AN ANALYSIS OF LOSS APPROXIMATIONS

FOR LINK SYSTEMS

by

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Summary

There is a large number of approaches to calculate time congestion and call congestion of multistage link systems. An analysis of the different papers revealed, that the methods of calculation can be reduced to a few basic principles. These basic principles will be presented in this paper.

Introduction

The switching networks of modern telephone arrangements are often built up as multistage arrays which are controlled by conjugated selection, i.e. links are only occupied if they can reach a free outlet.

There is a large scale of procedures to calculate time congestion or call congestion of such "link systems". The accurate analysis of these procedures reveals, that we can distinguish a few basic principles of calculation, which will be explained in this paper. Of course it will be impossible to mention all the valuable contributions in this field. Instead of that some significant papers of each principle will be handled.

I. Exact Calculation

The exact calculation of link systems leads to a linear homogeneous system of equations for the probabilities of state. From the mathematical point of view the probabilities of state are the final probabilities of a one- or multi- dimensional Markoff-process with a finite number of states. If we can get the numerical solution of the system of equations of state, we find the probability of loss by linear combinations of certain probabilities of state.

Because of the immense number of possible states of a link system there are enormous difficulties to formulate the linear system of equations and especially to do its numerical evaluation. Even using relaxation methods for solving the systems of equations, only very small two-stage systems, which actually are not of practical importance, can be exactly calculated up to now.

There are three publications known /8/, /20/, /31/ concerning with the exact calculation of probability of loss.

B a s h a r i n /20/ calculates the probability of loss in two-stage link systems for p r e s e l e c t i o n if a pure chance traffic (Poisson input) is offered to the first switching stage (see fig. 1).

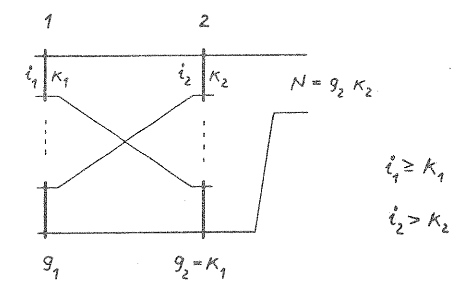

Fig. 1

To describe the states of occupation the system of link-lines is represented by a matrix (fig. 2). One row of the matrix represents a multiple of the first selector stage and contains k_1 elements according to the k_1 outlets of a multiple. There are all together g_1 rows. The elements of the matrix be "0" or "1" according to the corresponding link being idle or busy. The matrix of a certain state of occupation is denoted by $\|s_{ij}\|$.

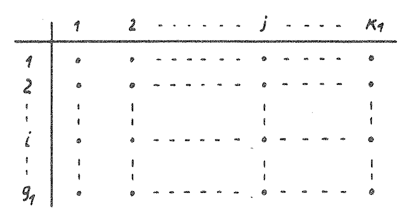

Fig. 2

Further notations:

c_{Ai}: Average number of calls offered per unit of time to the multiple No.i in the first stage, $1 \leq i \leq g_1$.

h_i: Mean holding time of a line being occupied by calls originating from multiple No.i in the first stage.

A_i: Traffic offered to multiple No.i in the first stage, $A_i = c_{Ai} \cdot h_i$.

v_i: Number of free outlets in multiple No.i of the first stage having internal blocking.

$s_{i.}$: Number of occupied outlets in multiple No.i of the first stage.

For the probabilities of state $p(\|s_{ij}\|)$

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Basharin gets the following linear homogeneous system of equations:

$$\sum_{i,j} p(\|s_{i1},\dots,s_{ij}-1,\dots,s_{iK_1}\|)\cdot\frac{c_{Ai}\,s_{ij}}{K_1-s_{i.}-V_i+1}$$

$$+\sum_{i,j} p(\|s_{i1},\dots,s_{ij}+1,\dots,s_{iK_1}\|)\cdot(1-s_{ij})\frac{1}{h_i}$$

$$-p(\|s_{ij}\|)\cdot\Big(\sum_{i,j}\frac{s_{ij}}{h_i}+\sum_{i}^{*}c_{Ai}\Big) = 0 \qquad (1.)$$

$$\sum_{i,j} p(\|s_{ij}\|) = 1 \qquad (2.)$$

Explanation: The first expression in equation (1) is the probability of transition into the state $\|s_{ij}\|$ from any neighbouring "lower" state if a new successful call occurs. The second term of the sum expresses the probability of transition into the state $\|s_{ij}\|$ from any neighbouring "higher" state by ending of an occupation. The probability that the state $\|s_{ij}\|$ already exists and there happens neither a successful call ($\sum_i^{*} c_{Ai}$) nor the end of an occupation is represented by the third term.

