupied) is wired with multiple (m,S) and this multiple is blocked to trunk group r; i.e. ${}^m_r\sigma_e^{(S-1)} = 1$

$$
{}^m_r\sigma_e^{(S-1)} \begin{cases} = 1 \\ = 0 \end{cases} \text{if } \sum_{\varsigma=1}^{k_{S-1}} (1 - s_{e\varsigma}^{(S-1)*}) \cdot \delta_{\varphi_{e\varsigma}^{(S-1)}, \varphi_{vw}^{(S-1)}} \cdot {}_r\tau_{vw}^{(S-1)} \begin{cases} > 0 \\ = 0 \end{cases}
\tag{4.34.a}
$$

with $m = \varphi_{vw}^{(S-1)}$

Thus

$$
{}^m_r\chi_e^{(S-1)*} = {}^m_r\chi_e^{(S-1)} \cdot {}^m_r\sigma_e^{(S-1)}
\tag{4.35}
$$

Multiple (v,ν), $\nu \in [1,S-2]$: The number of idle outlets to $r|(m,s)$ is given by (4.32) where ${}^m_r\beta_{vw}^{(\nu)}$ has the same content as ${}^m_r\beta_{vw}^{(S-1)}$ but the formal determination must be modified.

$$
{}^m_r\beta_{vw}^{(\nu)} = \sum_{\varsigma=1}^{k_\nu} (1 - s_{v\varsigma}^{(\nu)*}) \cdot \left[\delta_{\varphi_{v\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}} \cdot {}^m_r\chi_{\varphi_{vw}^{(\nu)}}^{(\nu+1)} + (1 - \delta_{\varphi_{v\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}}) \cdot {}^m_r\chi_e^{(\nu+1)*}\right]
\tag{4.33.b}
$$

where ${}^m_r\chi_e^{(\nu+1)*}$ acc. to (4.35) with $S-1 \neq \nu+1$; $e_{\nu+1} = \varphi_{v\varsigma}^{(\nu)}$

Multiple $(e \neq v, \nu)$, $\nu \in (1,S-2)$: The same conditions are valid as for (e,S-1); therefore is

${}^m_r\chi_e^{(\nu)*}$ acc. to (4.35) with $S-1 \neq \nu$

${}^m_r\chi_e^{(\nu)}$ acc. to (4.14)

$$
{}^m_r\sigma_e^{(\nu)} \begin{cases} = 1 \\ = 0 \end{cases} \text{if } \sum_{\varsigma=1}^{k_\nu} (1 - s_{e\varsigma}^{(\nu)*}) \cdot \left[\delta_{\varphi_{e\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}} \cdot {}^m_r\chi_v^{(\nu+1)} + (1 - \delta_{\varphi_{e\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}}) \cdot {}^m_r\chi_e^{(\nu+1)*}\right] \begin{cases} > 0 \\ = 0 \end{cases}
\tag{4.34.b}
$$

with

$e_{\nu+1} = \varphi_{e\varsigma}^{(\nu)}$; $v_{\nu+1} = \varphi_{vw}^{(\nu)}$

The probability that multiple (v,1) and the marked outlet in multiple (m,S) to trunk group r will be connected via the path (4.25) becomes

$$
{}^m_r\eta_{v,w_1 \cdots w_{S-1}} = \left[\prod_{\nu=1}^{S-1} {}^m_r\pi_{vw}^{(\nu)}\right]^{-1}
\tag{4.36}
$$

B. Sequential Hunting

Two conditions must be regarded for each multiple (v,ν), $\nu \in [1,S)$. In multiple (v,ν) is no outlet ς which

- will be hunted before the outlet w and
- is backwardly blocked to $r|(m,S)$ in $\{\tilde{x}\}$ but idle in the lower pattern.

