PROBABILITY OF LOSS OF DATA TRAFFICS WITH DIFFERENT BIT RATES HUNTING ONE COMMON PCM CHANNEL

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ABSTRACT

This paper deals with the calculation of the probabilities of loss for data traffics with different bit rates, hunting one common data link, e.g. one PCM channel.
This PCM channel is divided up into subchannels, so a superframe structure is obtained. A data channel for a data connection is constituted by allocating one or more subchannels, corresponding to the bit rate, to this connection. This allocation of subchannels to data connections is done dynamically during call establishment. According to the multiplexing techniques, two different principles of allocating subchannels to one data connection are considered:

Arbitrary subchannel allocation:
The necessary subchannels for one data connection are selected arbitrarily out of all idle subchannels without regarding their time slot position within the superframe.
Regular subchannel allocation:
The necessary subchannels for one data connection are selected out of all idle subchannels such that only time slot positions are selected, which are equally spaced within the superframe.

As the data traffics with different bit-rates suffer different probabilities of loss, the paper investigates means to obtain equal probabilities of loss for all types of traffic. This is done by introducing a limited availability for calls with lower bit-rates. The paper concludes with a comparison between the probabilities of loss in case of the two dynamic allocation principles and with the case of permanent allocation (separate "trunkgroups" for all types of traffic).

1. INTRODUCTION

In modern data networks or service integrated PCM networks for data and telephone, data traffic with different bit rates has to be transmitted and switched. Between the data exchanges bitstreams with 64 kbps will be used as a basic data link /1/. This bit rate can be transmitted either in one PCM channel or, by using group band modems, within one primary group /2/.
To make efficient use of this 64 kbps bit rate several low speed data connections should be aggregated to one multiplex bit rate. This can be performed, according to CCITT recommendations, by dividing up the 64 kbps data link into 8o subchannels, each carrying 8oo bps, where 6oo bps are available for data transmission (the remainder bit-rate is necessary for signalling and synchronizing).
Thus, we obtain a superframe which comprises 8o subchannels. Data channels for the standardized bit-rate classes with o.6, 2.4 and 9.6 kbps will be obtained by allocating 1, 4 or 16 subchannels, resp. to one data connection.
Three possibilities regarding the multiplexing of data traffic with different bit rates into one 64 kbps data link are of interest :

- Only data channels with homogeneous bit rates are multiplexed (e.g. 5 data channels with 9.6 kbps or 2o with 2.4 kbps or 8o with o.6 kbps).
- Data channels with heterogeneous bit rates are multiplexed (e.g. 2 with 9.6 kbps and 8 with 2.4 kbps and 16 with o.6 kbps, occupying a total number of 8o subchannels). The allocation of subchannels to the data channels is permanent; the available number of data channels for the different bit-rate classes is fixed.
- Data channels with heterogeneous bit rates are multiplexed. The allocation of subchannels is dynamic; the available number of data channels for the different bit-rates is variable.

The first two multiplexing schemes do not lead to new problems concerning traffic theory, because the 8o subchannels are allocated permanently to the data channels. Thus the 64 kbps data link represents separate "trunkgroups" for each bit-rate class. Therefore, the paper will deal with the efficiency of a multiplexing scheme concerning the third case, where all subchannels can be occupied by all traffics with different bit rates.

Two different principles of allocating subchannels to a data connection will be considered (see Fig.1) :
- arbitrary subchannel allocation: the necessary subchannels for one data connection (e.g. four for a 2.4 kbps connection) are selected out of all idle subchannels without regarding their position within the superframe.
- regular subchannel allocation: the necessary subchannels for one data connection are selected out of all idle subchannels in such a way that only time slot positions are selected, which are equally spaced within the superframe.

As later will be shown, the regular subchannel allocation will lead to a less efficient channel utilisation than the arbitrary subchannel allocation. The amount of equipment (buffer memories) for multiplexing will be smaller, however.

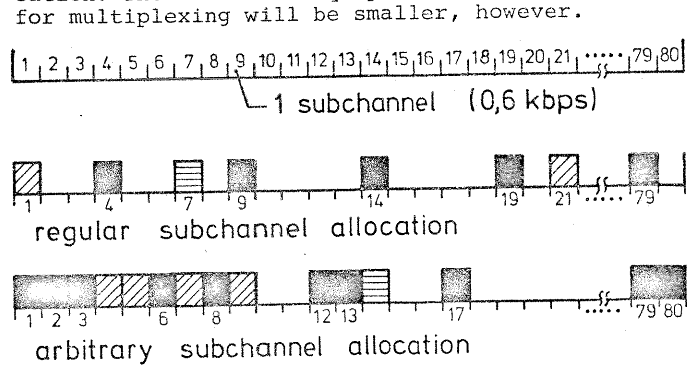

Fig. 1: The principles of subchannel allocation

The calculations demonstrate that for both allocation principles the different bit-rate classes suffer a different probability of loss, where the differences can reach the range of magnitudes. Therefore in the paper new methods will be shown to equalize the probabilities of loss for both principles of subchannel allocation. The paper is concluded by a comparison between the two subchannel allocation principles and the case of permanent allocation of subchannels, to show the advantage of dynamic allocation with regard to channel utilisation.