The system of equations (1) for the probabilities of states $p(\|s_{ij}\|)$ contains

$$n = \Big[\sum_{\nu=0}^{K_2}\binom{g_1}{\nu}\Big]^{g_2} \qquad (3.)$$

linear homogeneous equations.

When the system of equations of state has been solved, one gets the probability of loss as a linear function of certain probabilities of state.

The number of unknowns can be considerably reduced by introducing conditions of symmetry: symmetrical loading and random hunting of the free link-lines. Thereby all states $\|s_{ij}\|$, differing only in the transposition within rows or columns, become equally probable.

E l l d i n /8/ investigates two-stage systems for g r o u p selection with Poisson- or Engset-input. His method contains some slight approximations. Each state of the system is fully described by four parameters u, x_1, x_2, s and is denoted by $\{u, x_1, x_2, s\}$, its probability by $[u, x_1, x_2, s]$. Figure 3 shows a "link-unit" of the system.

u: Number of unusable connecting paths from stage 1 and stage 2 to the considered outgoing trunk group.

x_1: Number of occupied outgoing lines of the multiple in the first stage.

x_2: Number of occupied lines in the considered outgoing trunk group.

s: That part of x_1 occupations which is connected to the considered outgoing trunk group.

$\alpha(x_1)$: Probability density for the arrival of a further call in the considered multiple, if x_1 links of the unit are busy.

$\beta(x_2-s)$: Probability density for calls, originating from any of the remaining other g_1-1 multiples 1 a n d occupieing successfully the considered outgoing trunk group.

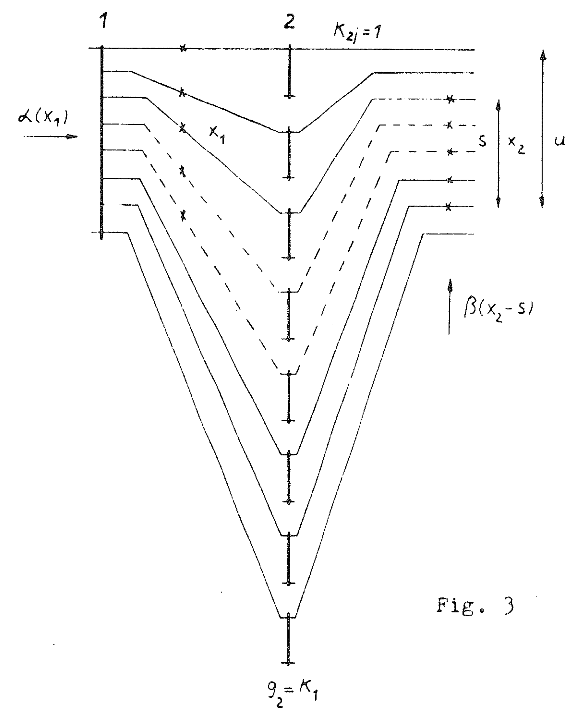

Fig. 3

To simplify the evaluation random hunting and symmetrical offered traffic are presumed.

Now we can calculate probabilities of transition from the state $\{u, x_1, x_2, s\}$ into neighbouring states and therewith a general system of equations of state can be set up. If statistical equilibrium is assumed, we get a linear system of equations for the probabilities of state $[u, x_1, x_2, s]$

Blocking occurs for arriving calls if $u=g_2$, i.e. in the state $\{g_2, x_1, x_2, s\}$. By summing up over all possible values x_1, x_2, s we get an expression for the time congestion E:

$$E = \sum_{x_1}\sum_{x_2}\sum_{s}[g_2, x_1, x_2, s] \qquad (4.)$$

To describe link systems with more than two selector stages further parameters are needed. In consequence the theoretical and numerical difficulties increase.

Therefore, as a rule, time congestion and probability of loss must be calculated using approximations. The numerical results of such approximate solutions have to be checked by tests with artificial traffic.

II. Approximations

II A. Functional Dependence and Statistical Independence

First of all we investigate the methods without using the "hypothesis of independence". This means the functional dependence must be taken into account between the probability distributions describing the number of simultaneously occupied links in the individual intermediate stages as well as in the outgoing trunk groups.