Then the lower pattern is a LNS-pattern. With regard to ${}^m_r\mu_v^{(\nu)}$ the following conditions are simplified:

According to A. the multiples must be distinguished:

multiple (v,S-1): There is no outlet ς blocked by the outlet w if $(1 - \beta_{vw}^{(S-1)}) = 1$

$$
\beta_{vw}^{(S-1)} \begin{cases} = 1 \\ = 0 \end{cases} \text{if } \sum_{\varsigma=1}^{w-1} (1 - s_{v\varsigma}^{(S-1)*}) \cdot \delta_{\varphi_{v\varsigma}^{(S-1)}, \varphi_{vw}^{(S-1)}} \begin{cases} > 0 \\ = 0 \end{cases}
\tag{4.37.a}
$$

multiple $(e \neq v, S-1)$: It has an outlet blocked by outlet (w,v,s-1) if $\sigma_e^{(S-1)} = 1$

$$
\sigma_e^{(S-1)} \begin{cases} = 1 \\ = 0 \end{cases} \text{if } \sum_{\varsigma=1}^{k_{S-1}} (1 - s_{e\varsigma}^{(S-1)*}) \cdot \delta_{\varphi_{e\varsigma}^{(S-1)}, \varphi_{vw}^{(S-1)}} \begin{cases} > 0 \\ = 0 \end{cases}
\tag{4.38.a}
$$

multiple (v,ν), $\nu \in [1, S-1)$: No outlet ς is blocked by the outlet w if $(1-\beta_{vw}^{(\nu)}) = 1$

$$\beta_{vw}^{(\nu)} \begin{array}{l} = 1 \\ = 0 \end{array} \Bigg\} \text{ if } \sum_{\varsigma=1}^{w-1} (1-s_{v\varsigma}^{(\nu)*}) \left[\delta_{\varphi_{v\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}} + (1-\delta_{\varphi_{v\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}}) \cdot G_e^{(\nu+1)}\right] \begin{cases} > 0 \\ = 0 \end{cases} \tag{4.37.b}$$

with
$e = \varphi_{v\varsigma}^{(\nu)}$

multiple $(e \neq v, \nu)$, $\nu \in (1, S-1)$: According to (4.38.a) $G_e^{(\nu)}$ becomes

$$G_e^{(\nu)} \begin{array}{l} = 1 \\ = 0 \end{array} \Bigg\} \text{ if } \sum_{\varsigma=1}^{k_\nu} (1-s_{e\varsigma}^{(\nu)*}) \cdot \left[\delta_{\varphi_{e\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}} + (1-\delta_{\varphi_{e\varsigma}^{(\nu)}, \varphi_{vw}^{(\nu)}}) \cdot G_e^{(\nu+1)}\right] \begin{cases} > 0 \\ = 0 \end{cases} \tag{4.38.b}$$

with
$e_{\nu+1} = \varphi_{e\varsigma}^{(\nu)}$

By ${}^m_r\eta_{v_1 w_1 \dots w_{S-1}}$ it is denoted, whether the connection is built up via the path (4.25):

$${}^m_r\eta_{v_1 w_1 \dots w_{S-1}} = \prod_{\nu=1}^{S-1} (1-\beta_{vw}^{(\nu)}) \tag{4.39}$$

The LNS-patterns $\{\tilde{x}-1\}$ are given by (4.28) multiplied with (4.30) and with

$${}^m_rE^*_{\cdot\cdot} = {}^m_rE^*_{v,1} = {}^m_rE^*_{v_1 w_1 \dots w_{S-1}} = {}^m_r\Gamma^* \cdot {}^m_r\eta_{v_1 w_1 \dots w_{S-1}} \tag{4.40}$$

4.5.1.2 PR-MODE

The first attempt is successful with ${}^m_rE^*_{v,1}$. A second attempt arises if

- the first marked outlet not part of the multiple (m,S). This takes place with $(1-{}^m_r\Gamma^*)$. Then with probability

$$\left.\begin{array}{c} \dfrac{{}^m_r\gamma+1}{{}_r\gamma} \\ 0 \end{array}\right\} \text{ if } {}_r\gamma \begin{cases} > 0 \\ = 0 \end{cases}$$

now an outlet of the multiple (m,S) will be marked and this second attempt will be successful with

$$\left.\begin{array}{l} {}^m_rE^{*\prime}_v = (1-{}^m_r\Gamma^*) \dfrac{{}^m_r\gamma+1}{{}_r\gamma} \cdot {}^m_r\eta_{v_1 w_1 \dots w_{S-1}} = {}^m_rE^*_{v,1} \dfrac{{}_r\gamma - {}^m_r\gamma}{{}_r\gamma} \\ = 0 \end{array}\right\} \text{ if } {}_r\gamma \begin{cases} > 0 \\ = 0 \end{cases} \tag{4.41}$$

