EXAKT CALCULATION OF THE PROBABILITY OF LOSS FOR TWO-STAGE LINK SYSTEMS WITH PRESELECTION AND GROUPSELECTION

Wolfgang Lörcher
Technical University Stuttgart
Stuttgart, Federal Republic of Germany

ABSTRACT

Two-stage link systems for groupselection are considered, having gradings between the outlets of stage 1 or/and stage 2 respectively. Some special cases as preselection or systems without gradings are also described in detail.

Assuming stationarity of the traffic, the state pattern probabilities can be determined by a system of equations for these probabilities. This system of equations is derived.

The structure of the link system is described by means of matrices. The state patterns of the established calls in the system are also denoted by matrices.

Equations for the determination of the probability of loss, the carried traffic and the offered traffic are derived. Thus these quantities can be determined for two types of traffic : PCT 1 and PCT 2, i.e. Pure Chance Traffic of Type 1 (Poisson input, infinite number of sources, negative exponential holding time distribution) and Pure Chance Traffic of Type 2 (binomial input, finite number of sources, neg. ex. holding time distribution).

1. INTRODUCTION

In modern telephone systems the switching networks are normally built up as multistage arrays with conjugated selection : If a call occurs then the central control establishes a path between an inlet and an outlet of the network only if a free trunk of the wanted outgoing group is accessible. Such systems are called "link systems".

In addition to a large number of approximate procedures to calculate time congestion or call congestion /1/ there exist some publications /2/,/3/,/4/,/5/ et al. which deal with the exact calculation of special two-stage link systems w i t h o u t gradings.

The following paper is based on publications of G.P.Basharin and gives an extension of these methods to two-stage link systems for groupselection w i t h gradings. (The special cases of preselection and of systems without gradings are included.) The presented structures are of practical importance.

Let be given the types of offered traffic, the structure of the system, and the hunting method the loss probability is determined by means of the system of equations of the state probabilities.

Basharin has already shown /2/ that the number of unknowns can be considerably reduced by introducing conditions of symmetry. This question is however not investigated in this paper.

2. TRAFFIC, STRUCTURE OF THE SYSTEM, HUNTING METHOD

2.1. Traffic

Two types of offered traffic are distinguished :

a. Pure chance traffic of type 1 (PCT 1)
An infinite number of sources produces the offered traffic; the call intensity λ is constant and independent of the number of occupied sources (Poisson Input).

b. Pure chance traffic of type 2 (PCT 2)
A finite number of sources produces the offered traffic. Each idle source has a call intensity α; the total call intensity λ of all sources depends on the instantaneous number of occupied sources.

In both cases, the distribution of the holding times is assumed to be negative exponential with the mean value h.

2.2. System

Two-stage link systems with the following structure (fig. 1) will be considered.

i_1, i_2 number of inlets per multiple of stage 1,2
k_1, k_2 number of outlets per multiple of stage 1,2
${}_rk_2$ number of outlets per multiple of stage 2 to the group r $(r \in [1,R])$
g_1, g_2 number of multiples of stage 1,2 per linkblock

G number of linkblocks
M_1, M_2 number of multiples of stage 1,2
${}_r n_2$ number of outgoing trunks to group r
R number of outgoing groups (No.1,2,..r..R)

All G linkblocks have the same structure. The number of outlets ${}_r k_2$ per multiple to group r is allowed to be different for each group, but is the same one for each multiple.

We obtain $g_1 k_1 \geq g_2 i_2$, $M_2 \cdot {}_r k_2 \geq {}_r n_2$

where

$>$: grading of the outlets of stage 1 or/and of the outlets to group r, resp.

$=$: without grading

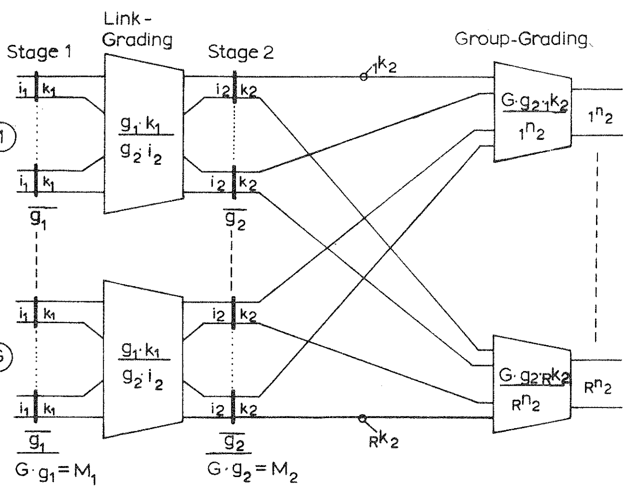


Fig.1 Two-stage link system with grading behind the outlets of stage 1 and of stage 2

2.3. Hunting mode

Two types of hunting modes of the outlets in the multiples of stage 1 have to be distinguished :

a. random hunting
The probability that an idle outlet is occupied is equal for all idle outlets in this multiple.

b. sequential hunting
Starting at a fixed home position the outlets will be hunted in sequential order.

The outlets in the multiples of stage 2 will always be sequentially hunted. However, an extension of the calculation method to "random hunting" of the outlets per multiple in stage 2 could be easily made, analogously to the considerations of random hunting of the outlets per multiple in stage 1.

3. THE SYSTEM OF EQUTIONS FOR THE STATE PROBABILITIES

3.1. State pattern $\{\tilde{x}\}$

Considering the stationary traffic process, the probabilities p(x) for the states

$$\{x \mid x \in [\,0,\ \sum_{r=1}^{R} {}_r n_2]\},$$

i.e. "x linklines are busy", are time invariant and the principle of statistical equilibrium can be applied.

Then the state probabilities p(x) can be calculated by means of the transition probabilities from and to neighbouring states. If a successful call arrives or if an established call terminates, each state transits into a neighbouring state.

Each state $\{x\}$ consists of a number of state patterns $\{\tilde{x}\}$. Two state patterns $\{\tilde{x}'\}$ and $\{\tilde{x}''\}$ have the same number of x busy linklines and outgoing trunks, however different patterns.

Except the boundary states each state pattern $\{\tilde{x}\}$ has some "higher" neighbouring state (HNS) patterns $\{\tilde{x}+1\}$ and some "lower" neighbouring state (LNS) patterns $\{\tilde{x}-1\}$ with (x+1) resp. (x-1) busy linklines.

(These state patterns are treated in more detail in section 4.2.)