However, to avoid a too complicated process of calculation it is assumed, certain states of occupation in the different sections of the link system to be statistically independent of each other.

This leads to a set of equations which can be solved only by iteration. The results are - as far as known - rather accurate up to large values of loss.

But because statistical independence of certain states of occupation is assumed, the results will be somewhat too pessimistic, i.e. the loss will be on the safe side.

Elldin investigates in his paper /24/ two-stage link systems for group selection. We denote by $\{x_1\}$ the state for x_1 link-lines to be occupied in the considered link-unit and by $\{x_2\}$ the state that x_2 lines in the considered outgoing trunk group are busy (cf. fig. 3). The corresponding probabilities be $[x_1]$ and $[x_2]$. Consequently one gets:

$$[x_1] = f([x_2]) \; ; \; [x_2] = g([x_1]) \tag{5}$$

i.e. the probability distributions $[x_1]$ and $[x_2]$ are dependent of each other.

For the practical evaluation a set of starting distributions $[x_1]_0$ and $[x_2]_0$ is given, e.g. Erlang's-distribution or Erlang's-Bernoulli-distribution for full-access trunk groups.

By means of $[x_2]$ and a combinatorial statement the blocking factor $\sigma(x_1)$ and the "passage factor" $\mu(x_1) = 1 - \sigma(x_1)$ for the state $\{x_1\}$ can be calculated. $\mu(x_1)$ means the probability that a call arriving in the state $\{x_1\}$ can be connected. Similarly one gets from $[x_1]$ the values $\sigma(x_2)$ and $\mu(x_2)$, where $\mu(x_2)$ means the probability that a call arriving in the state $\{x_2\}$ will be successful.

Now the following process of iteration must be done:

1. The set of starting distributions $[x_1]_0$, $[x_2]_0$ is assumed.
2. From $[x_2]_0$ and a combinatorial statement the values $\sigma(x_1)_0$, $\mu(x_1)_0$ can be calculated.
3. Statistical equilibrium using $\mu(x_1)_0$ leads to an improved distribution $[x_1]_1$.
4. From $[x_1]_1$ and a combinatorial statement the values $\sigma(x_2)_1$, $\mu(x_2)_1$ can be calculated.
5. Statistical equilibrium using $\mu(x_2)_1$ leads to an improved distribution $[x_2]_1$.
6. Therewith calculation of $[x_1]_2$ according to 2. and 3. and so on.

The process of iteration is continued until a prescribed accuracy is reached. One demands:

$$\begin{aligned} &|[x_1]_\nu - [x_1]_{\nu-1}| < \varepsilon_1 \\ &|[x_2]_\nu - [x_2]_{\nu-1}| < \varepsilon_2 \\ &|\sum_{x_1} x_1 [x_1]_\nu - F \cdot \sum_{x_2} x_2 [x_2]_\nu| = |y_1 - F \cdot y_2| < \varepsilon_3 \end{aligned} \tag{6}$$

The traffics carried y_1 and y_2 may differ from each other by a constant factor F which takes into account the relation between the total traffic y_1 per link-unit and the traffic y_2 of the considered outgoing trunk group.

Employing now the evaluated distributions $[x_1]$ and $[x_2]$ time congestion and call congestion can be calculated.

Bininda and Daisenberger /39/ propose a similar method for calculating link systems with an arbitrary number S of selector stages. They investigate the case that a certain subscriber forming one inlet of the first selector stage cannot be linked to a certain subscriber connected with one outlet of the last selector stage ("Point to Point Loss"). Assuming appropriate configurations one can also calculate with this method the loss of an outgoing trunk group in a system with S-1 selector stages.

The necessary processes of iteration are suitable to be programmed for a digital computer.

II B. Functional and Statistical Independence

The calculation of time congestion is essentially simplified, if we don't postulate the functional dependence according to section II A.

Instead of this we set up the hypothesis that the probability distributions be independent from each other (hypothesis of independence). For example the probability that x_1 link-lines are busy is assumed to be independent from the probability that x_2 lines of the considered outgoing trunk group are busy.

The probability distributions for each stage are presumed to be well-known distributions used for single-stage switching arrays, such as Erlang's-distribution, Bernoulli-distribution, Erlang's-Bernoulli-distribution etc. According to this choice one will get rather different results.