For the calculation of the probabilities of loss the following assumptions are made:

- each input stream of R different bit-rate classes is Poissonian with the arrival rate λ_r. (r = 1,2,3..R)
- the holding times of all data connections are negative exponentially distributed with the mean h_r. A data connection of bit-rate class r requires a data channel with m_r subchannels.
- the offered traffic for each bit-rate class is $A_r = \lambda_r/h_r$.
- lost calls are cleared.

Regarding the numerical results, special emphasis is put on the multiplexing scheme according to CCITT recommendations with 8o subchannels, R= 3 and $m_1 = 1$, $m_2 = 4$, $m_3 = 16$ (corresponding to the bit rates o.6, 2.4 and 9.6 kbps).

2. CALCULATION OF THE PROBABILITY OF LOSS IN CASE OF ARBITRARY SUBCHANNEL ALLOCATION.

2.1. FULL ACCESS TO THE SUBCHANNELS BY ALL BIT-RATE CLASSES.

The calculation of the probability of loss in case of arbitrary subchannel allocation and full access to all subchannels by R bit-rate classes was presented by Enomoto and Miyamoto /1/ and Frederikson /2/. In /1/ general formulas for an arbitrary number of R bit-rate classes are derived. Therefore in this paper only the special case for R = 3 is shown, with the assumption

$$n/m_r \quad \text{and} \quad m_{r+1}/m_r$$

being integer values (which holds for most practical cases and especially in the case of bit rates o.6, 2.4 and 9.6 kbps). We get the probability of state $p(x_1,x_2,x_3)$ where x_r is the number of occupied data channels by the bit-rate class r:

$$p(x_1,x_2,x_3) = \frac{\prod_{r=1}^{3} A_r^{x_r}/x_r!}{\sum_{\xi_3=0}^{\frac{n}{m_3}} \sum_{\xi_2=0}^{\frac{n-m_3\xi_3}{m_2}} \sum_{\xi_1=0}^{\frac{n-m_3\xi_3-m_2\xi_2}{m_1}} \prod_{r=1}^{3} \frac{A_r^{\xi_r}}{\xi_r!}} \qquad (1)$$

The probability of loss B_r for a call of bit-rate class r can be derived from the probability of state less complicated than shown in /1/. All those probabilities of state $p(x_1,x_2,x_3)$ with a number of $x = m_1x_1 + m_2x_2 + m_3x_3$ occupied subchannels greater or equal than $n - m_r + 1$ are summed up.
We obtain :

$$B_r = \sum_{x_3=0}^{\frac{n}{m_3}} \sum_{x_2=0}^{\frac{n-m_3x_3}{m_2}} \sum_{x_1=c}^{\frac{n-m_3x_3-m_2x_2}{m_1}} p(x_1,x_2,x_3) \qquad (2)$$

$$c = \mathrm{Max}\left\{\frac{n-m_r+1-m_3x_3-m_2x_2}{m_1},\ 0\right\}$$

In Fig. 2a,b the probability of loss B_r for the three bit-rate classes 0.6, 2.4, 9.6 kbps and n = 80 subchannels are shown. In the first case B_r is a function of the total offered traffic

$$A = A_1m_1 + A_2m_2 + A_3m_3 \qquad (3)$$

The ratio of the three offered traffics $A_1{:}A_2{:}A_3$ is fixed to 16:4:1, i.e. each traffic would occupy the same number of subchannels if an unlimited number were available. In the second case B_r is a function of the offered traffic A_1 with fixed offered traffics A_2,A_3.

Both diagrams show the significant differences in the probabilities of loss for the different bit-rate classes, which amount to a factor greater than 1o. If the admissible offered traffics are chosen such that the probability of loss B_1 does not exceed a prescribed value, e.g. 1%, B_3 will be greater than 2o% . If, on the other hand, B_3 is about 1%, B_1 and B_2 are unnecessarily small in the range of o.1% and below.

2.2. LIMITED ACCESS TO THE SUBCHANNELS BY THE LOWER BIT-RATE CLASSES.

In the following, two methods will be investigated to reduce the probability of loss for the higher bit-rate classes at the expence of the unnecessarily small losses of the lower bit-rate classes.This can be achieved as follows:
The lower bit-rate classes are allowed to occupy simultaneously only a limited number out of all subchannels. Thus for the higher bit-rate traffics, on the average, more idle subchannels are available. Two ways are investigated in order to realize this limited access to the subchannels for the lower bit-rate traffics.