The call intensity λ_v (PCT 1) resp. α_v per idle source (PCT 2) as well as the termination rate $\mu_v = 1/h_v$ are allowed to be <u>not</u> uniform for the multiples $\{v \mid v \in [1, M_1]\}$ of stage No. 1.

3.2. Transitions

To achieve a simpler presentation the deduction given in this chapter is explained by means of the case of sequential hunting of the outlets in the multiples of stage 1 and for preselection with offered PCT 1.

3.2.1. Transition $\{\tilde{x}+1\} \longrightarrow \{\tilde{x}\}$

The probability that the considered state pattern $\{\tilde{x}\}$ originates by termination of a certain established call in multiple v in stage 1 in a <u>certain</u> HNS-pattern $\{\tilde{x}+1\}$ during the time interval $(t, t+\Delta t)$ is given by

$$p(\tilde{x}+1)\frac{1}{h_v}\Delta t + o(\Delta t)$$

The function $o(\Delta t)$ contains terms of higher order in Δt only.

In the general case there exists more than one HNS-pattern $\{\tilde{x}+1\}$. The probability that the considered state pattern originates from <u>any</u> HNS-pattern is given by the summation

$$\sum_A p(\tilde{x}+1)\frac{1}{h_v}\Delta t + o(\Delta t) \tag{3.1}$$

where

$\sum_A$: Sum over all HNS-patterns $\{\tilde{x}+1\}$

3.2.2. Transition $\{\tilde{x}\} \longrightarrow \{\tilde{x}+1\}$

The probability that a call arrives in a multiple v of stage 1 during the time intervall $(t, t+\Delta t)$ is $\lambda_v \Delta t$. The probability that the considered state pattern $\{\tilde{x}\}$ disappears by the arrival of a successful call (transition into the HNS-patterns $\{\tilde{x}+1\}$) is given by

$$p(\tilde{x})\sum_B \lambda_v \cdot \Delta t + o(\Delta t) \tag{3.2}$$

where

$\sum_B$: Sum over all multiples v of stage 1 where a new call can be established in the considered state pattern $\{\tilde{x}\}$.

3.2.3. Transition $\{\tilde{x}\} \longrightarrow \{\tilde{x}-1\}$

The probability that a certain established call in multiple v of stage 1 terminates during $(t, t+\Delta t)$ is $\Delta t/h_v$.

Therefore, the probability that the considered state pattern $\{\tilde{x}\}$ disappears by termination of an established call (transition into the LNS-patterns $\{\tilde{x}-1\}$) is

$$p(\tilde{x})\sum_C \frac{x_v}{h_v}\Delta t + o(\Delta t) \tag{3.3}$$

where

$\sum_C$: Sum over all those multiples v of stage 1 where outlets are occupied in the considered state pattern $\{\tilde{x}\}$.

x_v : Number of occupied outlets in multiple v of stage 1 in the considered state pattern $\{\tilde{x}\}$, where $x_v \in [0, x]$.

3.2.4. Transition $\{\tilde{x}-1\} \rightarrow \{\tilde{x}\}$

The probability that the considered state pattern $\{\tilde{x}\}$ originates by the arrival of a successful call in a definite multiple v of stage 1 in a certain LNS-pattern $\{\tilde{x}-1\}$ during $(t,t+\Delta t)$ is

$$p(\tilde{x}-1)\lambda_v \Delta t + o(\Delta t)$$

In the general case there exists more than one LNS-pattern $\{\tilde{x}-1\}$ which can transit into the considered state pattern $\{\tilde{x}\}$ if a successful call occurs in a definite multiple v of stage 1 (regarding sequential hunting). The probability that the considered state pattern $\{\tilde{x}\}$ arises from any LNS-pattern $\{\tilde{x}-1\}$ is given by summation

$$\sum_D p(\tilde{x}-1)\lambda_v \Delta t + o(\Delta t) \qquad (3.4)$$

where

$\sum_D$: Sum over all of these and only these LNS-patterns $\{\tilde{x}-1\}$ which can transit into the considered state pattern $\{\tilde{x}\}$ if a successful call is established in a certain multiple of stage 1 (where sequential hunting is regarded).

3.3. The equations of the state pattern probabilities

Considering the stationary traffic process, the probability that a certain state pattern appears is equal to the probability that it disappears. From (3.1),...,(3.4) we obtain

$$\sum_A p(\tilde{x}+1)\frac{\Delta t}{h_v} + \sum_D p(\tilde{x}-1)\lambda_v \Delta t - p(\tilde{x})\Big[\sum_B \lambda_v \Delta t + \sum_C \frac{x_v \Delta t}{h_v}\Big] = o(\Delta t)$$

Because of the time independence of the process we obtain

$$\sum_A p(\tilde{x}+1)\frac{1}{h_v} + \sum_D p(\tilde{x}-1)\lambda_v - p(\tilde{x})\Big[\sum_B \lambda_v + \sum_C \frac{x_v}{h_v}\Big] = 0 \qquad (3.5)$$

with the normalizing condition

$$\sum_U p(\tilde{x}) = 1 \qquad (3.6)$$

where

$\sum_U$ comprises all state patterns of the system

3.4. Number of unknowns

The number of unknowns U (different state patterns) depends on the inividual structure of the gradings in use.

By modification of the formula given by Basharin in /4/ we receive an upper boundary value U_u for the number of unknowns :

$$U_u = \Big[\sum_{\nu=0}^{i_2^*} \binom{k_1^*}{\nu} \frac{k_2!}{(k_2-\nu)!}\Big]^{M_2}$$

where :

$i_2^* = \inf(i_2, k_2) \qquad k_1^* = \frac{g_1 k_1}{g_2}$

4. FORMAL DESCRIPTION OF THE LINK SYSTEM STRUCTURE AND OF THE STATE PATTERN $\{\tilde{x}\}$

In the following we use the indices :

i,v,y	$\in [1,M_1]$	number of a multiple in stage 1
a,c,e	$\in [1,M_2]$	number of a multiple in stage 2
I	$\in [1,g_1]$	number of a multiple within its own linkblock
j,w,z	$\in [1,k_1]$	number of an outlet in a multiple of stage 1
b,d,f,ς	$\in [1,{}_rk_2]$	number of an outlet in a multiple of stage 2 to group r
χ	$\in [1,i_2]$	number of an inlet in a multiple of stage 2
r,ν	$\in [1,R]$	number of an outgoing group
m	$\in [1,{}_rn_2]$	number of a trunk in an outgoing group r

we introduce the following abbreviated notations:
outlet(j,i,1) : outlet j in multiple i of stage 1
multiple(i,1) : multiple i of stage 1

4.1. Description of the gradings

4.1.1. Grading between the outlets of stage 1

All G linkblocks have the same size and structure. Therefore, we need only one "linkblock matrix" (Θ) with the dimension $(g_1 \times k_1)$ to describe their structure. Any type of grading between stage 1 and 2 is possible.