A large number of publications is based on this principle. The expressions for time congestion in the case of two-stage link systems can often be transformed into simple formulae which can be evaluated with well-known function-tables, such as Palm's Erlang-Tables etc.

It must be pointed out, that for this reason Jacobaeus in his fundamental contribution /1/ and many studies published later by other authors use the further approximation: offered traffic A = carried traffic y. Otherwise time congestion can only be evaluated by iteration carried out on a digital computer.

Therefore all these methods yield sufficient accurate values of loss only up to about 2%. But the accuracy can be improved significantly if the approximation for the offered traffic, A=y, is replaced by a generating offered traffic $A_0(y,N)$.

The same principle can also be applied to link systems with an arbitrary number of selector stages.

Now we follow the publication of J a c o b a e u s /1/ to explain the fundamental principle. For this purpose let us consider the link-unit in figure 4.

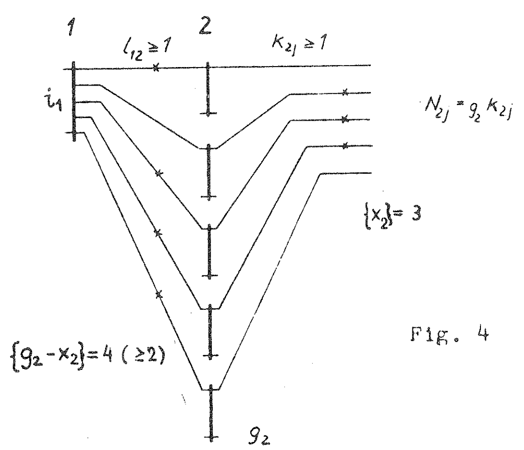

Fig. 4

Notations:

$[x_2]$: probability that exactly in x_2 arbitrary multiples of the second selector stage all k_{2j} outlets to the considered outgoing trunk group No. j are busy.

$[g_2-x_2]$: Momentary loss factor, i.e. probability that at least $(g_2-x_2)l_{12}$ certain links forming a certain pattern are busy (namely these, which lead to those multiples 2 still having free outlets to the direction No. j in consideration).

The states $\{x_2\}$ and $\{g_2 - x_2\}$ corresponding to $[x_2]$ and $[g_2 - x_2]$ are assumed to be independent. By means of presumed distributions, e.g. Erlang's-distribution, Bernoulli-distribution etc., and of combinatorial statements the momentary loss factor $[g_2 - x_2]$ can be derived.

A call arriving in multiple 1 of the considered link-unit cannot be successful if the states $\{x_2\}$ and $\{g_2 - x_2\}$ exist. The time congestion E is calculated from:

$$E = \sum_{x_2=0}^{g_2} [x_2]\cdot[g_2 - x_2] \qquad (7.)$$

Let us consider two examples (already given by Jacobaeus).

<u>Example 1</u>

a) Two-stage link system (cf. fig.4) with $l_{12}=1$, $k_{2j}=1$ and $i_1=k_1$,
b) Bernoulli-distribution in the link-unit between stage 1 and 2; Erlang-distribution on the considered outgoing trunk group,
c) Traffic offered to the considered outgoing trunk group A_{2j} and traffic carried per link p_1.

Remark: Traffic offered and traffic carried are denoted in this paper according to their meaning. However, the approximation A=y is always assumed.

From equation (7):

$$E = \sum_{x_2=0}^{g_2} \frac{A_{2j}^{x_2}/x_2!}{\sum_{\nu=0}^{g_2} A_{2j}^{\nu}/\nu!}\, p_1^{g_2-x_2} \qquad (8.)$$

After a short transformation we get from (8):

$$E = \frac{E_{1,g_2}(A_{2j})}{E_{1,g_2}\left(\frac{A_{2j}}{p_1}\right)} \qquad (9.)$$

$E_{1,N}(A)$ being Erlang's formula.

<u>Example 2</u>

a) The same as in example 1, but Erlang-distribution both on the links and on the considered outgoing trunk group,
b) Traffic offered per multiple in the first stage A and to the outgoing trunk group of direction No.j A_{2j}.

