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Point-to-Point Loss in Link Systems: Model and Calculation Methods

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Abstract—The switching networks of modern telephone exchanges are often built as multistage arrays which are controlled by conditional selection. Thereby, the strategy is often applied that a certain inlet in stage 1 is to be connected with a certain outlet in the last stage. The traffic loss due to nonavailability of an idle path between the two points is called point-to-point loss or point loss. A traffic model for the calculation of the point loss is defined. Then approximate methods of calculating the point loss in three- and four-stage link systems are derived. The results of calculation are compared with those obtained with a full-scale simulation of the systems investigated.

I. INTRODUCTION

THE speaking paths of modern telephone arrangements often consist of several selector stages in series, which are controlled by a centralized device in conditional selection. Idle links between the stages will only be selected if they can be used in a path connecting the inlet to an idle outlet. Thereby, we often have the case where a certain inlet in stage 1 is to be connected with a certain outlet in stage S , i.e., with a certain outgoing line. The traffic loss due to nonavailability of an idle path between the two points is called point-to-point loss or short, point loss.

II. DEFINITION OF POINT LOSS

To be able to calculate the point loss we first have to define the general statement, "connecting between a certain inlet and a certain outlet." For that purpose we will develop a traffic model upon which we may base the calculation of the interesting quantity point loss.

Traffic Model

When a call arrives at a multiple in the first stage, i.e., when it marks a certain incoming line, out of all the outgoing lines which are idle at this moment, an arbitrary line is chosen at random and marked. Then a connecting path between the two marked points must be looked for and switched. The probability that the marked free goal cannot be occupied is denoted by B_{NF} .

Calls arriving in the state "all outgoing lines are busy" cannot be switched and get lost. Such calls only contribute

to the total loss B of the system but not to the point loss B_{NF} . Therefore, the total loss B comprises both calls which cannot be connected to the marked outgoing line which is idle and calls which get lost because all outlets of the system are busy. Thus, among the total loss B , the total traffic offered A , and the total traffic carried Y the following equation holds:

$$Y = A \cdot (1 - B).$$

Now two different definitions of the point loss B_{NF} are possible which are denoted by B_{NF1} and B_{NF2} .

Definition of B_{NF1}

The quantity B_{NF1} is defined by the ratio number C_{NF} of calls which could not occupy the idle outgoing line marked, and the number C_A^* of calls which have arrived in the state "at least one outgoing line is idle." Therefore,

$$B_{NF1} = C_{NF}/C_A^*. \quad (1)$$

Definition of B_{NF2}

The quantity B_{NF2} is defined by the ratio number C_{NF} of calls which could not occupy the idle outgoing line marked, and the number C_A of total calls presented to the network. Therefore,

$$B_{NF2} = C_{NF}/C_A. \quad (2)$$

The traffic model defined herewith will be applied to calculate the point loss in the following sections.

The two definitions given here are further discussed in Section III in connection with the results of the exact calculation.

III. EXACT CALCULATION

The exact calculation leads to a linear homogeneous system of equations for the probabilities of state. If we can solve numerically the system of equations of state, we will find the point loss by linear combinations of certain probabilities of state.

The difficulties and the limits of an exact calculation are well known. Therefore only a small three-stage link system with preselection (see Fig. 1) has been calculated exactly. In this case the system of equations of state could be set up in a relatively simple way by utilizing properties of symmetry such as an equally distributed traffic offered among the multiples in the first stage and random hunting of the outlets of these multiples.

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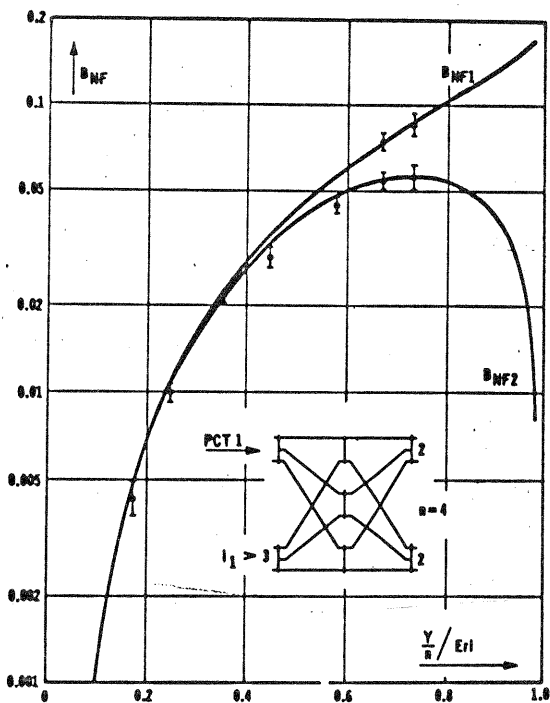