- the first marked outlet was in multiple (m,S) but the connection was not built up according to the considered path; this fact arises with ${}^m_r\Gamma^*(1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}})$. Now, the second attempt will be successful with

$$\left.\begin{array}{l} {}^m_rE^{*\prime\prime}_v = (1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}}) \cdot {}^m_r\Gamma^* \dfrac{{}^m_r\gamma}{{}_r\gamma} \cdot {}^m_r\eta_{v_1 w_1 \dots w_{S-1}} = {}^m_rE^*_{v,1} \dfrac{{}^m_r\gamma}{{}_r\gamma} (1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}}) \\ = 0 \end{array}\right\} \text{ if } {}_r\gamma \begin{cases} > 0 \\ = 0 \end{cases} \tag{4.42}$$

($^m_rE^{*\prime\prime}_v = 0$ for sequential hunting, because ${}^m_r\eta_{v_1 w_1 \dots w_{S-1}}$ is equal to 0 or 1)

The probability for success in the second attempt is

$$\left.\begin{array}{l} {}^m_rE^*_{v,2} = {}^m_rE^{*\prime}_v + {}^m_rE^{*\prime\prime}_v \\ = \dfrac{{}^m_rE^*_{v,1}}{\binom{{}_r\gamma}{1}} \left[\binom{{}_r\gamma - {}^m_r\gamma}{1} + (1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}})\binom{{}^m_r\gamma}{1}\right] \\ = 0 \end{array}\right\} \text{ if } {}_r\gamma \begin{cases} > 0 \\ = 0 \end{cases} \tag{4.43}$$

and for success in one of the first two attempts

$${}^m_rE^*_v = {}^m_rE^*_{v,1} + {}^m_rE^*_{v,2} \tag{4.44}$$

Analogously, the probability for success in the t-th attempt becomes

$$\left.\begin{array}{l} {}^m_rE^*_{v,t} = \dfrac{{}^m_rE^*_{v,1}}{\binom{{}_r\gamma}{t-1}} \cdot \Bigg[\binom{{}_r\gamma - {}^m_r\gamma}{t-1} + \binom{{}_r\gamma - {}^m_r\gamma}{t-2}(1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}})\binom{{}^m_r\gamma}{1} + \dots + \binom{{}_r\gamma - {}^m_r\gamma}{1} \\ \qquad \cdot (1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}})^{t-2}\binom{{}^m_r\gamma}{t-2} + (1-{}^m_r\eta_{v_1 w_1 \dots w_{S-1}})^{t-1}\binom{{}^m_r\gamma}{t-1}\Bigg] \\ = 0 \end{array}\right\} \text{ if } {}_r\gamma \begin{cases} > t-2 \\ \leq t-2 \end{cases} \tag{4.45}$$

and the probability for success within the first t attempts

$${}^m_rE^*_v = \sum_{l=1}^{t} {}^m_rE^*_{v,l} \tag{4.46}$$

4.5.1.3 PG-MODE

Here the condition $r|(m,S)$ has to be replaced by r, i.e. the sum over m and the index m vanish. Thus the LNS-patterns are given by (4.28) and (4.40)

$$\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{w_1=1}^{{}_r\mu_v^{(1)}} \cdots \sum_{w_{S-1}=1}^{{}_r\mu_v^{(S-1)}} \{\|s_{ij}^{(1)} - \kappa_1 \delta_{iv}^{(1)} \delta_{jw}^{(1)}\|, \dots, \|s_{ij}^{(S-1)} - \kappa_{S-1} \delta_{iv}^{(S-1)} \delta_{jw}^{(S-1)}\|\} \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)}, \kappa_\nu} \cdot {}_rE^*_v \tag{4.47}$$

where ${}_rE^*_v$ according to (4.40) and
${}_r\mu_v^{(\nu)}$ according to (4.29) with regard to condition $r|(m,S) \hat{=} r$