Within a linkblock an element ϑ_{Ij} characterizes the outlet(j,i,1). Its numerical value indicates the number of the connected inlet of stage 2 within this linkblock, i.e. $\vartheta_{Ij} \in [1, g_2 i_2]$.

From this linkblock matrix (Θ) we can also derive a "multiple matrix" (ϕ) with the dimension $(M_1 \times k_1)$.

The numerical value of the element φ_{ij} indicates the number of a multiple in stage 2 to which the outlet(j,i,1) is wired, i.e. $\varphi_{ij} \in [1, M_2]$.

We obtain

$$\varphi_{ij} = g_2\Big(\frac{i-1}{g_1}\Big)_{\text{rounded}} + \Big(\frac{\vartheta_{Ij}-1}{i_2} + 1\Big)_{\text{rounded}} \qquad (4.1)$$

where

$$I = (i)_{\text{mod } g_1}$$

If we would define G individual linkblock matrices, it would be possible to extend this description also to a system having linkblocks which are different from each other. In the following however, for reason of simplicity, only systems with uniform linkblocks are considered.

4.1.2. Grading between the outlets of stage 2

Each multiple of stage 2 has the same number ${}_rk_2$ of outlets to a certain outgoing group No.r. To describe the grading between the $M_2 \cdot {}_rk_2$ outlets and the group No.r a "grading matrix" $({}_r\Gamma)$ with the dimension $(M_2 \times {}_rk_2)$ is applied. Therefore R grading matrices are necessary. As in section 4.1.1. any type of grading is allowed.

The numerical value of the element ${}_r\gamma_{ab}$ indicates the number m of this outgoing trunk in group r to which the outlet(b,a,2) is wired.

$${}_r\gamma_{ab} = m \qquad m \in [1, {}_rn_2] \qquad (4.2)$$

If an outlet(j,i,1) is wired to an inlet in multiple(a,2) it holds

$$\varphi_{ij} = a \qquad (4.3)$$

and from (4.2)

$${}_r\gamma_{\varphi_{ij},b} = m$$

4.2. Description of the state patterns $\{\tilde{x}\}$

To desribe a certain state pattern $\{\tilde{x}\}$ (x established calls in the system) it is necessary to characterize the two states idle/busy of each outlet in the multiples of stages 1 and 2. This can be done by using "state matrices" (S) and $({}_rT)$.

4.2.1. Outlets of stage 1

The "link state matrix" $(S) = \|s_{ij}\|$ with the dimension $(M_1 \times k_1)$ denotes the occupation state of all outlets of stage 1.

The numerical value of the element s_{ij} indicates the number of a group to which the outlet(j,i,1) is connected. We obtain

$$s_{ij} = \begin{cases} 0 & \text{if the outlet(j,i,1) is not busy} \\ r & \text{if this outlet is busy to group r} \end{cases} \qquad (4.4)$$

To denote only the difference between "busy" and "not busy" it is usefull to introduce s^*_{ij} where

$$s^*_{ij} = 1 \text{ if } s_{ij} > 0; \quad = 0 \text{ if } s_{ij} = 0 \qquad (4.5)$$

4.2.2. Outlets of stage 2 to group r

To denote the occupation state of all outlets of stage 2 with respect to the outgoing group r we introduce R "group state matrices" $({}_rT)$.

Let:

$${}_rt_{ab} = 0 \text{ if the outlet}(b,a,2) \text{ to group } r \text{ is not busy}$$
$$= \chi \text{ if this outlet is busy and connected with the inlet } \chi \text{ of its multiple, } \chi \in [1, i_2]. \qquad (4.6)$$

where :

$$\chi = (\vartheta_{Ij})_{\text{mod } i_2}$$
$$I = (i)_{\text{mod } g_1} \qquad (4.7)$$

To denote the difference between "busy" and "not busy" we introduce

$${}_rt^*_{ab} = 1 \text{ if } {}_rt_{ab} > 0; \quad = 0 \text{ if } {}_rt_{ab} = 0 \qquad (4.8)$$

where a is given by (4.3)

Let :

$$\Delta = \{({}_\nu T) \mid \nu \in [1,R]\} = \{\|{}_\nu t_{ab}\| \mid \nu \in [1,R]\} \qquad (4.9)$$
$${}_r\Delta = \{({}_\nu T) \mid \nu \in [1,R], \nu \neq r\} \qquad (4.10)$$

4.2.3. Description of the state patterns $\{\tilde{x}\}$, $\{\tilde{x}+1\}$, $\{\tilde{x}-1\}$

Each considered state pattern $\{\tilde{x}\}$ is fully described by the matrices (S) and Δ.

The neighbouring state patterns can be characterized by means of the "Kronecker-symbol"

$$\delta_{xy} = 1 \text{ if } x = y; \quad = 0 \text{ if } x \neq y \qquad (4.11)$$

Let (according to (4.3)) :

$$\varphi_{vw} = c \qquad (4.12)$$

The matrices

$$\|s_{ij} + r\delta_{iv}\delta_{jw}\|, \|{}_rt_{ab} + \chi\delta_{ac}\delta_{bd}\|, {}_r\Delta$$

denote uniquely the HNS-patterns: The matrices $\|s_{ij} + r\delta_{iv}\delta_{jw}\|$ and $\|s_{ij}\|$ differ only in the element s_{vw}. This element has in the matrix $\|s_{ij} + r\delta_{iv}\delta_{jw}\|$ a numerical value which is by r higher than in the matrix $\|s_{ij}\|$. In the matrix $\|{}_rt_{ab} + \chi\delta_{ac}\delta_{bd}\|$ is the numerical value of the element ${}_rt_{ab}$ by χ higher than in the matrix $\|{}_rt_{ab}\|$; all other elements in this matrix are unchanged.

Accordingly we can denote the LNS-patterns by means of the matrices

$$\|s_{ij} - r\delta_{iv}\delta_{jw}\|, \|{}_rt_{ab} - \chi\delta_{ac}\delta_{bd}\|, {}_r\Delta .$$

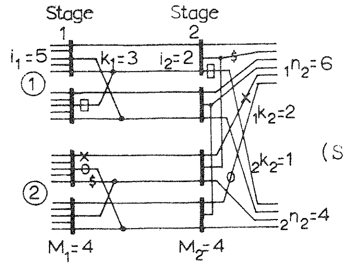
With this description of the state patterns it is possible to formulate the system of equations of the state pattern probabilities exactly.

