A Discrete-Time Queueing Model for ATM

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Extended Abstract

Realistic source models reflecting the main characteristics of traffic sources such as video codecs are essential in order to study the performance of ATM network components. For simulation purposes, the source models can in principle be sufficiently detailed (though complex), but for analytical investigations they are usually very simple. Unfortunately, the simulation technique has several limitations which reduce its applicability for the performance study of ATM networks. These limitations are due to the measurement of extremely small probabilities below 10^{-6} and the runtime requirements for complex traffic models. Hence, analytic models which enable a queueing analysis are useful, e.g., when dimensioning a statistical multiplexer to provide a loss probability of 10^{-10}.

Commonly used analytic source models are the Poisson process and the Bernoulli process. However, these models do not incorporate important characteristics of the traffic such as its correlation structure or periodicity. Therefore, more realistic models are required which include the discrete-time nature and other basic properties of the cell traffic, while still being analytically tractable.

The Discrete-Time Markovian Arrival Process (DMAP) [1] is a promising approach in this direction. It is a stochastic process which is based on an irreducible discrete-time Markov Chain with state space $\{1, \ldots, m\}$. If the process is in state i at time $k \cdot \Delta t$, it moves to state j at time $(k+1) \cdot \Delta t$ with probability $u_{ij} = c_{ij} + d_{ij}$. Here, c_{ij} is the probability that the transition from state i to state j occurs without an arrival, and d_{ij} is the probability that the same transition generates an arrival event. Thus, the process is completely defined in terms of the matrices $\mathbf{C}$ and $\mathbf{D}$, where $\mathbf{C} = [c_{ij}]$ and $\mathbf{D} = [d_{ij}]$. From this definition it is clear that the transition matrix of the underlying Markov Chain is $\mathbf{C} + \mathbf{D}$, and consequently $\sum_{j=1}^{m}(c_{ij} + d_{ij}) = 1, \ \forall\, i$. In the following, the state of the Markov Chain describing the DMAP will be called its phase to avoid confusion with the state of the queueing system considered later on. Note that the DMAP is the discrete-time analog of a stochastic process described in [8]. It was introduced in [1] and [12], and a detailed description of its characteristics can be found in [2]. It covers many discrete-time processes excluding batch arrivals. For instance, $\mathbf{D} = p$ and $\mathbf{C} = 1 - p$ implies a Bernoulli process with arrival probability p. A very general class of processes called General Modulated Deterministic Processes (GMDP) [6] can be modelled as a DMAP. This class includes the well-known burst silence model [5] and the Markov Modulated Deterministic Process (MMDP) as a special case [2]. In addition, a DMAP allows the modelling of an arbitrary discrete-time renewal process, which has a well-defined maximum interarrival time.

The queueing model studied first consists of a discrete-time single server queue with infinite capacity and FIFO service discipline. The arrival process is supposed to be a DMAP characterized by the transition matrices **C** and **D**. The service time has a general discrete-time distribution $h(k)$. Mutual independence is assumed with respect to the arrival process and the service times. Since we consider a discrete-time queueing system, an arrival and a departure may occur simultaneously. In this case the arriving customer enters the system immediately before the departing one leaves (arrival first, AF). The solution for this queueing model is based on a two-dimensional embedded Markov Chain at departure instants describing the state of the system by the number of customers and the phase of the DMAP. The state probabilities at departures and the stationary queue length distribution can be obtained using a matrix analytic approach, which can be found in several publications [8, 10, 11].

In order to obtain the loss probability of the system with finite buffer capacity s the stationary state probabilities at departures and the blocking probability are calculated. Due to the finite state space the exact values of these probabilities can be computed directly or iteratively by solving the system of equations given by the Markov Chain. However, these methods are either memory or time consuming, especially for large m and s. Therefore, an approximation for the state probabilities and the loss probability is given by taking into account the state probabilities at departures in the system with infinite capacity, thus reducing the computational effort. If an exact solution is necessary, it can be obtained very efficiently from an additional iteration starting with the approximated values. In this case the iteration converges rapidly, because its initial values are already close to the exact solution.

As an application, the characteristics of the Leaky Bucket algorithm used for source monitoring in ATM networks is investigated. This algorithm can be modelled as a finite capacity single server queueing system with deterministic service time [13].