4.5.2 TRANSITION

The transition rate from all LNS-patterns $\{\tilde{x}-1\}$ to the considered state pattern $\{\tilde{x}\}$ is given by

$$\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \sum_{m=1}^{g_S} \sum_{w_1=1}^{{}^m_r\mu_v^{(1)}} \cdots \sum_{w_{S-1}=1}^{{}^m_r\mu_v^{(S-1)}} p(\|s_{ij}^{(1)} - \kappa_1 \delta_{iv}^{(1)} \delta_{jw}^{(1)}\|, \dots, \|s_{ij}^{(S-1)} - \kappa_{S-1} \delta_{iv}^{(S-1)} \delta_{jw}^{(S-1)}\|) \cdot \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)}, \kappa_\nu} \cdot \delta_{\varphi_{vw}^{(S-1)}, m} \cdot {}^m_rE^*_v \cdot {}_r\lambda^*_v \tag{4.48}$$

with ${}^m_r\mu_v^{(\nu)}$ according to (4.29)
${}^m_rE^*_v$ according to (4.40) for PP-Mode
(4.46) for PR-Mode
(4.40) for PG-Mode with regard to condition $r|(m,S) \hat{=} r$

Additional for PG-Mode the sum over m and the index m vanish, $\delta_{\varphi_{vw}^{(S-1)}, m} = 1$.

4.6 SYSTEM OF EQUATIONS

From all transitions (4.8), (4.17), (4.26) and (4.48) the system of equations according to (4.1) is given by

$$\begin{aligned} &\sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \Bigg[\sum_{w_1=1}^{k_1} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} p(\|s_{ij}^{(1)} + \kappa_1 \delta_{iv}^{(1)} \delta_{jw}^{(1)}\|, \dots, \|s_{ij}^{(S-1)} + \kappa_{S-1} \delta_{iv}^{(S-1)} \delta_{jw}^{(S-1)}\|) \cdot \prod_{\nu=1}^{S-1} (1-s_{vw}^{(\nu)*})(1-{}_r\tau_{vw}^{(S-1)}) \cdot \psi_v \cdot {}_r\varepsilon_v \\ &+ \sum_{m=1}^{g_S} \sum_{w_1=1}^{{}^m_r\mu_v^{(1)}} \cdots \sum_{w_{S-1}=1}^{{}^m_r\mu_v^{(S-1)}} p(\|s_{ij}^{(1)} - \kappa_1 \delta_{iv}^{(1)} \delta_{jw}^{(1)}\|, \dots, \|s_{ij}^{(S-1)} - \kappa_{S-1} \delta_{iv}^{(S-1)} \delta_{jw}^{(S-1)}\|) \cdot \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)}, \kappa_\nu} \cdot \delta_{\varphi_{vw}^{(S-1)}, m} \cdot {}^m_rE^*_v \cdot {}_r\lambda^*_v \Bigg] \\ &- p(\|s_{ij}^{(1)}\|, \dots, \|s_{ij}^{(S-1)}\|) \sum_{r=1}^{R} \sum_{v_1=1}^{g_1} \Big[{}_rE_v \cdot {}_r\lambda_v + \sum_{w_1=1}^{k_1} \cdots \sum_{w_{S-1}=1}^{k_{S-1}} \prod_{\nu=1}^{S-1} \delta_{s_{vw}^{(\nu)}, \kappa_\nu} \cdot {}_r\varepsilon_v \Big] = 0 \end{aligned} \tag{4.49.a}$$

normalized by

$$\sum_{\tilde{x}} p(\|s_{ij}^{(1)}\|, \dots, \|s_{ij}^{(S-1)}\|) = 1 \tag{4.49.b}$$

5. CHARACTERISTIC TRAFFIC VALUES

5.1 PROBABILITY p(x)

From the probabilities of state $p(\tilde{x})$ the probability $p(x)$ "x paths occupied" can be determined as

$$p(x) = \sum_{\tilde{x}} [p(\|s_{ij}^{(1)}\|, \dots, \|s_{ij}^{(S-1)}\|) \cdot \delta_{x, s_{\cdot\cdot}^{(1)*}}] \tag{5.1}$$

with $s_{\cdot\cdot}^{(1)*} = \sum_{v=1}^{g_1} s_{v\cdot}^{(1)*}$

with corresponding equations other probabilities e.g. p(x outlets of multiple (v,ν) occupied) or p (x outlets to trunk group r occupied) can be calculated.