Fig. 1. Exact calculation of point loss. PCT 1—pure chance traffic 1 (unlimited number of independent traffic sources); n —number of outgoing lines, i_1 —number of inlets per multiple in stage 1, Y —total traffic carried in erlangs.

The value of an exact calculation of such small systems having no practical importance is that on the one hand we can check results obtained by tests with artificial traffic and on the other hand we can set up statements concerning the course of the curves of loss. Because we know in principle how to set up the equations of state by means of the diagram of states, the derivation is omitted.

The course of the curves for B_{NF1} and B_{NF2} is based on the definitions according to (1) and (2): At low loads both curves are practically identical, because the state "all outgoing lines are busy" only occurs with a negligible probability. With increasing load the curves diverge. The point loss B_{NF1} rises in a monotonous way, whereas the point loss B_{NF2} rises up to a maximum and then falls down to zero when the load is increased.

In link systems with a considerably greater number of outgoing lines of practical interest, the difference between B_{NF1} and B_{NF2} occurs at greater values of traffic carried and hence may be disregarded within the range of losses ($B_{NF} < 0.05$) considered.

IV. APPROXIMATE CALCULATION OF THREE- AND FOUR-STAGE LINK SYSTEMS WITH METHOD OF MULTIPLICATION AND SUMMATION OF PROBABILITIES

A. Structure and Graph of Connecting Paths

In Figs. 2 and 3 the structures of a three- and four-stage link system are shown. When no occupation exists in a system, only a certain number of paths will be suited to switch a call from the inlet being marked to the outlet

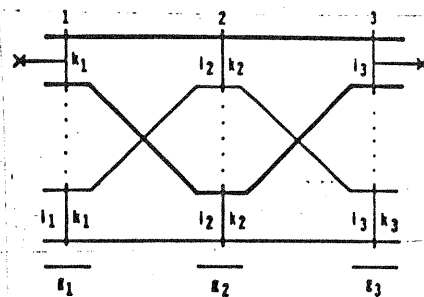


Fig. 2. Three-stage link system.

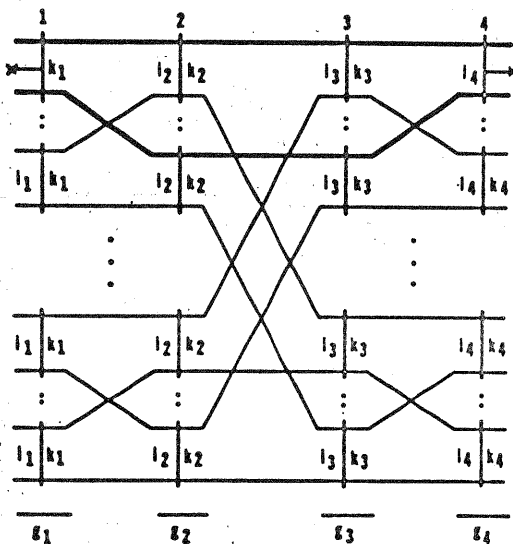


Fig. 3. Four-stage link system.

being marked. The entirety of these paths which are suited is called the graph of the system considered. The graph lying between the two points marked with crosses is displayed by heavy lines in the Figs. 2 and 3.