To illustrate this description of the system and the notation of the state patterns we regard a very small linksystem with gradings behind the outlets of stage 1 and of stage 2 (fig.2):

linkblock matrix $(\Theta) = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 2 & 4 \end{pmatrix}$

multiple matrix $(\phi) = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 4 & 3 \\ 4 & 3 & 4 \end{pmatrix}$

grading matrices $({}_1\Gamma) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 2 \\ 6 & 4 \end{pmatrix}$ $\quad ({}_2\Gamma) = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$



Example of a state pattern

$$(S) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad ({}_1T) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \\ 2 & 0 \end{pmatrix} \quad ({}_2T) = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Fig.2 Linksystem with G=2 linkblocks; formal description of the structure; notation of a possible state pattern of the system

5. SYSTEM OF EQUATIONS OF THE STATE PATTERN PROBABILITIES

Here we regard random- and sequential hunting of the outlets in the multiples of stage 1.

First we consider PCT 2 and groupselection. The system of equations for PCT 1 is described in section 5.2.2. Preselection is the special case of groupselection with one group (R = 1).

Let :

${}_r\alpha_i$ the call intensity to group r of each idle source connected with the multiple(i,1).

$$s^*_{i.} = \sum_{j=1}^{k_1} s^*_{ij} \quad \text{number of busy outlets in multiple}(i,1) \qquad (5.1)$$

$${}_r\lambda_i = {}_r\alpha_i(i_1 - s^*_{i.}) \qquad (5.2)$$

the instantaneous call intensity in multiple(i,1) to group r (where i_1 the number of inlets, i.e. sources per multiple in stage 1)

$${}_r\mu_i = 1/{}_rh_i \qquad (5.3)$$

the termination rate of each call established from multiple(i,1) to group r

In the last section we have distinguished between the two states "busy" and "not busy" of an outlet. Now we divide the state "not busy" in

idle, i.e. the outlet can be occupied by a call, offered to the wanted group r;

blocked, i.e. the outlet has no access to an idle outgoing trunk of group r.

5.1. Transitions

The formulae below are derived in the following way :

First (in section 5.1.1.) one considers a certain state pattern $\{\tilde{x}\}$ and determines all its HNS-patterns. Furthermore, one determines the corresponding transition coefficients to the considered pattern $\{\tilde{x}\}$ for each individual HNS-pattern.

Second (in section 5.1.2.) the same state pattern $\{\tilde{x}\}$ is regarded and one determines all the (individual) transition coefficients by which it can transit to one of the HNS-patterns $\{\tilde{x}+1\}$.

Third (in section 5.1.3.) the transition coefficients from $\{\tilde{x}\}$ to all LNS-patterns $\{\tilde{x}-1\}$ are determined.

Fourth (in section 5.1.4.) all LNS-patterns $\{\tilde{x}-1\}$ have to be determined, which can - by a successful call - transit to the considered state pattern $\{\tilde{x}\}$. This $\{\tilde{x}-1\}$ patterns depend on the hunting mode. Furthermore, the individual transition coefficients of each relevant LNS-pattern $\{\tilde{x}-1\}$ to $\{\tilde{x}\}$ are determined.

5.1.1. Transition $\{\tilde{x}+1\} \longrightarrow \{\tilde{x}\}$

With (4.4) up to (4.12) we obtain the transition rate from all HNS-patterns $\{\tilde{x}+1\}$ to a certain considered state pattern $\{\tilde{x}\}$ - by analogy to the expression in (3.1) - as

$$\sum \{\tilde{x}+1\} \longrightarrow \{\tilde{x}\} = \qquad (5.4)$$

$$\sum_{r=1}^{R}\sum_{v=1}^{M_1}\sum_{w=1}^{k_1}\sum_{d=1}^{{}_rk_2} p(\|s_{ij}+{}_r\delta_{iv}\delta_{jw}\|,\|{}_rt_{ab}+\chi\delta_{ac}\delta_{bd}\|,{}_r\Delta)\frac{(1-s^*_{vw})(1-{}_rt^*_{cd})\psi_v(1-\varrho_{vw})(1-{}_r\tau_{cd})}{{}_rh_v}$$

Remarks:

$(1-s^*_{vw})$ By multiplication with this factor only those state patterns $\{\tilde{x}+1\}$ are considered where the outlet(w,v,1) is not busy in the state pattern $\{\tilde{x}\}$.

$(1-{}_rt^*_{cd})$ By multiplication with this factor those state patterns are excluded where the outlet(d,c,2) to group r is already busy in the considered state pattern $\{\tilde{x}\}$.

ψ_v denotes whether the number of occupied outlets in multiple(v,1) is less or equal to the number i_1 of sources per multiple.

This factor is necessary for the case $i_1 < k_1$.

$$\psi_v \begin{matrix} = 1 \\ = 0 \end{matrix}\Big\} \text{ if } s^*_{v.} \begin{cases} < i_1 \\ = i_1 \end{cases} \qquad (5.5)$$

ϱ_{vw} is necessary to distinguish whether a certain inlet in multiple(c,2) (accessible from outlet(j,i,1)) is occupied via grading by another outlet of stage 1.

For that purpose one has to check the state of all outlets of stage 1 which are interconnected with the considered outlet.

$$\varrho_{vw} = \sum_{\substack{y=v^*+1 \\ y\neq v}}^{v^*+g_1} \sum_{z=1}^{k_1} s^*_{yz}\cdot\delta_{\vartheta_{Iw},\vartheta_{y^*z}} \qquad (5.6)$$

i.e.

$\varrho_{vw} = 1$ if the inlet of stage 2 is occupied by an outlet(z,y,1) which is wired with outlet(w,v,1)

$= 0$ otherwise

where :

$I = (v)_{\mathrm{mod}\ g_1}$ acc.to (4.7)

$y^* = (y)_{\mathrm{mod}\ g_1}$ $\qquad v^* = g_2\left(\frac{v-1}{g_1}\right)_{\mathrm{rounded}}$

Only the outlets of this linkblock which contains the multiple v must be checked.

${}_r\tau_{cd}$ denotes whether the outgoing trunk in group r to which the considered outlet(d,c,2) has access is occupied by another outlet of stage 2 via the grading.