5.2 PROBABILITY OF LOSS

5.2.1 POINT TO POINT LOSS

According to /6/ the point to point loss is defined as B_{NF1} or B_{NF2}, respectively.

$$B_{NF1} = \frac{\text{number } C_{NF} \text{ of calls which cannot be connected to the marked outlet}}{\text{number } C_A^* \text{ of calls which have arrived in the state "at least one outgoing line is idle"}}$$

$$B_{NF2} = \frac{C_{NF}}{\text{number } C_A \text{ of calls which have arrived all together}}$$

With these definitions the point to point loss ${}_rB_{v,NF1}$ per multiple (v,1) to group r is given by

$${}_rB_{v,NF1} = \frac{\sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\cdot(1-{}_rE_v)\cdot(1-{}_r\Gamma)\cdot{}_r\lambda_v\right]}{\sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\cdot(1-{}_r\Gamma)\cdot{}_r\lambda_v\right]} \qquad (5.2)$$

and

$${}_rB_{v,NF2} = \frac{\sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\cdot(1-{}_rE_v)\cdot(1-{}_r\Gamma)\cdot{}_r\lambda_v\right]}{\sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\cdot{}_r\lambda_v\right]} \qquad (5.3)$$

with

$$\left.\begin{array}{l}{}_r\Gamma = 1\\ \quad\;\; = 0\end{array}\right\} \text{if } {}_r\gamma \begin{cases} = 0\\ > 0\end{cases} \qquad (5.4)$$

${}_rE_v$ according to (4.16) for PP-mode
(4.22) for PR-mode
(4.24) for PG-mode

The probabilities ${}_rB_{NFi}$, B_{vNFi} or B_{NFi}, i=1,2 are determined according to (5.2) and (5.3) where in nominator and denominator it must be summed over v,r or r and v.

5.2.2 PROBABILITY OF LOSS

Additionally to C_{NF} the number of calls which have arrived in the state "all outgoing lines are occupied" must be regarded. Thus the probability of loss, ${}_rB_v$ for multiple (v,1) to group r is

$${}_rB_v = \frac{\sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\cdot(1-{}_rE_v)\cdot{}_r\lambda_v\right]}{\sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\cdot{}_r\lambda_v\right]} \qquad (5.5)$$

Other probabilities as ${}_rB$, B_v or B can be determined by corresponding summations in nominator and denominator of (5.5)

5.3 CARRIED TRAFFIC

The carried traffic ${}_rY_v$ per multiple (v,1) to group r is given by

$${}_rY_v = \sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\sum_{w_1=1}^{k_1}\cdots\sum_{w_{S-1}=1}^{k_{S-1}}\prod_{\nu=1}^{S-1}\delta_{s_{vw}^{(\nu)},\kappa_\nu}\right] \qquad (5.6)$$

corresponding to (5.6) ${}_rY$ is determined as

$${}_rY = \sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\sum_{v_{S-1}=1}^{g_{S-1}}\sum_{w_{S-1}=1}^{k_{S-1}}\delta_{s_{vw}^{(S-1)},r}\right] \qquad (5.7)$$

Other carried traffics e.g. $Y_v^{(\nu)}$, Y can be calculated by similiar expressions.

5.4 OFFERED TRAFFIC

The offered traffic ${}_rA_v$ per multiple (v,1) to group r is

$${}_rA_v = \sum_{\tilde{x}}\left[p(\|s_{ij}^{(1)}\|,\ldots,\|s_{ij}^{(S-1)}\|)\frac{{}_r\lambda_v}{{}_r\varepsilon_v}\right] \qquad (5.8)$$

the other offered traffics ${}_rA$, A_v and A can be calculated by corresponding summations of ${}_rA_v$.