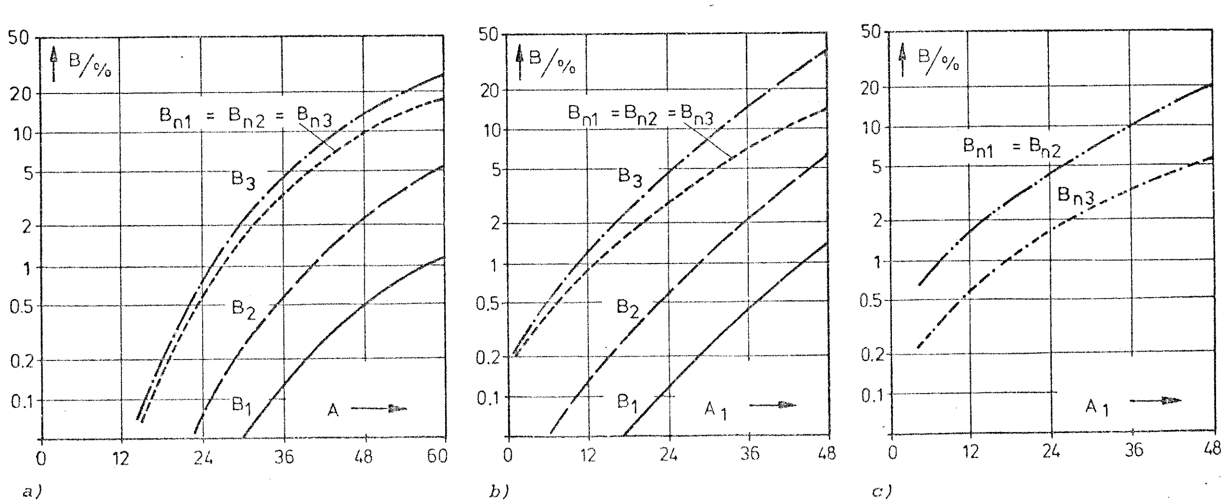

Fig. 2: The probabilities of loss B_r for three bit-rate classes as a function of
a) the offered traffic A with $A_1{:}A_2{:}A_3 = 1/m_1 : 1/m_2 : 1/m_3 = 16{:}4{:}1$
b and c) the offered traffic A_1 with $A_2 = 2$ and $A_3 = 0.5$.
(The curves for B_{nr} are valid in the case of limited availability SLM, cf. section 2.2.2 with
a) $n_1^* = n_2^* = n_3^* = 65$, b) $n_1^* = n_2^* = n_3^* = 65$, c) $n_1^* = n_2^* = 61$, $n_3^* = 65$

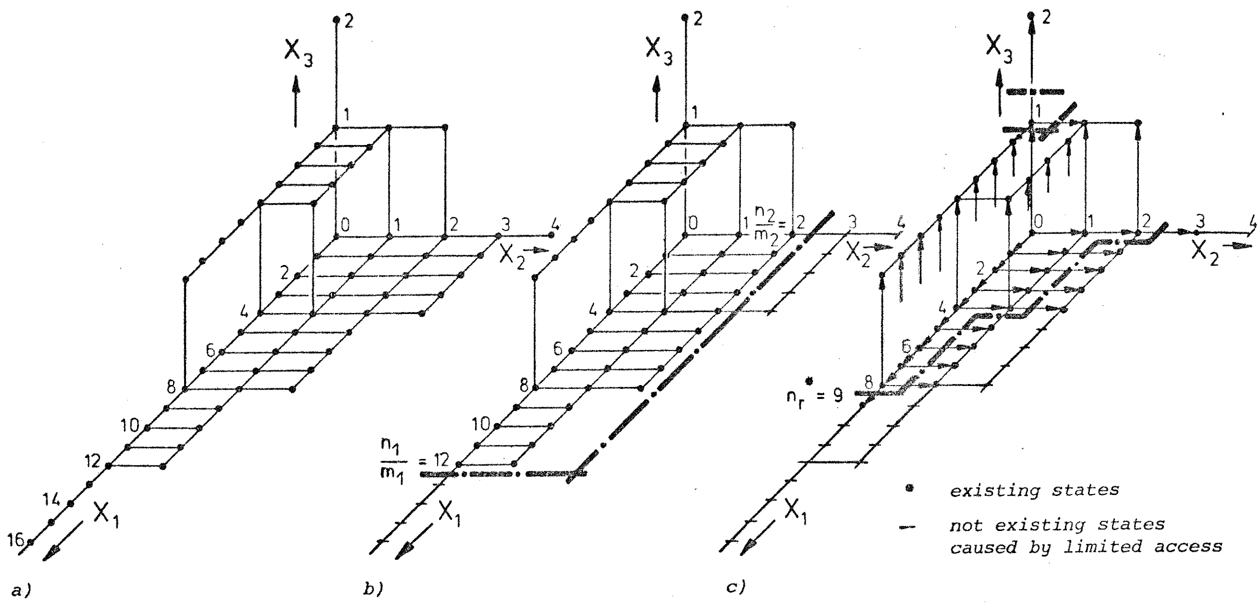

Fig. 3: State spaces in the case of a) full availability b) CLM and c) SLM.