Therefore we must check the state of all outlets of stage 2 which are wired with the considered outlet.

$${}_r\tau_{cd} = \sum_{\substack{e=1 \\ e\neq c}}^{M_2} \sum_{f=1}^{{}_rk_2} {}_rt^*_{ef}\cdot\delta_{{}_r\gamma_{cd},{}_r\gamma_{ef}} \qquad (5.7)$$

i.e.

${}_r\tau_{cd} = 1$ if the outgoing trunk is occupied by an outlet(f,e,2) which is wired with outlet(d,c,2)

$= 0$ otherwise

5.1.2. Transition $\{\tilde{x}\} \longrightarrow \{\tilde{x}+1\}$

With (4.4),...,(4.8),(4.12),(5.1),(5.2) we obtain the transition rate from the considered state pattern $\{\tilde{x}\}$ to all HNS-patterns - by analogy to expression (3.2) - as

$$\{\tilde{x}\} \longrightarrow \sum\{\tilde{x}+1\} = p(\|s_{ij}\|,\Delta) \sum_{r=1}^{R} \sum_{v=1}^{M_1} (1-{}_r\xi_v)\,{}_r\lambda_v \qquad (5.8)$$

where :

${}_r\lambda_v$ acc.to (5.2)

${}_r\xi_v$ takes into account whether a call offered to group r via multiple(v,1) can be established.

${}_r\xi_v = 1$ if all outlets in multiple(v,1) are blocked for group r or busy

$= 0$ if at least one outlet in multiple (v,1) has access to an idle trunk of group r.

Hence

$${}_r\xi_v \begin{matrix} = 1 \\ = 0 \end{matrix}\Big\} \text{ if } k_1 - s^*_{v.} - {}_r\Omega_v \begin{cases} = 0 \\ > 0 \end{cases} \qquad (5.9)$$

where

$s^*_{v.}$ acc.to (5.1) and

${}_r\Omega_v$ number of blocked outlets in multiple(v,1) to group r.

$${}_r\Omega_v = \sum_{w=1}^{k_1} {}_r\omega_{vw} \qquad (5.10)$$

${}_r\omega_{vw}$ denotes whether the outlet(w,v,1) is blocked to group r. There are two conditions :

① the outlet is not busy, i.e. $(1-s^*_{vw}) = 1$, and

② either
the inlet of stage 2 connected with this outlet is occupied by another outlet of stage 1, i.e. $\varrho_{vw}=1$ (ϱ_{vw} acc.to (5.6))
or
the accessible inlet of stage 2 is not busy, i.e. $\varrho_{vw}=0$ but all outlets to group r in multiple(c,2) are busy or blocked, i.e. ${}_r\sigma_{vw} = 1$ (see (5.12))

$${}_r\omega_{vw} = (1-s^*_{vw})\left[\varrho_{vw} + (1-\varrho_{vw})\,{}_r\sigma_{vw}\right] \qquad (5.11)$$

i.e.

${}_r\omega_{vw} = 1$ if the outlet(w,v,1) is blocked

$= 0$ if this outlet is idle

where:

${}_r\sigma_{vw} = 1$ if all these outgoing trunks in group r which are accessible from outlet (w,v,1) are busy $\qquad (5.12.1)$

$= 0$ otherwise

Therefore

$${}_r\sigma_{vw} \begin{matrix} = 1 \\ = 0 \end{matrix}\Big\} \text{if} \sum_{d=1}^{{}_rk_2} (1-{}_r\tau_{cd})(1-{}_rt^*_{cd}) \begin{cases} = 0 \\ > 0 \end{cases} \qquad (5.12.2)$$

5.1.3. Transition $\{\tilde{x}\} \longrightarrow \{\tilde{x}-1\}$

With (4.4) up to (4.8) we obtain the transition rate from $\{\tilde{x}\}$ to all LNS-patterns $\{\tilde{x}-1\}$ - by analogy to expression (3.3) - as

$$\{\tilde{x}\} \longrightarrow \sum\{\tilde{x}-1\} = p(\|s_{ij}\|,\Delta)\sum_{r=1}^{R} \sum_{v=1}^{M_1} \sum_{w=1}^{k_1} \frac{\delta_{s_{vw},r}}{{}_rh_v} \qquad (5.13)$$

where:

$\delta_{s_{vw},r} = 1$ if the outlet(w,v,1) is occupied to group r

$= 0$ if this outlet is not busy or busy to another group

5.1.4. Transition $\{\tilde{x}-1\} \longrightarrow \{\tilde{x}\}$

With (4.4),..,(4.12),(5.1),(5.2) we obtain the transition rate from all LNS-patterns $\{\tilde{x}-1\}$ to the certain considered state pattern $\{\tilde{x}\}$ - by analogy to expression (3.4) - as

$$\sum\{\tilde{x}-1\} \rightarrow \{\tilde{x}\} = \sum_{r=1}^{R}\sum_{v=1}^{M_1}\sum_{w=1}^{\varepsilon}\sum_{d=1}^{\pi} p(\|s_{ij}-{}_r\delta_{iv}\delta_{jw}\|,\|{}_rt_{ab}-\chi\delta_{ac}\delta_{bd}\|,{}_r\Delta)\frac{{}_r\lambda_v\cdot\delta_{s_{vw},r}\cdot\delta_{{}_rt_{cd},\chi}}{{}_rN_{vw}} \qquad (5.14)$$

where :

$\delta_{s_{vw},r}$ see section 5.1.3.

$\delta_{{}_rt_{cd},\chi} = 1$ if the outlet(d,c,2) is busy and connected with the inlet χ of this multiple

$= 0$ otherwise

π the last - sequentially hunted - outlet in multiple(c,2) which is not idle for group r in the considered state pattern.

Let :

$q_f = {}_rt^*_{cf} + {}_r\tau_{cf}$ if $f \in [1, {}_rk_2]$

$= 0$ if $f = {}_rk_2+1$

we obtain

$$\pi = \inf\left(\pi^* \,\Big|\, \sum_{f=1}^{\pi^*+1} q_f < \pi^*+1,\ \pi^* \in [0, {}_rk_2]\right) \qquad (5.15)$$

ε, ${}_rN_{vw}$ must be distinguished in the following manner :

a. Sequential hunting of the outlets in the multiples of stage 1 :

We obtain

ε the last sequentially hunted outlet in multiple(v,1) which is not idle to group r in the considered state pattern $\{\tilde{x}\}$; it holds

$$\varepsilon = \inf(\varepsilon^* \mid \sum_{z=1}^{\varepsilon^*+1} u_z < \varepsilon^*+1,\ \varepsilon^* \in [0,k_1]) \qquad (5.16)$$

where :

$$u_z = s^*_{vz} + {}_r\omega_{vz} \quad \text{if } z \in [1,k_1]$$
$$= 0 \quad \text{if } z = k_1+1$$

$$_rN_{vw} = \frac{1}{_r\eta_{vw}} \qquad (5.17)$$

where

$_r\eta_{vw}$ denotes whether the outlet w is - in the LNS-pattern - the first idle outlet in multiple(v,1) to group r.