6. EXAMPLES

The figures 3 - 5 show some results for a small linkssystem with S=3 stages for groupselection (R=2) with PP-mode (fig.2).

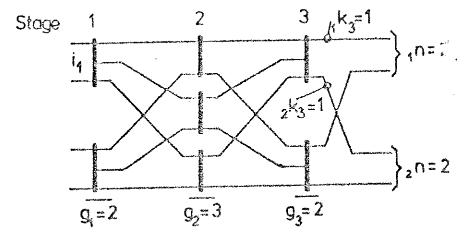

Fig.2 : Link system with S=3 stages and R=2 outgoing trunk groups
Selection mode : PP-mode
Hunting mode : Sequential hunting

This system has 817 different state patterns $\{\tilde{x}\}$.

The above described algorithm had been programmed in FORTRAN. It was partitioned in three parts :

- determination of the state patterns $\{\tilde{x}\}$
- determination of the system of equations for the probabilities of state $p(\tilde{x})$
- solving of the system of equations, i.e. calculation of the probabilities of state $p(\tilde{x})$ and calculation of the characteristic traffic values.

The first program is used only once for each link system. The second and third program are used for each traffic value.

The system of equations for the probabilities of state $p(\tilde{x})$ is solved by the Successive Overrelaxation Method. Normally, less then 25 iteration cycles are necessary for a accuracy EPS $\leq 10^{-9}$ /10/.

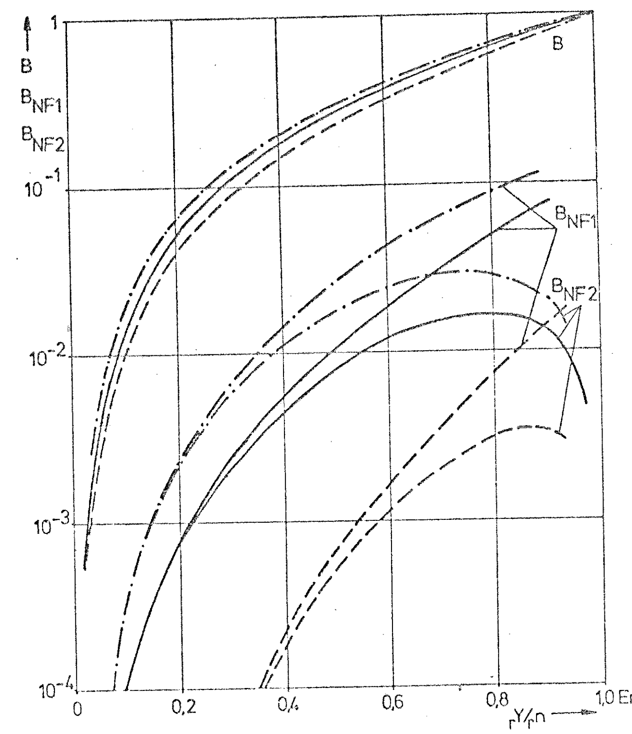

Fig.3 : Point to point loss B_{NF1} and B_{NF2} and the probability of loss B as a function of the carried traffic ${}_rY/{}_rn$ per outgoing trunk (r=1,2) for balanced offered traffic
Parameter : —·— PCT1
——— PCT2, i_1 = 6
— — — PCT2, i_1 = 3
System : see fig.2

Figure 3 shows the point to point loss ${}_rB_{NF1}$ and ${}_rB_{NF2}$ as a function of the carried traffic ${}_rY/{}_rn$ per trunk group (r=1,2). Furthermore, the probability of loss B, which includes the states "all outgoing trunks occupied" acc. to (5.5), is shown. By reason of balanced offered traffic it is ${}_rB = B_y = B$ and the corresponding properties for the point to point losses yield. This figure shows the transition from PCT2 with $i_1 = k_1$ to PCT1.

Figure 4 shows for the same conditions the probabilities p(x), x=0,1,...,4 for "x paths occupied" but only for PCT2 with i_1 = 6.