2.2.1. CLASS LIMITATION METHOD (CLM)

Def: An arriving call of bit-rate class r is only accepted, if the number of occupied subchannels $m_r x_r$ of its class is less than an upper limit n_r

$$m_r x_r < n_r \leq n$$

and at least m_r subchannels are idle.

In Fig.3a on the left side the state space (x_1, x_2, x_3) is shown in the case of a multiplexing scheme with 16 subchannels and three bit-rate classes with $m_1 = 1$, $m_2 = 4$, $m_3 = 8$ and full availability. The transitions between states caused by arriving or terminating calls of class r run parallel to the corresponding x_r - axis.
If CLM is applied, e.g. with the upper limits $n_1/m_1 = 12$, and $n_2/m_2 = 2$, we obtain the state space as shown in Fig. 3b. States with $x_1 > 12$ and $x_2 > 2$ do no longer exist. The calculation of the probability of loss B_{nr} can then simply be done by replacing the upper summation indices in eq.(1) and (2) for x_2 and x_1 by the expression

$$\mathrm{Min}\ \{\frac{n-m_3x_3}{m_2}, n_2\} \quad \text{and} \quad \mathrm{Min}\ \{\frac{n-m_3x_3-m_2x_2}{m_1}, n_1\}, \text{resp.}$$

Numerical results for the case with $n_1 = 24$, $n_2 = 32$ and $n_3 = 80$ are shown in Fig. 4a as a function of the total offered traffic A and in Fig. 4b as a function of the offered traffic A_1 with A_2 and A_3 = const. Fig.4a demonstrates that B_{n1} and B_{n2} increase considerably by the limited access, whereas B_3 decreases only insignificantly. Fig. 4b points out that in the case of overload, e.g. caused by bit-rate class 1, its probability of loss B_{n1} increases much stronger than in the case of full access and exceeds the values of B_{n2} and B_{n3}. That means, the traffic causing the overload has the highest probability of loss and the other traffics are protected. The application of CLM does not lead to a significant reduction of the probability of loss B_3, therefore other limitation methods will be investigated:
Let us consider again the example of CLM. The number of available subchannels for class 1 and 2 are limited to 24 and 32,resp. Therefore 80 - 56 = 24 subchannels are always available for class 3. The same number of 24 subchannels remains reserved for class 3, if we allow class 1 and 2 to occupy together up to 56 subchannels (Sum Limitation Method). Each of the 56 subchannels, not used by class 1 can be occupied by class 2 and vice versa. So we can expect smaller probabilities of loss by using SLM.

2.2.2 SUM LIMITATION METHOD (SLM)

Def: An arriving call of bit-rate class r is only accepted, if the total number of occupied subchannels is less than an upper limit n_r^*.

$$x = \sum_i m_i x_i < n_r^* \leq n - m_r + 1$$

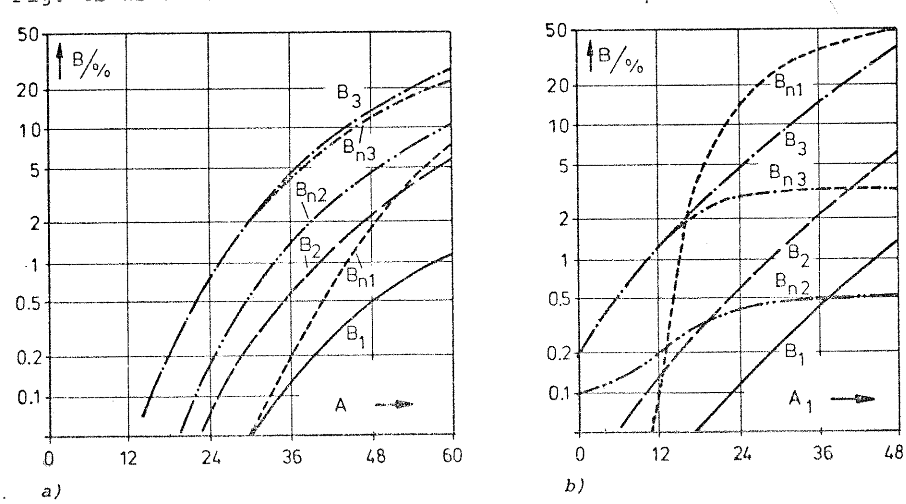

Fig.4 The probabilities of loss B_{nr} for three bit-rate classes in case of CLM as a function of
a) the offered traffic A with $A_1:A_2:A_3$ = 16:4:1 and $n_1 = 24$, $n_2 = 32$, $n_3 = 80$
b) the offered traffic A_1 with $A_2 = 2$, $A_3 = 0.5$ and $n_1 = 24$, $n_2 = 32$, $n_3 = 80$.
For comparison the probabilities of loss B_r in case of full availability are shown, too.

As a call, requiring m_r subchannels, is still accepted if the number of occupied subchannels is $x = n_r^* - 1$, we get the maximum number of available subchannels for class r to $n_r^* - 1 + m_r$. For class R should be chosen

$$n_R^* = n + 1 - m_R \qquad (4)$$

The probabilities of loss B_r can be made identical for all r = 1,2,3..R, if for all classes the same $n_r^* = n_R^*$ is prescribed, then all classes get identical states, which contribute to the probabilities of loss.