From equation (7) we get:

$$E = \sum_{x_2=0}^{g_2} \frac{A_{2j}^{x_2}/x_2!}{\sum_{\nu=0}^{g_2} A_{2j}^{\nu}/\nu!} \cdot \frac{E_{1,g_2}(A)}{E_{1,x_2}(A)} \rightarrow$$

$$E = \frac{A\cdot E_{1,g_2}(A) - A_{2j}\cdot E_{1,g_2}(A_{2j})}{A - A_{2j}} \qquad (10.)$$

Similarly Jacobaeus deals with systems having expansion, $i_1 < k_1$, or concentration, $i_1 > k_1$, in the first selector stage. To get simple formulae which can be evaluated with well-known function-tables as in the examples 1 and 2, we must introduce arithmetical approximations if transformations starting from (7) are made. In doing so the different cases Bernoulli-distribution both on the links and on the outgoing trunk group, called "Bernoulli/Bernoulli", furthermore "Bernoulli/Erlang" and "Erlang/Erlang" are treated.

Moreover two-stage systems with a grading behind the last selector stage are investigated. That calculation method combines the methods mentioned above with O'Dell's formula.

Using the principle of Jacobaeus, F o r t e t /9/, /11/ and P r ö h l /29/ calculate the most general case, i.e. the number l_{12} of links leading from one multiple in stage 1 to a certain multiple in stage 2 being greater or equal unity, $l_{12} \geq 1$; furthermore the number k_{2j} of outlets in one multiple of the second selector stage to the desired outgoing group No. j being greater or equal unity, $k_{2j} \geq 1$. For the blocking factor $[g_2 - x_2]$ a slightly more accurate expression is used.

An extension of the calculation to systems with an arbitrary number of selector stages - as already Jacobaeus has shown in his paper for three- and four-stage link systems - is contained in the contributions of L e e /7/, L e G a l l /10/, /12/ and v a n B o s s e /32/. However, they all use the approximation A=y.

As already mentioned, the accuracy of the results can be improved significantly if the approximation for the traffic offered, A=y, is replaced by a generating traffic offered A_0 as proposed by J e n s e n /2/, /3/, L o t z e /17/ in The Hague and later on by H u b e r /35/ at the fourth ITC in London. Assuming Erlang-distribution (equation (11))

$$[x] = \frac{A_0^x / x!}{\sum_{\nu=0}^{N} A_0^\nu / \nu!} \qquad (11)$$

and starting with the actually carried traffic y we look for a fictitious "generating" offered traffic A_0 defined by:

$$A_0 = \frac{y}{1 - \frac{A_0^N / N!}{\sum_{\nu=0}^{N} A_0^\nu / \nu!}} \qquad (12)$$

The effect of that modification is that especially for high loads the computed blocking probabilities will be reduced, thus better fitting reality. The process of iteration to find A_0 can easily be done by a digital computer.

B i n i n d a / D a i s e n b e r g e r / D i d l a u k i s /38/ use this modified principle to calculate blocking probabilities of systems having an arbitrary number of selector stages and arbitrary probability distributions on each intermediate link group as well as on the outgoing trunk groups.

The solutions in the following sections C - F compromise between the total dependence and the total independence of the probability distributions for each selector stage.

II C. Partial Functional Dependence

For the outgoing trunk group a probability distribution according to Erlang's interconnection formula or a corresponding expression for the case of a limited number of traffic sources is assumed. Therewith the state distribution on the first intermediate links is calculated. The blocking factor needed for this distribution depends on the carried traffic of the system and on the distributions of the following link groups and outgoing trunk groups. These distributions in their turn depend on the probability distribution on the first link group. Therefore, iterations are required

The results are sufficiently accurate from very small up to large values of loss.

We will explain the fundamental ideas of the principle - according to a paper of B r e t s c h n e i d e r /23/ - in calculating the probability of loss for a two-stage link system with preselection to which a pure chance traffic is offered (Poisson input).

As already mentioned, we assume for the outgoing trunk group the probability distribution $[x_2]$ to be the same as for an ideal Erlang-grading (Erlang's interconnection formula, EIF). The availability k of the ideal grading has to be chosen such, that with the same number of outgoing trunks it handles the same traffic with the same loss as the considered two-stage link system (cf. section II E). However, k can only be determined if the probability of loss is already known. Therefore, we must start with

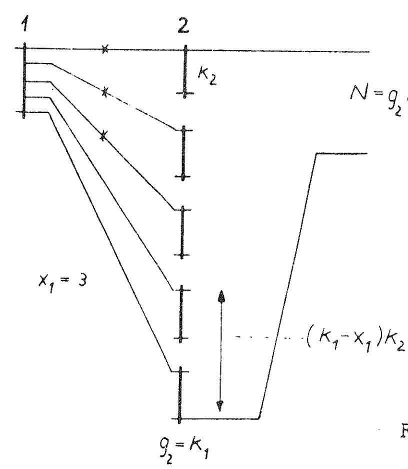

Fig. 5

an estimated value $\tilde{k}$ for k.