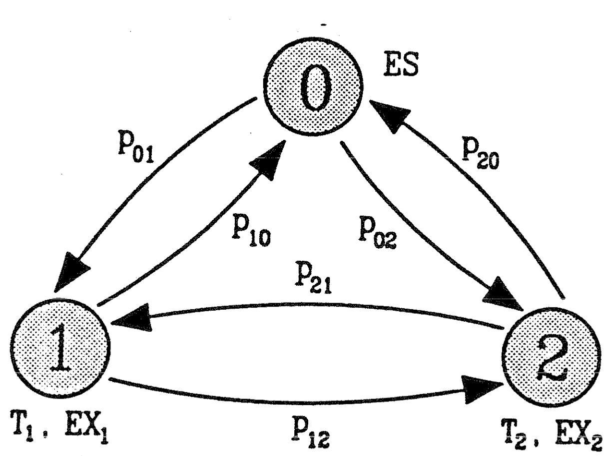

Figure 1: State transition diagram of an MMDP with 3 states

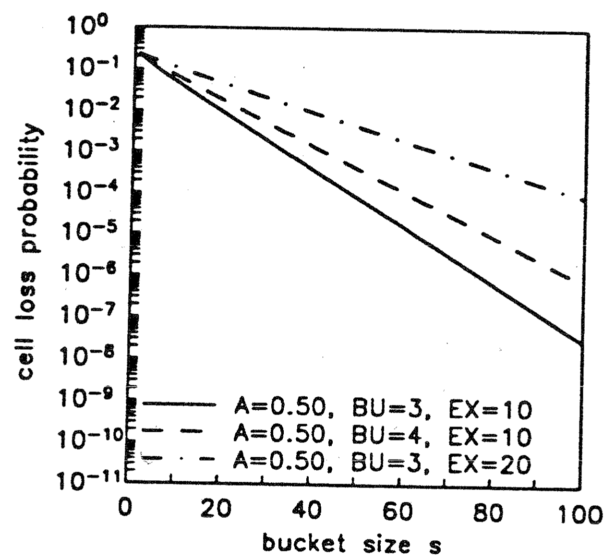

Figure 2: Cell loss probability versus bucket size

As shown in Figure 1 the input process is assumed to be an MMDP with 3 states which may describe the cell generation process of a variable bitrate source (e.g. video codec for ATM networks). In the states 1 and 2 cells have a constant interarrival time T_1 and T_2, respectively. While being in state 0 no cells are emitted. The number of cells generated in states 1 and 2 and the sojourn time of the silence state have a shifted geometric distribution with mean $EX_1 = EX_2 = EX$ and ES. State changes occur with probability $p_{vw} = 0.5$, $v \neq w$.

Figure 2 depicts the cell loss probability of the Leaky Bucket mechanism versus bucket size for different source parameters. Higher burstiness BU (= max. cell rate / mean cell rate) and longer burst duration lead to higher cell loss probabilities.

In all cases considered here the proposed approximation yields exact results. In general, the approximation error is small and almost independent of the buffer size s according to our experience.

References

[1] C. BLONDIA, "A Discrete-time Markovian Arrival Process", *Document RACE 1022*, PRLB_123_0015_CC_CD, August 1989.

[2] C. BLONDIA, T. H. THEIMER, "A Discrete-time Model for ATM Traffic", *Document RACE 1022*, PRLB_123_0018_CC_CD / UST_123_0022_CC_CD, October 1989.

[3] C. BLONDIA, "Finite Capacity Vacation Models with Non-Renewal Input", to appear in *Journal of Applied Probability*, vol. 28, 1991.

[4] U. BRIEM, T. H. THEIMER, H. KRÖNER, "A General Discrete-Time Queueing Model: Analysis and Application", *Proceedings of the 13th International Teletraffic Congress (ITC)*, Copenhagen, June 1991, pp. 13—19.

[5] H. HEFFES, D. M. LUCANTONI, "A Markov Modulated Characterization of Packetized Voice and Data Traffic and Related Statistical Multiplexer Performance", *IEEE Journal on Selected Areas in Communications*, vol. SAC-4, no. 6, September 1986, pp. 856–868.

[6] P. J. KÜHN, "Input Process Model GMDP", *Document RACE 1022*, UST_123_0003_-CC_CD, June 1988.

[7] D. M. LUCANTONI, V. RAMASWAMI, "Efficient Algorithms for Solving the Non-Linear Matrix Equations Arising in Phase Type Queues", *Communications in Statistics — Stochastic Models 1*, no. 1, 1985, pp. 29–51.

[8] D. M. LUCANTONI, K. S. MEIER-HELLSTERN, M. F. NEUTS, "A Single Server Queue with Server Vacations and a Class of Non-Renewal Arrival Processes", *Working Paper 88-008*, Systems and Industrial Engineering Department, The University of Arizona, 1988.

[9] M. F. NEUTS, "A Versatile Markovian Point Process" *Journal of Applied Probability*, vol. 16, 1979, pp. 764–779.

[10] M. F. NEUTS, *Structured Stochastic Matrices of M/G/1 Type and their Applications*, Marcel Dekker Inc., New York and Basel, 1989.

[11] V. RAMASWAMI, "The N/G/1 Queue and its Detailed Analysis", *Adv. Appl. Probability 12*, 1980, pp. 222–261.

[12] V. RAMASWAMI, G. LATOUCHE, "Modeling Packet Arrivals From Asynchronous Input Lines", *Proceedings of the 12th International Teletraffic Congress (ITC)*, Torino, June 1988, Paper 4.1A.5.

[13] E. P. RATHGEB, "Policing Mechanisms for ATM Networks — Modelling and Performance Comparison", *ITC Specialist Seminar*, Morristown, October 1990.

[14] J. SOLÈ, J. DOMINGO, J. GARCÌA, "Traffic Burstiness Influence on ATM Switching Behaviour", *Fourth International Conference on Data Communication Systems and their Performance*, Barcelona, June 1990, pp. 133–147.