Fig. 3c shows the state space for an example with n = 16 subchannels, $m_1 = 1$, $m_2 = 4$, $m_3 = 8$; n_r^* is obtained according to (4). The dashed-dotted line separates the states with $x \geq n_r^*$ from the states with $x < n_r^*$, if n_r^* is chosen to 9. Transitions to higher states caused by the offered traffic A_r are only possible, as long as x is smaller than n_r^*. This leads to irregularities in the set of state equations and eq.(1) holds no longer. As no explicit solution could be found the state probabilities were calculated by solving the set of linear state equations. The number of unknowns z (including those states which will become zero because of the limited access) is given by:

$$z = \sum_{x_3=0}^{\frac{n}{m_3}} \left(1+\frac{n-m_3x_3}{2m_1}\right)\left(1+\frac{n-m_3x_3}{m_2}\right) \qquad (5)$$

(In our interesting case with n = 80, $m_1 = 1$, $m_2 = 4$ and $m_3 = 16$ we obtain z = 1946.
The corresponding set of linear equations is computed in some minutes by using the "Relaxation Method" on a medium speed computer).
The probability of loss B_{nr} is obtained for each class, if all those probabilities of state are summed up for which holds:

$$\sum_i m_i x_i \geq n_r^* \quad \text{or} \quad x_1 \geq \frac{n_r^* - x_2m_2 - x_3m_3}{m_1}$$

Therefore in eq.(2) the lower summation index has to be replaced by :

$$\text{Max}\left\{\frac{n_r^* - m_2x_2 - m_3x_3}{m_1}, 0\right\} \qquad (6)$$

In case of $n_r^* = n_1^* = n_2^* = n_3^* = 65$ numerical results are shown in Fig. 2a - c and compared with those for the case of full access. In Fig.2a the probability of loss $B_{n1} = B_{n2} = B_{n3}$ is drawn as a function of A, in Fig.2b as a function of A_1, with A_2 and A_3 fixed.
If, for example, none of the three probabilities of loss is allowed to exceed a value of 1%, we can increase the offered traffic A of about 7% or the offered traffic A_1 of about 20% by applying SLM. If desired, B_3 can be chosen smaller than B_1 and B_2 by choosing n_1^* and n_2^* smaller than n_3^*, e.g. $n_1^* = n_2^* = 61$ and $n_3^* = 65$. The probablities of loss in this case are depicted in Fig.2c.

3. CALCULATION OF THE PROBABILITY OF LOSS IN THE CASE OF REGULAR SUBCHANNEL ALLOCATION.

3.1. FULL ACCESS TO THE SUBCHANNELS BY ALL BIT - RATE CLASSES.

Data channels with higher bit rates, which need more than one subchannel, require now an equally spaced sequence of m_r subchannels within the superframe, e.g. the sequence 2, 7, 12...77 for a 9.6 kbps connection, see Fig.5. Five data channels with 0.6 kbps which occupy the subchannels No.1 to 5 would block all 9.6 kbps data channels. To avoid unnecessary blocking, special hunting strategies in case of regular channel allocation have to be applied. For this purpose the subchannels will be rearranged in Fig.5. Each 9.6 kbps channel is divided up into four 2.4 kbps channels with four equally spaced subchannels each, e.g. 7, 27, 47, 67. Each 2.4 kbps data channel comprises four 0.6 kbps data channels. If this rearranged sequence of subchannels is hunted from "left to right", it will be defined as sequential hunting. This hunting mode guarantees that an arriving low speed call does not occupy one of the last data channels, with regard to hunting direction, as long as it can be switched via one of the first data channels. Thus the last data channels are kept free for high speed data calls. On the other hand, as Fig.6 shows, an arriving call of class 1 may occupy a subchannel of an idle data channel of class 2 although there exists another class 2 data channel where already three subchannels are occupied. Therefore a second hunting mode will be investigated:

If a call of class r arrives, hunting of subchannels is done in two steps:
- to look for those data channels of class r + 1 which have at least m_r idle subchannels and momentarily the maximum number of occupied subchannels.
- to select one of these data channels of class r + 1 and to occupy m_r of its idle subchannels.

A call of bit-rate class 0.6 kbps will look at first for a 2.4 kbps data channel with three occupied subchannels, if it finds none, it looks for one with two occupied subchannels and so on (Fig.6). This hunting mode will be defined as " gap hunting ".