B. General Derivation

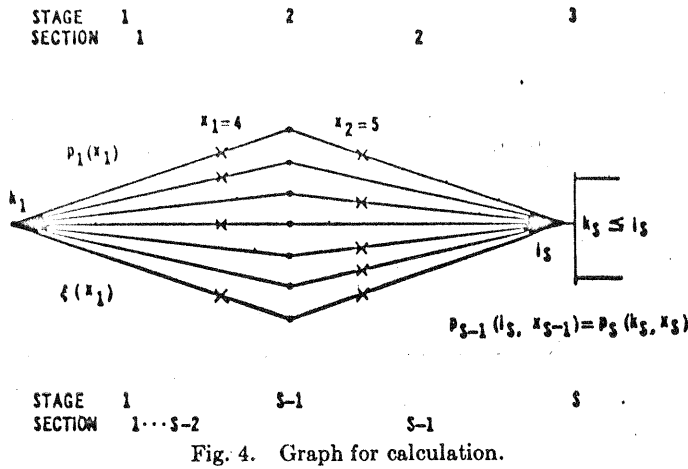
The method denoted by "multiplication and summation of probabilities" starts from the assumption that the probabilities for x lines to be occupied in a section of the graph are known for each section. Besides this the probability distribution for the whole outgoing group shall be known.

Remark: To state the formulas in a general form, we denote the number of stages by S ; in the examples given S will be 3 or 4.

The calculation is based upon the graph represented in Fig. 4. At first the procedure is explained assuming $S = 3$ (see Fig. 4 upper part) and then it is extended to an arbitrary number $S > 3$.

In the first section x_1 arbitrary links will be occupied with probability $p_1(x_1)$. The idle outgoing line which is marked in stage 3 will not be reached if at least those $(k_1 - x_1) = (i_3 - x_1)$ links which are still idle in the first section are occupied in section 2. The probability therefore is denoted by

$$w(k_1 - x_1) = w(i_3 - x_1).$$



In the case of networks with more than three stages ($S > 3$) x_1 represents the number of connecting paths between sections 1 and $(S - 2)$, which are occupied with probability $\xi(x_1)$. Now the idle outgoing line marked in stage S will not be reached if at least those $(k_1 - x_1) = (i_S - x_1)$ links are occupied in section $(S - 1)$, which correspond to the idle paths running from section 1 through to section $(S - 2)$. The probability therefore is denoted by

$$w(k_1 - x_1) = w(i_S - x_1).$$

Provided that all the patterns of occupation of a certain state $\{x_S\}$ occur with an equal probability, we obtain in the well-known way

$$w(k_1 - x_1) = \sum_{x_S=k_1-x_1}^{k_S} \frac{\binom{x_S}{i_S - x_1}}{\binom{i_S}{i_S - x_1}} \cdot p_S(k_S, x_S). \quad (3)$$

The quantity $p_S(k_S, x_S)$ is the probability that x_S out of k_S outgoing lines in the multiple considered in stage S are busy.

Furthermore, we have to take into account that only idle outgoing lines get marked, i.e., there must be at least one idle line in the last section of the graph. Besides this we have to take into consideration the influence of the common control, because it is supposed that one out of all the idle outgoing lines is marked at random. This means that the probability of having a certain multiple in the last stage as goal is proportional to the number of outlets which are idle in it at that time. Both demands are taken into account by a weighting function $G(x_S)$ as follows:

$$G(x_S) = \frac{k_S - x_S}{k_S - Y_S} \quad (4)$$

with

$$Y_S = \sum_{x_S=1}^{k_S} x_S \cdot p_S(k_S, x_S). \quad (5)$$

Now we obtain from (3)

$$w^*(k_1 - x_1) = \sum_{x_S=k_1-x_1}^{k_S} G(x_S) \cdot \frac{\binom{x_S}{i_S - x_1}}{\binom{i_S}{i_S - x_1}} \cdot p_S(k_S, x_S). \quad (6)$$

Finally, if we denote by $p(x)$ the probability distribution of the whole outgoing group, we will obtain for the point loss the following equations in the case of pure chance traffic 1 (Poisson input, PCT 1): ...

$$B_{NF1} = \frac{\sum_{x_1=i_S-k_S}^{i_S} \xi(x_1) \cdot w^*(k_1 - x_1)}{1 - p(n)} \quad (7a)$$

$$B_{NF2} = \sum_{x_1=i_S-k_S}^{i_S} \xi(x_1) \cdot w^*(k_1 - x_1). \quad (7b)$$

As was already pointed out in Section III, the difference between B_{NF1} and B_{NF2} may be disregarded within the range of loss which is of practical interest, because then $p(n)$ will be much smaller than unity.

As to the calculation or assumption of the probabilities $p_i(x_i)$, there exist different possibilities similar to the calculation of route blocking in link systems [22].