For this purpose we must check the state of all outlets (ς<w) which are blocked in the considered state pattern.

$_r\eta_{vw}$=1 if the outlet w is the first hunted idle outlet in multiple(v,1) to group r in the LNS-pattern
=0 otherwise

Hence:

$$_r\eta_{vw} = \begin{cases}1 \\ 0\end{cases} \text{ if } \sum_{\varsigma=1}^{w-1} {}_r\beta_{vw,\varsigma} \begin{cases}= 0 \\ > 0\end{cases} \qquad (5.18)$$

where:

$_r\beta_{vw,\varsigma}$ =1 if the outlet(ς,v,1) is blocked in the considered state pattern $\{\tilde{x}\}$ however idle in the LNS-pattern because the outlet(w,v,1) has terminated its occupation
=0 otherwise

Hence:

$$_r\beta_{vw,\varsigma} = (1-s^*_{v\varsigma})(1-\varrho_{v\varsigma})\,\delta_{ce} \qquad (5.19)$$

where:

$e = \varphi_{v\varsigma}$ acc.to (4.3)

b. Random hunting of the outlets in the multiples of stage 1

We obtain

$$\varepsilon = k_1 \qquad (5.20)$$

$_rN_{vw}$ the number of idle outlets in multiple (v,1) to group r in the LNS-pattern

$$_rN_{vw} = k_1 - s^*_{v.} + 1 - {}_r\Omega_v + {}_r\beta_{vw} \qquad (5.21)$$

where :

$s^*_{v.}$ acc.to (5.1)

$_r\Omega_v$ acc.to (5.10)

$_r\beta_{vw}$ the number of outlets in multiple(v,1) which are blocked in the considered state pattern $\{\tilde{x}\}$ however idle in the LNS-pattern because the outlet(w,v,1) has terminated its occupation.

$$_r\beta_{vw} = \sum_{\substack{\varsigma=1 \\ \varsigma \neq w}}^{k_1} {}_r\beta_{vw,\varsigma} \qquad (5.22)$$

where :

$_r\beta_{vw,\varsigma}$ acc.to (5.19)

5.2. System of equations

5.2.1. System of equations for PCT 2

Analogously to (3.5) and (3.6) we obtain with (5.4),(5.8),(5.13),(5.14) :

$$\sum_{r=1}^{R}\sum_{v=1}^{M_1}\sum_{w=1}^{k_1}\sum_{d=1}^{_rk_2} p(\|s_{ij} + r\delta_{iv}\delta_{jw}\|, \|_rt_{ab} + \chi\delta_{ac}\delta_{bd}\|, {}_r\Delta)\frac{(1-s^*_{vw})(1-{}_rt^*_{cd})\psi_v(1-\varrho_{vw})(1-{}_r\tau_{cd})}{_rh_v}$$

$$+\sum_{r=1}^{R}\sum_{v=1}^{M_1}\sum_{w=1}^{\varepsilon}\sum_{d=1}^{\pi} p(\|s_{ij} - r\delta_{iv}\delta_{jw}\|, \|_rt_{cd} - \chi\delta_{ac}\delta_{bd}\|, {}_r\Delta)\frac{_r\lambda_v \cdot \delta_{s_{vw},r} \cdot \delta_{_rt_{cd},\chi}}{_rN_{vw}}$$

$$-p(\|s_{ij}\|,\Delta)\left[\sum_{r=1}^{R}\sum_{v=1}^{M_1}\left[(1-{}_r\xi_v)\,{}_r\lambda_v + \sum_{w=1}^{k_1}\frac{\delta_{s_{vw},r}}{_rh_v}\right]\right] = 0 \qquad (5.23)$$

normalized by the condition

$$\sum_{U} p((S),\Delta) = 1 \qquad (5.24)$$

where $\sum_U$ comprises all state patterns of the system,

with :

Δ	acc.to	(4.9)	$_r\Delta$	acc.to	(4.10)	a	acc.to	(4.3)
c	"	(4.12)	s_{ij}	"	(4.4)	s^*_{ij}	"	(4.5)
$s^*_{i.}$	"	(5.1)	$_rt_{ab}$	"	(4.6)	$_rt^*_{ab}$	"	(4.8)
δ_{xy}	"	(4.11)	ψ_v	"	(5.5)	ϱ_{vw}	"	(5.6)
$_r\tau_{cd}$	"	(5.7)	$_r\xi_v$	"	(5.9)	χ	"	(4.7)
π	"	(5.15)	ε	"	(5.16) resp. (5.20)			
			$_rN_{vw}$	"	(5.17) " (5.21)			

5.2.2. System of equations for PCT 1

Here, the call intensity $_r\lambda_v$ is constant and independent of the number of busy sources.

Therefore we obtain the system of equations (5.23) and (5.24) with

$$_r\lambda_v = \text{const.} \qquad (5.25)$$
$$\psi_v = 1$$

6. SPECIAL CASES WITH SIMPLIFIED STRUCTURES

6.1. Groupselection (R>1), G>1, $g_1k_1 = g_2i_2$, Grading behind stage 2 ($M_2 \cdot {}_rk_2 > {}_rn_2$)

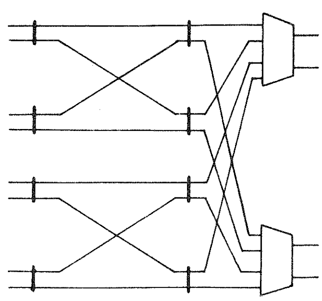


Fig. 3

Here the structure is without grading between the outlets of stage 1 (fig.3).

The matrices (Θ) and (ϕ) may be helpful if the linklines between stage 1 and stage 2 are wired in any manner.

These matrices are not necessary if each multiple of stage 1 is connected with each multiple of stage 2 in the same linkblock by one linkline, i.e. $k_1 = g_2$:

1. Linklines wired sequentially, i.e. outlet j in each multiple of stage 1 in a certain linkblock is connected with multiple(j,2) in the same linkblock.