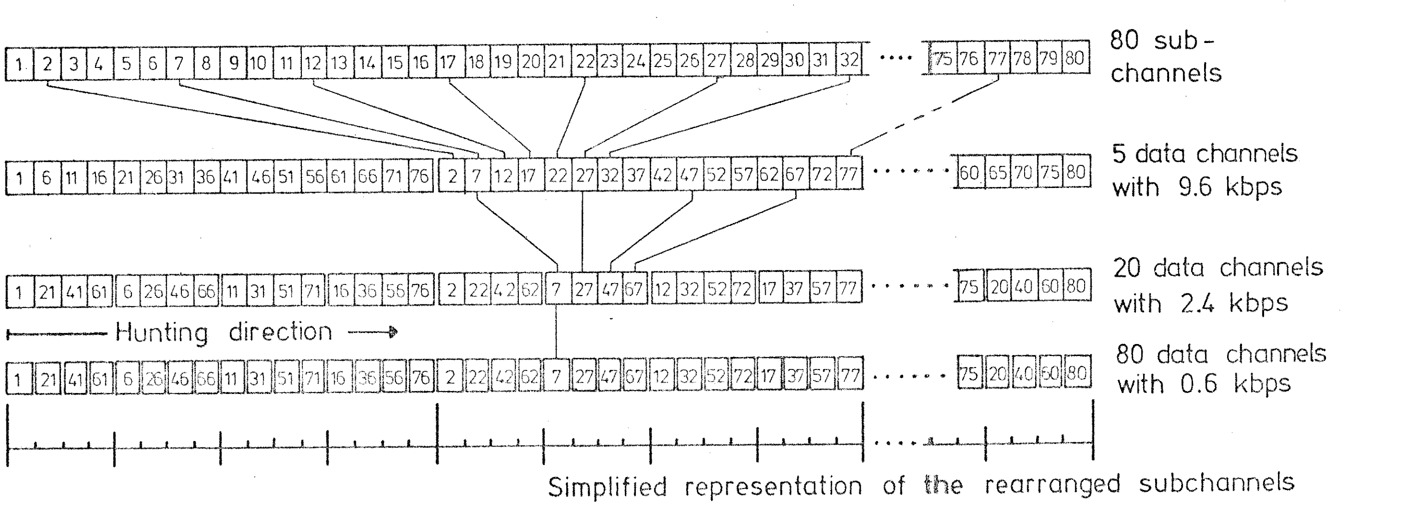

Fig. 5: Allocation of subchannels to data connections with different bit-rates (regular subchannel allocation)

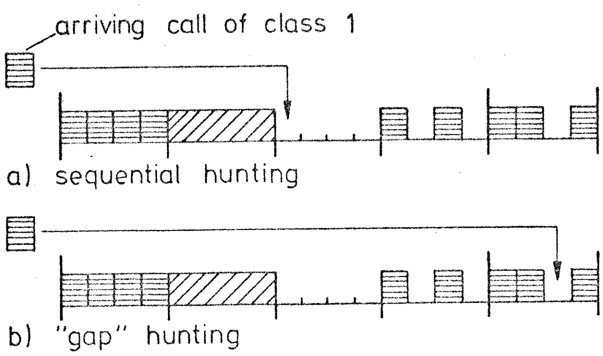

Fig. 6: Hunting strategies for calls of bit-rate class 1

3.1.1. SEQUENTIAL HUNTING

As the probabilities of loss of the above mentionend hunting modes will be compared and probably only small differences will be obtained, an exact solution is presented. This is possible, however, only for a small number of subchannels with two bit-rate classes with e.g. $m_1 = 1$ and $m_2 = 4$. As within one data channel of class 2 four calls of class 1 can be established, the state of this data channel can be defined by a state variable x having the values o,1,2,3,4,5, with the following meaning:

- 0 : the channel is free
- 1,2,3,4 : the channel is occupied by 1,2,3 or 4 calls of class 1
- 5 : the channel is occupied by one call of class 2

A data link with s data channels of class 2 is therefore characterized by a state space with 6^s states. To obtain the state probabilities, a set of linear equations with $z = 6^s$ unknowns has to be solved. For this reason the exact calculation of the probabilities of loss is only possible in practice up to s = 5 (z = 7776). However, the problem with n = 8o and only the two bit-rate classes 2.4 kbps and 9.6 kbps can be calculated exactly, as it can be reduced to the problem with n = 2o, $m_1 = 1$, $m_2 = 4$, which leads to the above mentioned problem with s = 5. Results of the exact calculation are given in Fig.2a-c (c.f. section 3.1.2.).

3.1.2. GAP HUNTING

The exact calculation of the probability of loss for this hunting mode can again only be done by solving the set of linear state equations. Although the number of relevant states can be reduced in the case of gap hunting, the interesting problem with n = 8o, $m_1 = 1$, $m_2 = 4$, $m_3 = 16$ cannot be calculated exactly in practice, too.

Numerical results are shown in Fig.7a-c.Fig.7a,b show an example for the probability of loss with n = 16, $m_1 = 1$, $m_2 = 4$ as a function of the total offered traffic A and as a function of the offered traffic A_1, resp. Fig.7c shows the example with n = 2o, $m_1 = 1$, $m_2 = 4$ (the problem if only 2.4 kbps and 9.6 kbps data connections have to be switched via 8o subchannels).