If x_1 intermediate lines in the link-unit are busy (see figure 5) an arriving call is only lost if at least those $(k_1-x_1)k_2$ outgoing trunks are busy which can be hunted by the (k_1-x_1) idle links. The loss factor $\sigma(x_1)$ or the passage factor $\mu(x_1)=1-\sigma(x_1)$ can be calculated using the EIF-distribution assumed on the outgoing trunk group.

$$\mu(x_1) = f(x_1, \tilde{K}, [x_2]) \qquad (13)$$

The passage factor $\mu(x_1)$ is a function of the number x_1 of busy links, of the estimated availability $\tilde{k}$ and of the distribution $[x_2]$ on the outgoing trunk group.

Furthermore, we apply the well-known recurrent formula (14) to the link group:

$$[x_1]\cdot x_1 = A[x_1-1]\cdot\mu(x_1-1) \qquad (14)$$

The quantity A means the traffic offered to one multiple of the first selector stage.

Using $\mu(x_1)$ according to equation (13) we can evaluate the probability distribution $[x_1]$ from equation (14). Therewith we get the probabilities E and B for time congestion respectively call congestion of the link system.

$$E = \sum_{x_1} [x_1]\sigma(x_1) \qquad (15)$$

According to the assumption made above the probability E (or B) of the link system should be the same as for the ideal reference grading, i.e. $\tilde{B}_{EIF}(A,\tilde{k},N)$. Because the estimated availability $\tilde{k}$ will not be fitting we find:

$$E \neq \tilde{B}_{EIF}(A,\tilde{k},N) \qquad (16)$$

Therefore, $\tilde{k}$ has to be changed by iteration until a prescribed accuracy is reached:

$$|E - \tilde{B}_{EIF}| < \varepsilon \qquad (17)$$

II D. Passage Factor

The principle discussed in this section also uses a recurrent formula corresponding to Erlang's statistical equilibrium. On the contrary to the principle investigated in section II C, this formula is applied not for the distribution $[x_1]$ on the first link group but for the distribution $[x_s]$ on the outgoing trunk group of stage No. S.

First of all the passage factor $\mu(x_s)$ of the whole link system (inlet - outgoing group) has to be calculated. If random hunting and equal loading of all link-lines and outgoing trunks is assumed, $\mu(x_s)$ and the blocking factor $\sigma(x_s)$ respectively can be given by an explicit combinatorial formula depending only on the structural parameters of the multistage system and on the number x_s of simultaneously occupied lines in the outgoing route. Using this passage factor $\mu(x_s)$ we can calculate the probability distribution $[x_s]$ on the outgoing trunk group according to the well-known recurrent formula of statistical equilibrium. It is:

$$[x_s+1](x_s+1)\varepsilon = \alpha(q-x_s)[x_s]\mu(x_s) \qquad (18)$$

ε: Probability density for termination of one occupation,
α: Probability density for calls from a free source,
q: Number of traffic sources,
$\mu(x_s)$: Passage factor if there are x_s lines in the outgoing trunk group busy.

The probability distributions of the intermediate link groups don't have to be calculated. Therefore no iteration is needed.

II E. Effective and Average Availability

A further way in calculating the grade of service of link systems is to use the e f f e c t i v e a c c e s s i b i l i t y of a multistage system, which was proposed by B i n i n d a / W e n d t /18/ and independently by K h a r k e v i c h /19/.

The link system is compared with a single-stage switching array having the same number of outgoing trunks and handling the same traffic at the same rate of loss.

In each state of occupation $\{x_1\}$ (cf.fig. 6) a certain momentary availability $k(x_1)$ exists from an inlet of the first stage with regard to the considered outgoing trunk group.