Remark: If we assign the weight function $G(x_S)$ (6) the value unity for all values x_S and extend the summation in the case of $i_S = k_S$ up to $(k_S - 1)$ instead of k_S , (7b) with (6) will yield the following relation:

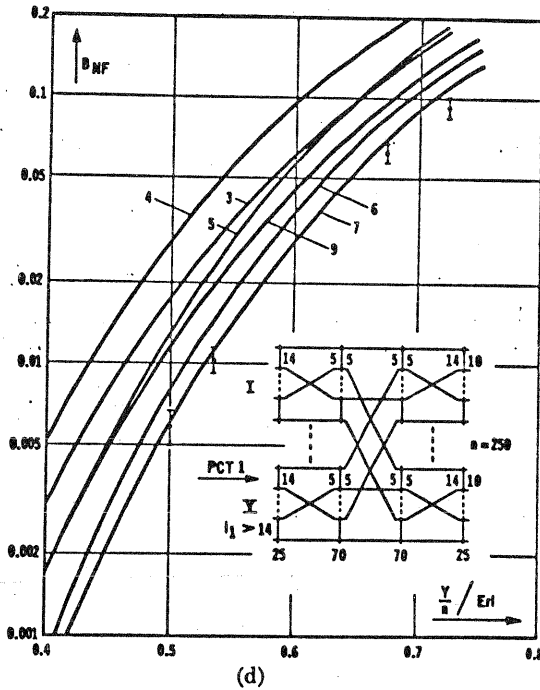
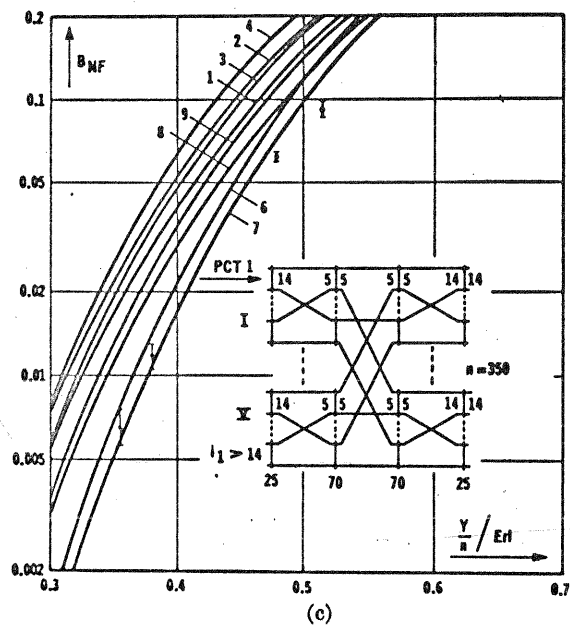
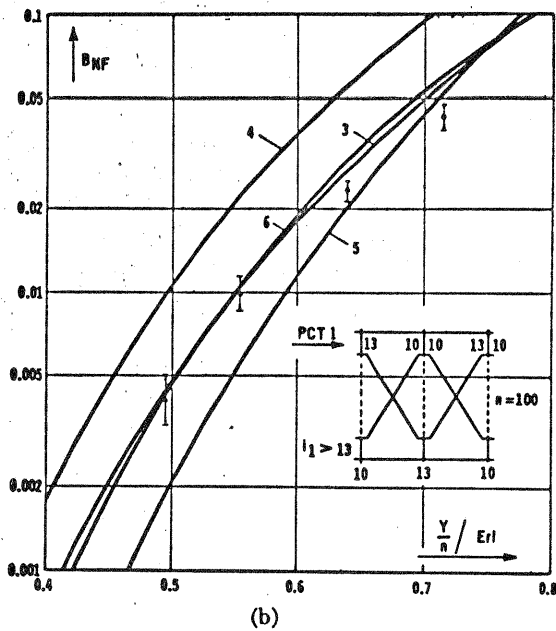
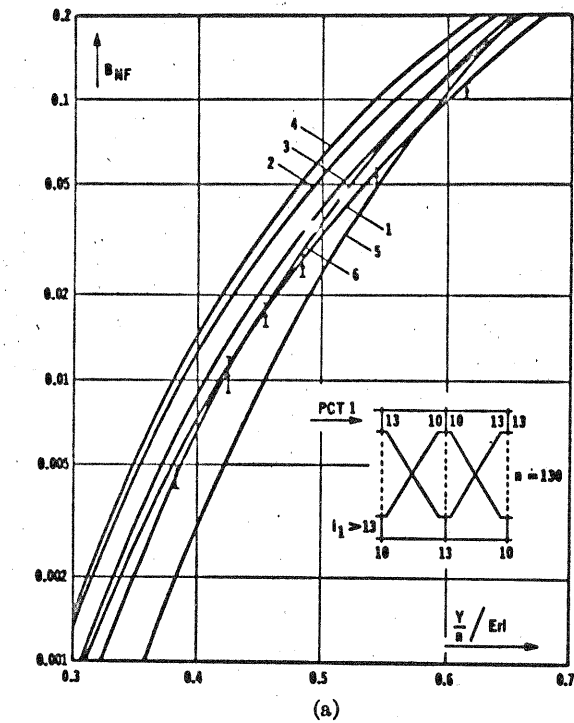
$$B_{NF2}^* = \sum_{x_1=0}^{i_S} \xi(x_1) \cdot w^*(k_1 - x_1) - p_S(k_S, k_S). \quad (8)$$