The results are drawn for both hunting modes, sequential and gap hunting. One can see that the probability of loss B_2 is reduced and B_1 is increased by using gap hunting.

As B_2 is higher than B_1 (more than one decade) the absolute increase of the overflowing traffic of class 1 can be neglected compared with the decrease of class 2. Fig. 7a,b show that especially for small offered traffics, B_2 is decreased by gap hunting without significant increase of B_1. An interesting fact of Fig.7b is the minimum of the probability of loss B_1. If A_1 tends to zero, B_1 will be equal to B_2, as calls of class 1 and class 2 will be lost, if all subchannels are occupied by calls of class 2. If A_1 increases now, an established call of class 1 blocks three further subchannels. These three subchannels remain reserved for class 1, and this causes the reduction of the probability of loss B_2.

3.1.3. APPROXIMATE CALCULATION OF THE PROBABILITY OF LOSS IN CASE OF SEQUENTIAL HUNTING .

As already mentioned, an exact calculation of the probablities of loss is only possible for data links with a comparatively small number of subchannels. So an approximate calculation method was developped, which is presented in the Appendix. Fig.8a-c show the probabilities of loss B_1, B_2 and B_3 as a function of the total offered traffic A (Fig.8a), as a function of the offered traffic A_1 (Fig.8b) and as a function of the offered traffic A_3 (Fig.8c). In addition, simulation results with 95% confidence intervals are drawn, which show in all three figures a good accordance with the approximate calculations. It is demonstrated that regular subchannel allocation leads to differences in the probabilities of loss which exceed the range of more than two decades.

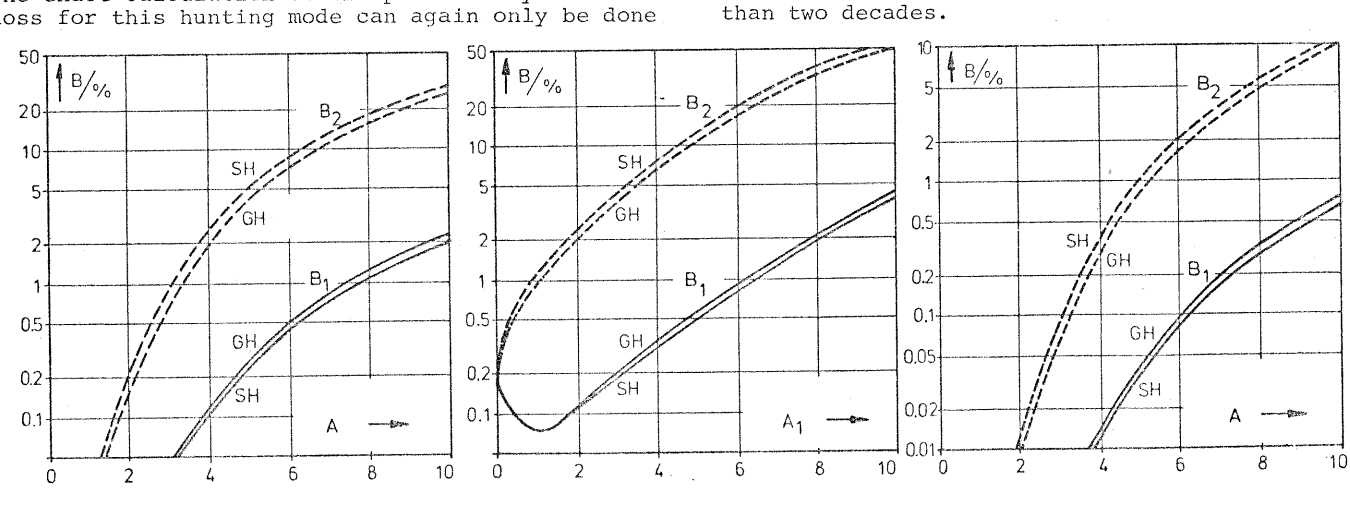

Fig. 7: The probabilities of loss B_r for two bit-rate classes in the case of gap (GH) and sequential hunting (SH) as a function of
a) offered traffic A with $A_1:A_2 = 4:1$ and with $n = 16$, $m_1 = 1$, $m_2 = 4$
b) offered traffic A_1 with $A_2 = 0.5$ and with $n = 16$, $m_1 = 1$, $m_2 = 4$
c) offered traffic A with $A_1:A_2 = 4:1$ and with $n = 20$, $m_1 = 1$, $m_2 = 4$.