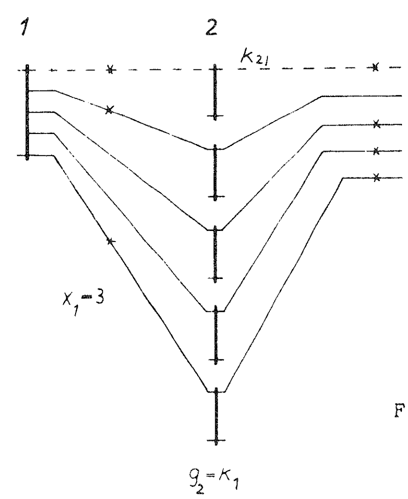

Fig. 6

It is:

$$K(x_1) = (K_1-x_1)K_{2j} + S \qquad (19)$$

The letter s denotes the number of occupations out of x_1, which just in this moment is connected to the direction No. j. The quantity s in equation (19) should be added, because an outlet of the considered outgoing trunk group belongs to the available outlets too, if it is occupied from the considered multiple in the first selector stage. (Kharkevich omits s).

A momentary probability of loss $B(x_1)$ belongs to the momentary availability $k(x_1)$. We presume $B(x_1)$ to be equal to the probability of loss of a single-stage switching array with accessibility $k(x_1)$ having the same number of outgoing lines and handling the same traffic with the same loss. The probability that the accessibility $k(x_1)$ exists if a new call occurs is denoted by $[k(x_1)]$. Therewith, we can calculate the mathematical expectation B of the momentary loss $B(x_1)$:

$$B = \sum_{x_1} B(x_1)\,[k(x_1)] \qquad (20)$$

As already mentioned, the effective accessibility k_{eff} is the accessibility of a single-stage reference connecting array, which with the same number of outgoing trunks handles the same traffic with the same loss B. Consequently, we may calculate the effective availability, if the function $B = f(A,N,k)$ is known (A: traffic offered, N: number of outgoing trunks, k: accessibility).

The effective accessibility k_{eff} can be replaced by the average availability k_m if we approximately assume a linear relation between $B(x_1)$ and $k(x_1)$. Furthermore, we neglect the fact that there may exist different values of $k(x_1)$ for the same x_1, caused by statistical variations. Therefore the momentary accessibility $k(x_1)$ is replaced by the average momentary accessibility $k(x_1)_m$:

$$k(x_1)_m = (k_1 - x_1)k_{2j} + \eta_{2j} \cdot x_1 \qquad (21)$$

η_{2j} denotes the fraction of the total traffic to the considered direction No. j, i.e. $\eta_{2j} x_1$ means the average value of s.

$[x_1]$ be the probability for x_1 lines in the intermediate trunk group being busy. Consequently we get for the average accessibility k_m:

$$k_m = \sum_{x_1} k(x_1)_m \cdot [x_1]$$

$$k_m = \sum_{x_1} \left((k_1 - x_1)k_{2j} + \eta_{2j} x_1\right)[x_1]$$

$$k_m = (k_1 - y_1)k_{2j} + \eta_{2j}\, y_1 \qquad (22)$$

Equation (22) is illustrated by figure 7.

Having evaluated the the numerical values of k_{eff} or k_m we can use loss tables for single-stage switching arrays, e.g. the EIF-table or the MPJ-table.

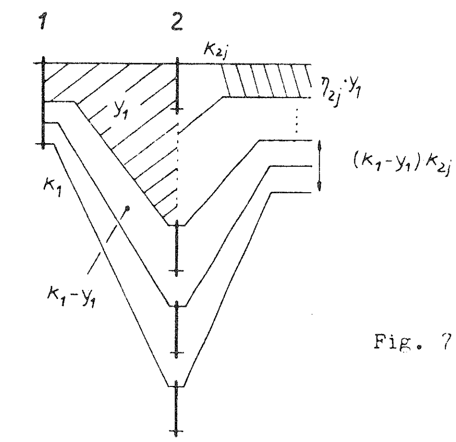

Fig. 7

II F. Method CIRB

A further development of the principle described in section II E is the calculation with the method of the combined inlet-route blocking, C I R B, by L o t z e /30/.

The approximate method CIRB permits to calculate time congestion and call congestion for link systems with an arbitrary number of selector stages and with preselection or group selection. The traffic offered may be pure chance traffic with Poisson input or traffic originating from a finite number of sources (Engset input). Gradings are allowed between the selector stages and behind the last stage.

The evaluation starts from the actually carried traffic of the system and thus yields results which are sufficiently close to reality from very small up to extremely large values of probability of loss. It is assumed, that the carried traffic is spread as evenly as possible among all multiples of each selector stage.