   We obtain (cf. (4.1),(4.3)and(4.7))

$$a = g_2\left(\frac{i-1}{g_1}\right)_{\text{rounded}} + j \;;\; c = g_2\left(\frac{v-1}{g_1}\right)_{\text{rounded}} + w$$
$$\chi = (i)_{\text{mod } g_1} = I \qquad (6.1)$$

2. Linklines wired cyclically, i.e. outlet(j,I,1) is connected with multiple($(I+j-1)_{\text{mod } g_2}$,2) in the same linkblock.

   We obtain

$$a = g_2\left(\frac{i-1}{g_1}\right)_{\text{rounded}} + (i+j-1)_{\text{mod } g_2}$$

$$c = g_2\left(\frac{v-1}{g_1}\right)_{rounded} + (v+w-1)_{mod\ g_2}$$
$$\chi = (i)_{mod\ g_1} = I \qquad (6.2)$$

In these two cases each outlet in a multiple of stage 1 is connected with another multiple of stage 2; therefore we can replace ${}_rN_{vw}$ by ${}_rN_v$. We obtain

$$ {}_rN_v = 1 \quad \text{(sequential hunting)}$$
$$ = k_1 - s^*_{v.} + 1 - {}_r\Omega_v \quad \text{(random hunting)} \qquad (6.3)$$

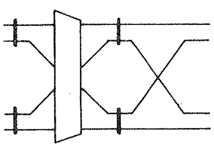
From (5.6) we get

$$\varrho_{vw} = 0 \qquad (6.4)$$

This simplifies the determination of blocked outlets ${}_r\omega_{vw}$ acc.to (5.11) and the equations(5.23).

6.2. Groupselection ($R>1$), $G=1$, $g_{2\,r}k_2 = {}_rn_2$

6.2.1. Grading between the outlets of stage 1 ($g_1k_1 > g_2i_2$)



Because there is no grading between the outlets of stage 2 (fig.4) the matrices (${}_r\Gamma$) and Δ can vanish.

Therefore we obtain the following modifications :

Fig. 4

${}_r\mathfrak{G}_{vw}$ the number of busy inlets in multiple(c,2) to group r must be checked to be less then the number of outlets ${}_rk_2$ in this multiple.

$$ {}_r\sigma_{vw} = \begin{cases} 1 \\ 0 \end{cases} \text{ if } \sum_{y=1}^{g_1}\sum_{z=1}^{k_1} \delta_{s_{yz},r}\cdot\delta_{ce} \begin{cases} = {}_rk_2 \\ < {}_rk_2 \end{cases} \qquad (6.5)$$

where :
$e = \varphi_{yz}$ acc.to (4.3)

i.e.
${}_r\sigma_{vw} = 1$ if all outgoing trunks of group r which are accessible from outlet (w,v,1) are busy (cf.(5.12.1)), i.e. all outlets(${}_rk_2$,c,2) are busy
$= 0$ otherwise

Then, ${}_r\omega_{vw}$ is determined by (5.11) with ${}_r\sigma_{vw}$ acc.to (6.5); by the transition $\{\tilde{x}+1\} \longrightarrow \{\tilde{x}\}$ acc.to (5.4) the multiplication with $(1-{}_r\sigma_{vw})$ is necessary.

From (5.23) we obtain

$$\sum_{r=1}^{R}\sum_{v=1}^{g_1}\sum_{w=1}^{k_1} p(\|s_{ij}+{}_r\delta_{iv}\delta_{jw}\|)\frac{(1-s^*_{vw})\gamma_v(1-\varrho_{vw})(1-{}_r\sigma_{vw})}{{}_rh_v}$$
$$+\sum_{r=1}^{R}\sum_{v=1}^{g_1}\sum_{w=1}^{\varepsilon} p(\|s_{ij}-{}_r\delta_{iv}\delta_{jw}\|)\frac{{}_r\lambda_v\cdot\delta_{s_{vw},r}}{{}_rN_{vw}} \qquad (6.6)$$
$$- p(\|s_{ij}\|)\Big[\sum_{r=1}^{R}\sum_{v=1}^{g_1}\Big[(1-{}_r\xi_v)\cdot{}_r\lambda_v + \sum_{w=1}^{k_1}\frac{\delta_{s_{vw},r}}{{}_rh_v}\Big]\Big] = 0$$

6.2.2. Without grading ($g_1k_1 = g_2i_2$), $k_1 = g_2$

Missing the grading between the outlets of stage 1 (fig.5) we obtain

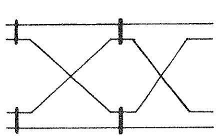


Fig. 5

$\varrho_{vw} = 0$ acc.to (6.4)

1. Linklines wired sequentially
As shown in section 6.1 all outlets j in the multiples of stage 1 are connected with the same multiple(j,2).Therefore, replacing of ${}_r\sigma_{vw}$ by ${}_r\sigma_w$ is possible.

We obtain

$$ {}_r\sigma_w = \begin{cases} 1 \\ 0 \end{cases} \text{ if } \sum_{v=1}^{g_1} \delta_{s_{vw},r} \begin{cases} = {}_rk_2 \\ < {}_rk_2 \end{cases} \qquad (6.7)$$

2. Linklines wired cyclically (cf. section 6.1)
From (6.2) we obtain

$$a = (i+j-1)_{mod\ g_2}$$
$$c = (v+w-1)_{mod\ g_2} \qquad (6.8)$$

and therefore we get ${}_r\sigma_c$ acc.to (6.5):

$$ {}_r\sigma_c = \begin{cases} 1 \\ 0 \end{cases} \text{ if } \sum_{y=1}^{g_1}\sum_{z=1}^{k_1} \delta_{s_{yz},r}\cdot\delta_{ce} \begin{cases} = {}_rk_2 \\ < {}_rk_2 \end{cases} \qquad (6.9)$$

where :
$e = (y+z-1)_{mod\ g_2}$

The calculation of ${}_r\omega_{vw}$ is given acc.to (5.11) where ${}_r\sigma_w$ acc.to (6.7), ${}_r\sigma_c$ acc.to (6.9)resp.; ${}_rN_v$ (cf. section 6.1) is determined acc.to(6.3). With these regards the system of equations (6.6) normalized by (5.24) is simplified also.