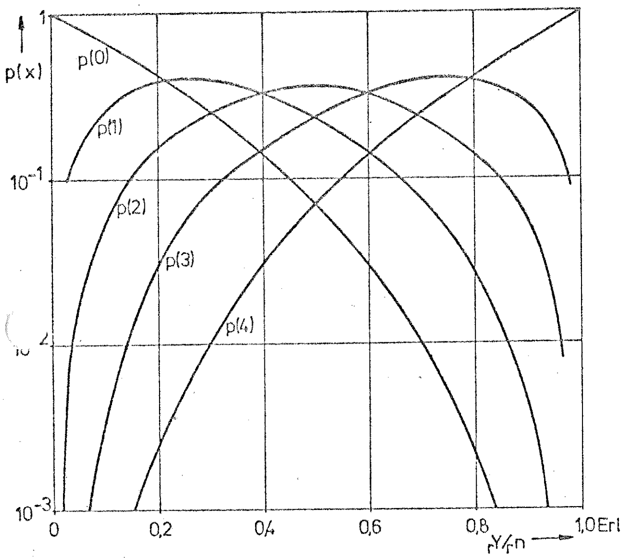

Fig.4 : Probabilities p(x), x=0,1,...,4 for "x paths occupied" as a function of the carried traffic ${}_rY/{}_rn$ per outgoing trunk (r=1,2) for balanced offered traffic
Parameter : PCT2, i_1 = 6
System : see fig.2

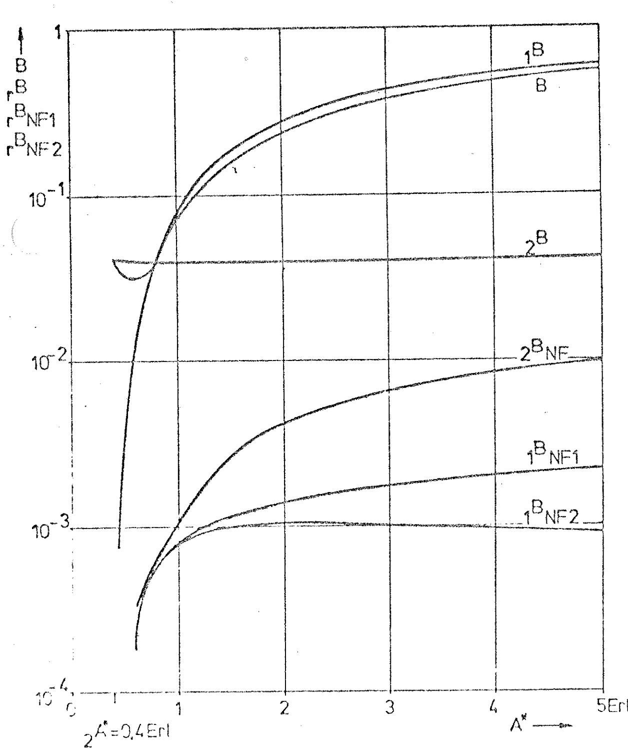

Fig.5 : Probabilities of loss B, ${}_rB$ (r=1,2) and point to point loss ${}_rB_{NF1}$ and ${}_rB_{NF2}$ as a function of the no-load offered traffic A^* and unbalanced carried traffic
Parameter : PCT2, i_1 = 6
${}_2A^*$ = 0,4 Erl = const.
System : see fig.2

Figure 5 shows an example of unbalanced carried traffic, with ${}_2A^*$ = 0,4 Erl = const.

$${}_rA^* = \sum_{\tilde{x}} \left[p(\tilde{x}) \sum_{v=1}^{g_1} \frac{{}_r\alpha_v \cdot i_1}{{}_r\varepsilon_v} \right] ; \; A^* = {}_1A^* + {}_2A^*$$

Unbalanced carried traffic implies the difference between the two trunk groups. The point to point loss ${}_2B_{NF1}$ is higher then ${}_2B_{NF2}$, but the difference is very small in the whole range of figure 5, therefore these two curves are drawn as one.

Results for PG-mode will be published in /10/.

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