The point loss B_{NF2}^* according to (8) is very similar to [15] and [16].

C. Numerical Evaluation

1) *Functional Dependence and Statistical Independence:* As is well known, the probability distributions $p_i(x_i)$ on the different sections of the graph are dependent on each other functionally. This means, for example (Fig. 4), that the probability for x_1 lines being occupied in the first section is a function of the probability for x_2 lines being occupied in the last section, and vice versa. The consideration of this functional dependence of the probability distributions leads to iterative methods for the calculation of the $p_i(x_i)$. Here we calculate these probabilities by means of the iterative procedure according to [16]. Then we obtain the probability $\xi(x_1)$ starting from the probabilities $p_i(x_i)$, $i = 1, 2, \dots, S - 2$, by means of the algorithm according to [15]. Now the point loss B_{NF} may be calculated with (7a) and (7b).

Remark: The functional dependence between the probability distributions $p_i(x_i)$ could only be taken into consideration in the case $i_S = k_S$, i.e., if there was no concentration in the last stage, because the iterative procedure in [16] is only planned for this case.



Curve	Explanations	Equations
1	Functional dependence; statistical independence	(4)-(7)
2	Same as curve 1; influence of common control not taken into account	(8)
3	Functional and statistical independence; Erlang distributions on each section	(4)-(7)
4	Same as curve 3; influence of common control not taken into account	(8)
5	Functional and statistical independence; Bernoulli distributions on each section; method of Leo [4]	(9)
6	Average availability k_m	(10), (13)-(15)
7	Modified average availability k_m^*	(12)-(15)
8	Same as curve 1; correction according to Daisenberger [20], [21]	(4)-(7)
9	Same as curve 3; correction according to Daisenberger [20], [21]	(4)-(7)

Fig. 5. Point loss versus traffic carried per outgoing line.

However, to avoid too complicated a process of calculation, in the iterative procedure described, as well as in (7a) and (7b), a further simplifying approximate assumption is introduced, namely, the assumption of the statistical independence of the states of occupation in the different sections of the graph.

2) *Functional and Statistical Independence*: If on the contrary the functional independence of the probability distributions of the different sections of the graph is assumed, we will have to prescribe probability distributions of well-known types. That means, for the evaluation of (7a) and (7b), that the distributions $p_i(x_i)$ are to be prescribed, e.g., Erlang distribution, Bernoulli distribution, and so forth. Apart from the weighting function G , the calculation method is in this case identical to the principle of Jacobaeus [1].

Remark 1: If we presume independent Bernoulli distributions on each section of the graph and if we assign to the weighting function $G(x_s)$ in (6) the value of unity for all values of x_s , we will get, in the case of $i_s = k_s$, the same results as with the method of Lee [4]. If we denote by Y_i , $i = 1, 2, \dots, S - 1$, the traffic carried on the different sections of the graph, we will obtain

$$B_{Lee} = \left(1 - \prod_{i=1}^{S-1} \left(1 - \frac{Y_i}{k_i} \right) \right)^{k_1} \quad (9)$$

It must be pointed out, however, that the method of Lee will yield such simple and clear relations only if it is applied to graphs with a series-parallel structure, as considered here. Applied to more complicated structures, such as meshed graphs, the method of Lee will be very unwieldy and in most cases not applicable.