The fundamental ideas of the method will be explained now for a two-stage link system with group selection. A link-unit of the system is represented in figure 7.

The total time congestion E is divided in two parts: inlet blocking and outlet blocking.

a) Inlet blocking: Time congestion of a multiple in the first stage occurs if either all k_1 outlets of the multiple are busy in the case $i_1 > k_1$ or all i_1 inlets of the multiple are occupied in the case $i_1 \leq k_1$. These states of occupation will be denoted by $\{k_1\}$ or by $\{i_1\}$, furthermore the corresponding probabilities by $[k_1]$ or by $[i_1]$ respectively.

b) Outlet blocking: As long as $\{k_1\}$ or $\{i_1\}$ doesn't occur, congestion can also occur if the idle $(k_1 - x_1)$ intermediate lines have no access through the system to free outgoing lines of the desired trunk group. This "outlet blocking" probability is denoted by [p] (Instead of calculating with (k_1-x_1) the method makes use of the average number (k_1-y_1) of free outlets of the considered multiple 1 - cf. fig.7).

If the events corresponding to $[k_1]$ respectively to $[i_1]$ on the one side and to [p] on the other side are assumed to be independent, we get the following expressions for time congestion E with regard to the considered trunk group No. j.

$$E = [K_1] + (1-[K_1])[p] \quad ; \; i_1 > K_1$$
$$E = [i_1] + (1-[i_1])[p] \quad ; \; i_1 \leq K_1 \qquad (23.)$$

The call congestion B will be calculated by means of the time congestion E (Poisson input: B=E if $i_1 > k_1$, B<E if $i_1 \leq k_1$; Engset input: B<E if $i_1 \gtreqless k_1$).

Calculation of Inlet Blocking: The carried traffic y_1 on the outlets of a multiple in the first selector stage be prescribed. Using probability distributions for full-access trunk groups, e.g. Erlang's distribution in the case of an unlimited number of sources and Erlang's-Bernoulli-distribution in the case of a limited number of traffic sources, we can evaluate $[k_1]$ or $[i_1]$:

$$[K_1] = E_{1K_1}(A_0)$$
$$A_0 = \frac{y_1}{1-E_{1K_1}(A_0)} \qquad q \to \infty \qquad (24.)$$

$$[K_1] = \frac{\binom{i_1}{K_1}\cdot\alpha_{10}^{K_1}}{\sum_{\nu=0}^{K_1}\binom{i_1}{\nu}\cdot\alpha_{10}^{\nu}} \qquad q \neq \infty \qquad (25.)$$

$$\alpha_{10} \text{ such that } \sum_{\nu=1}^{K_1} \nu\cdot[\nu] = y_1 \text{ is fulfilled}$$

The iterations to find the fitting values A_0 or α_{10} for prescribed load y_1 can easily be done by a digital computer.

Calculation of Outlet Blocking: As an example we consider the most simple case that from one multiple in the first stage at most one link leads to one multiple 2, i.e. $l_{12}=1$; furthermore there may exist no grading.

Analogously to the average accessibility k_m defined in section II E we now introduce an average "hunting number" p such, that p equals to the average number of lines in the considered outgoing trunk group which can be hunted free or busy by the inlets of one multiple 1

$$p = (K_1 - y_1)K_{2j} + \eta_{2j}y_1 \qquad (26.)$$

Analogously we get for a system with S > 2 selector stages:

$$p = (K_1-y_1)(K_2-y_2)\cdots(K_{S-1}-y_{S-1})K_{Sj} + \eta_{Sj}y_1 \qquad (27.)$$

If $p > N_{Sj}$, we set $p = p_{max} = N_{Sj}$.

The time congestion [p] means the probability for at least those p lines in the considered outgoing trunk group to be busy. We get its value for example from the "modified Palm-Jacobaeus-Formula" (MPJ) setting p = k in the tables:

$$[p] = \frac{E_{1,N_{Sj}}(A_{0j})}{E_{1,N_{Sj}-p}(A_{0j})}$$
$$y_{Sj} = A_{0j}\left(1-E_{1,N_{Sj}}(A_{0j})\right) \qquad (28.)$$

Remark: The term $\eta_{Sj}\cdot y_1$ in (27) can often be neglected (as a rule if $\eta_{Sj}\cdot y_1 \ll p$).

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