6.3. Preselection ($R=1$), $G=1$, $g_{2\,r}k_2 = {}_rn_2$

Because of $R=1$, we obtain $s_{ij} = s^*_{ij}$ (cf.(4.4) and (4.5)). Therefore, $\delta_{s_{vw},r}$ may be replaced by s_{vw}.

6.3.1. Grading between the outlets of stage 1 ($g_1k_1 > g_2i_2$) (fig.6)

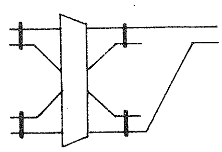


Fig. 6

From (6.6) we obtain

$$\sum_{v=1}^{g_1}\sum_{w=1}^{k_1} p(\|s_{ij}+\delta_{iv}\delta_{jw}\|)\frac{(1-s_{vw})\gamma_v(1-\varrho_{vw})(1-\sigma_{vw})}{h_v}$$
$$+\sum_{v=1}^{g_1}\sum_{w=1}^{\varepsilon} p(\|s_{ij}-\delta_{iv}\delta_{jw}\|)\frac{\lambda_v s_{vw}}{N_{vw}} \qquad (6.10)$$
$$- p(\|s_{ij}\|)\Big[\sum_{v=1}^{g_1}\Big[(1-\xi_v)\lambda_v + \sum_{w=1}^{k_1}\frac{s_{vw}}{h_v}\Big]\Big] = 0$$

6.3.2. Without grading ($g_1k_1 = g_2i_2$), $k_1 = g_2$

Missing the grading between the outlets of stage 1 (fig.7) we obtain

$\varrho_{vw} = 0$ acc.to (6.4)

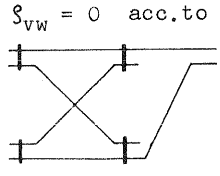


Fig. 7

If the linklines are wired sequentially resp. cyclically σ_w resp. σ_c must be determined acc.to (6.7) resp. (6.9) with regard to R=1. N_v is determined acc.to(6.3). The system of equations(6.10) is simplified.

This system of equations is also given for sequential wired linklines in /2/ for PCT 1 and random hunting of the outlets of stage 1 and in /1/,/6/ additional for PCT 2.

7. PROBABILITY OF LOSS B, CARRIED TRAFFIC Y, OFFERED TRAFFIC A

The deduction given here is done for the general case (section 5). In the special cases the description of the state patterns and of the coefficients is simplified (section 6).

with the state pattern probabilities calculated acc. to (5.23) and (5.24) we can determine the probability of loss, the carried traffic and the offered traffic.

7.1. Probability of loss B

The probability of loss ${}_rB$ to group r is

$$ {}_rB = \frac{\sum_U \left[p((S),\Delta) \sum_{v=1}^{M_1} {}_r\xi_v \cdot {}_r\lambda_v \right]}{\sum_U \left[p((S),\Delta) \sum_{v=1}^{M_1} {}_r\lambda_v \right]} \qquad (7.1) $$

where

$\sum_U$ comprises all state patterns of the system

${}_r\xi_v$ acc.to (5.9)

${}_r\lambda_v$ acc.to (5.2) for PCT 2
(5.25) for PCT 1

The total loss B is given by

$$ B = \frac{\sum_U \left[p((S),\Delta) \sum_{r=1}^{R} \sum_{v=1}^{M_1} {}_r\xi_v \cdot {}_r\lambda_v \right]}{\sum_U \left[p((S),\Delta) \sum_{r=1}^{R} \sum_{v=1}^{M_1} {}_r\lambda_v \right]} \qquad (7.2) $$

7.2. Carried traffic Y

The carried traffic ${}_rY$ to group r is

$$ {}_rY = \sum_U \left[p((S),\Delta) \sum_{v=1}^{M_1} \sum_{w=1}^{k_1} \delta_{s_{vw},r} \right] \qquad (7.3) $$

and the total carried traffic Y is given by

$$ Y = \sum_U \left[p((S),\Delta) \sum_{v=1}^{M_1} s^*_{v.} \right] \qquad (7.4) $$

where

$s^*_{v.}$ acc. to (5.1)

7.3. Offered Traffic A

The offered traffic ${}_rA$ to group r is

$$ {}_rA = \frac{{}_rY}{1-{}_rB} \qquad (7.5) $$

and the total offered traffic A is given by

$$ A = \sum_{r=1}^{R} {}_rA = \frac{Y}{1-B} \qquad (7.6) $$

8. EXAMPLE

In /6/ resp. /7/ ALGOL resp. FORTRAN programs are presented for calculating the probability of loss by means of the state pattern probabilities for systems according to section 6.3.2. resp. 6.2. and 6.3.

Diagram 1 shows the probability of loss B as a function of the carried traffic per outgoing trunk Y/n_2 for two systems. The values are calculated by means of the above programs.

System	G	g_1	k_1	i_2	g_2	k_2	R	
①	1	4	3	4	3	3	1	linklines wired sequentially; random- or sequential hunting
②	1	3	4	4	2	3	1	grading between the outlets of stage 1: $(\Theta) = \begin{pmatrix} 1 & 5 & 4 & 8 \\ 2 & 6 & 4 & 8 \\ 3 & 7 & 4 & 8 \end{pmatrix}$; sequential hunting

The offered traffic is PCT 1 where $\lambda_v = \lambda$.

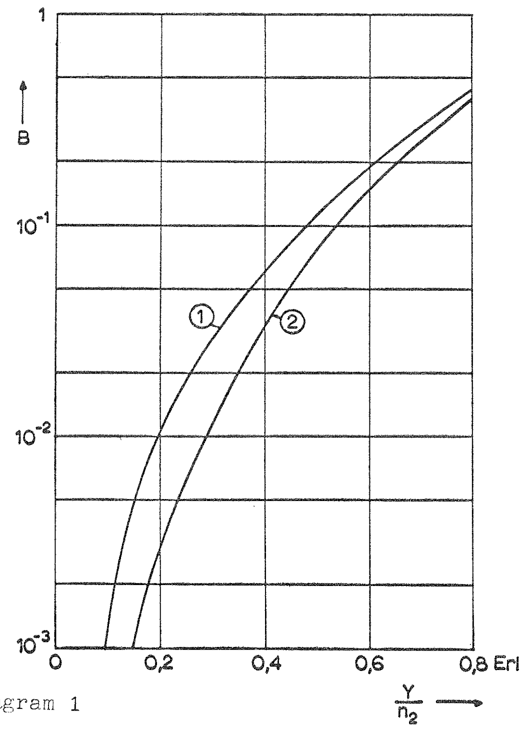


Diagram 1

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