Remark 2: To determine the point loss defined according to Lee in such complex structures, Grantges and Sinowitz [14] developed the simulation method NEASIM. The characteristic feature of this simulation method is that not the whole link system and the real traffic are simulated, but the graph with independent Bernoulli distributions on each section is used upon which the calculation method is based. This means, that with NEASIM not the whole link system is simulated, but the calculation method of Lee is used. Therefore the results of simulation with NEASIM naturally correspond to the numerical results of Lee. This has to be pointed out strongly, because the results of simulation with NEASIM are often used in practice for the comparison with and for the estimation of approximate calculation methods. However, the significance of such results of simulation cannot be expected to be greater than the accuracy of the calculation method of Lee.

D. Examples

To check the accuracy and utility of the method of multiplication and summation of probabilities for the calculation of the point loss, we calculate three- and four-stage systems in this section and compare the numerical results with the results of the simulation. When simulating, the whole link system was realized (full-scale simula-

tion). The confidence limits have been calculated for a significance level of 0.05. Fig. 5 and an explanation of the curves follow.

E. Discussion of Curves

First we will examine Fig. 5(a) and (b). The point loss, calculated according to (7a) and (7b), taking into account both the functional dependence of the probability distributions and the influence of the common control (see Fig. 5(a), curve 1) corresponds very well within the whole range of loss to the simulation results.

If we calculate the point loss according to the same equations but without taking into consideration the functional dependence, we will obtain results which are still sufficiently accurate (see Fig. 5(a) and (b), curve 3). When starting from (8) and (9), the point loss does not correspond to reality (see curves 2, 4, and 5).

Next we examine Fig. 5(c) and (d). The point loss calculated according to (7a) and (7b), taking into account both the functional dependence of the probability distributions and the influence of the common control (see Fig. 5(c), curve 1) lies within the whole range of loss considered above the simulation results. The divergences are so considerable, between 60 and 170 percent, that in any case we can only call the method a rough estimation. Even if the statistical dependence is taken into account partially according to [20], [21] (Fig. 5(c), curve 8), we will have no significant improvement. To achieve an essential improvement of the accuracy, the statistical dependence has to be taken into consideration more accurately.

If we calculate the point loss according to the same equations but without taking into account the functional dependence, we will obtain curve 3 (of Fig. 5(c) and (d)) and, with the improvement from [20] and [21], curves 9. The accuracy is not satisfactory. When starting from (8) and (9), the point loss does not correspond to reality (see curves 2, 4, and 5).

V. APPROXIMATE CALCULATION OF THREE- AND FOUR-STAGE LINK SYSTEM WITH METHOD OF AVERAGE AVAILABILITY

A. General Derivation

In this section we will calculate the point loss for three- and four-stage link systems (see Figs. 2 and 3), when the calculation starts from the method of combined inlet and route blocking (CIRB) of Lotze [12]. Thereby essentially the considerations occurring with the calculation of the route blocking by means of the average availability will be applied.

1) *Average Availability in Three-Stage Systems*: Fig. 6 shows the graph suitable for a connection between the two marked points of the three-stage link system according to Fig. 2. If on the average Y_2 of i_3 lines in the second section are occupied, because the remaining $(i_3 - Y_2)$ lines are idle, then $(i_3 - Y_2)$ lines in the first section will be "visible." That means, the average number k_m of lines

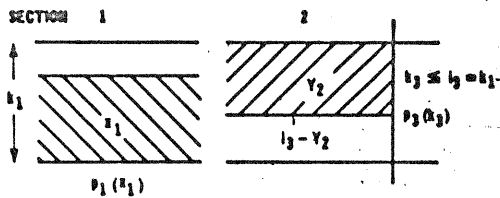


Fig. 6. Graph of three-stage system.

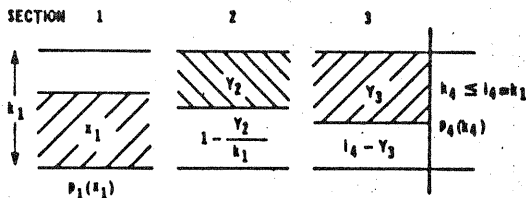


Fig. 7. Graph of four-stage system.

in the first section, which can be searched from the considered idle outlet of the system, is

$$k_m = i_3 - Y_2. \quad (10)$$

2) *Average Availability in Four-Stage Systems:* Fig. 7 gives the graph suitable for a connection between the two marked points of the four-stage link system of Fig. 3. If on the average Y_3 of i_4 lines in the third section are occupied, because the remaining $(i_4 - Y_3)$ lines are idle and because the lines in the second section are idle with probability $(1 - Y_2/k_1)$, on the average $(i_4 - Y_3) \cdot (1 - Y_2/k_1)$ lines in the first section will be "visible." Therefore, the mean number k_m of lines in the first section which can be searched from the considered idle outlet of the system, or in other words the mean number k_m of paths idle behind the first section up to the goal in the last stage, is

$$k_m = (i_4 - Y_3) \cdot \left(1 - \frac{Y_2}{k_1}\right). \quad (11)$$

Instead of the average availability k_m according to (11) we can use, with reference to an investigation by Daisenberger [20], [21], a modified availability k_m^* taking approximately into account the statistical dependence in consecutive sections of the system:

$$k_m^* = (i_4 - Y_3) \cdot \left(1 - \frac{Y_2}{k_1}\right) \cdot \frac{1}{1 - Y_3/i_4 i_3}. \quad (12)$$

3) *Point Loss B_{NF} :* A call arriving in the considered multiple in the first stage will not be able to occupy an idle line in section 2, if in the first section at least those k_m "visible" lines are occupied. In the case of pure chance traffic 1 (PCT 1) we presume an erlang distribution on the k_1 lines of the first section. According to the MPJ formula [10] we obtain for the probability $w(k_m)$ that at least certain k_m lines are occupied in the first section:

$$w(k_m) = \frac{E_{k_1}(A_0)}{E_{k_1 - k_m}(A_0)} \quad (13)$$

with

$$Y_1 = A_0(1 - E_{k_1}(A_0)). \quad (14)$$

If k_m is not an integer, we will to have interpolate.

Remark: Contrary to (7a) and (7b) we do not apply a weighting function to take into account the influence of the common control in this case, because the calculation starts from average values and hence such a function does not appear to be justified.

To determine the point loss we have to take into consideration furthermore that in the goal under consideration there must be at least one idle outlet. If we denote the probability that in the goal all k_s outlets will be occupied by $p_s(k_s)$ and if we assume the states of occupation corresponding to the probabilities $w(k_m)$ and $p_s(k_s)$ to be statistically independent, we will get for the point loss

$$B_{NF1} = \frac{w(k_m) \cdot (1 - p_s(k_s))}{1 - p(n)}. \quad (15a)$$

$$B_{NF2} = w(k_m) \cdot (1 - p_s(k_s)). \quad (15b)$$

As already mentioned (see (7a) and (7b)), $p(n)$ is the probability that all outgoing lines are occupied. Since within the range of losses considered, $p(n) \ll 1$, the difference between B_{NF1} and B_{NF2} can be disregarded.

B. Examples

To check the accuracy of the method derived in Section V-A, we calculated the point loss for some three- and four-stage systems and compared the results with tests made with artificial traffic (see Fig. 5). We can state generally that the method of the average availability yields results sufficiently close to reality for three- and four-stage link systems (see curves 6 and 7 of Fig. 5).

VI. CONCLUSION

Both the principle of multiplication and summation of probabilities and the principle of average availability have been applied to derive new formulas for calculating the point loss in three- and four-stage link systems with series-parallel linking patterns. The comparison of the numerical results with the results of a full-scale simulation shows that the first method yields accurate results for three-stage systems when a weighting function is introduced taking into account the influence of the common control. The results obtained with the average availability method were sufficiently close to reality in all of the cases investigated.

The difficulties of developing general methods which can be applied to a variety of systems such as meshed networks, or to different operational procedures such as reselection, are obvious. Therefore, it is not surprising that some methods (see [22] or the results for three-stage systems in this paper) yield more accurate results for certain networks than the average availability method. But those methods are much more complicated, and it is difficult, if not impossible, to extend them; whereas the

latter one probably can be extended for the application to more complicated cases due to the fact that it is working with average values.

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