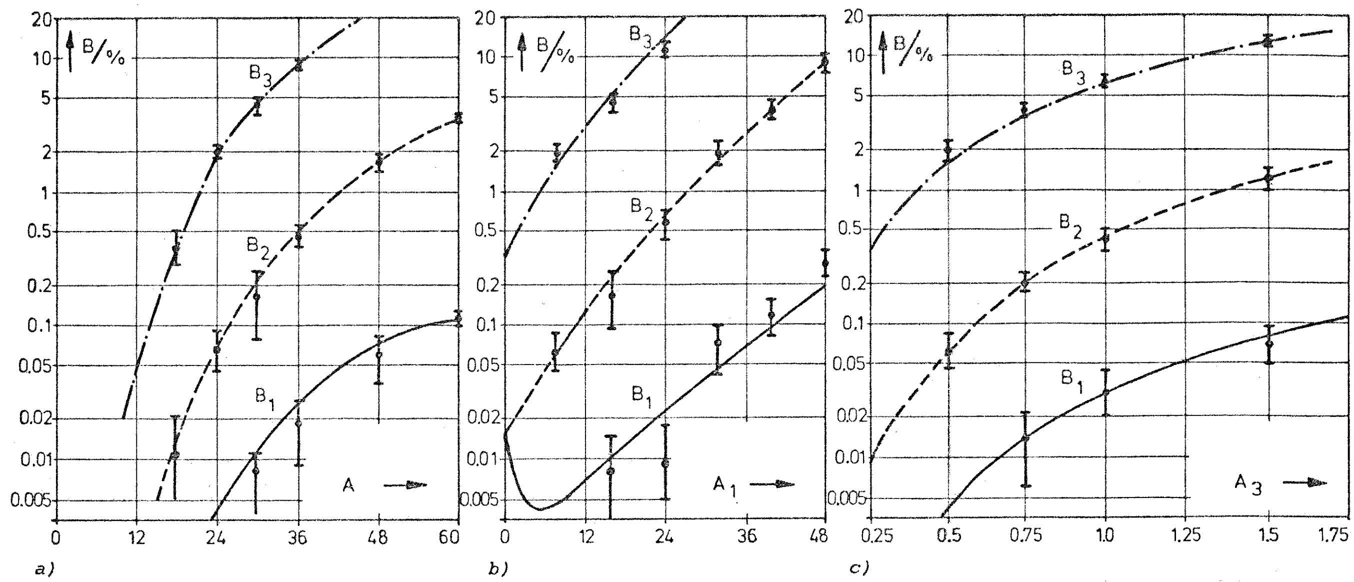

Fig. 8: Approximately calculated probabilities of loss B_r for three bit-rate classes as a function of
a) the offered traffic A with $A_1:A_2:A_3$ = 16:4:1 and n = 80
b) the offered traffic A_1 with A_2 = 2, A_3 = 0.5 and n = 80
c) the offered traffic A_3 with A_1 = 8, A_2 = 2 and n = 80
Additionally simulation results with 95% confidence interval are drawn.

3.2. LIMITED ACCESS TO SUBCHANNELS FOR LOWER BIT - RATE CLASSES IN CASE OF SEQUENTIAL HUNTING .

It will be investigated now, by the aid of traffic simulation, whether the different probabilities of loss can be equalized by limiting the access to the subchannels for the lower classes. The limitation of access to only n_r subchannels means that beginning from a fixed starting point, only n_r subchannels can be sequentially hunted and occupied (Hunting Limitation, see Fig.9).

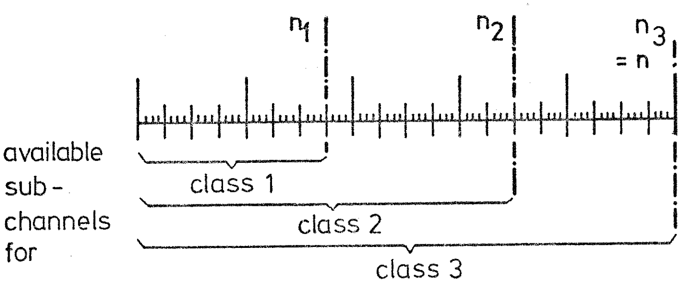

Fig. 9: Limited access to subchannels for the different bit-rate classes

In Fig.1oa an example is shown with n_1 = 38 and n_2 = 52 available subchannels for calls of class 1 and class 2, resp. The probabilities of loss of all three bit-rate classes are equalized to about 1%, when a total offered traffic of 24 Erl. is offered. The probability of loss B_3 is reduced from about 2% to 1.25%, whereas B_1 and B_2 are increased. This loss reduction of B_3 is obtained on the whole range of the offered traffic A; it is even possible to obtain a probability of loss B_3 smaller than B_1 or B_2 (see Fig.1oa for A = 2o Erl.).
Fig. 1ob shows the probabilities of loss again with n_1 = 28 and n_2 = 52 as a function of the offered traffic A_2. The probability of loss B_2 exceeds B_1 and B_3 if the offered traffic A_2 increases considerably (overload caused by traffic A_2). The traffic A_2 which causes the overload, and traffic A_1, suffer a probability of loss, amounting to 1oo% with increasing A_2.
The traffic A_3, however, tends to a probability of loss $E_{1,1}(A_3)$ = 33%, as at least one data channel for class 3 is always available.

Fig.1oc shows the same example, except n_2 = 52 is changed now into n_2 = 48. Then 8o - 48 = 32 subchannels remain available for class 3 only, i.e. two data channels. With increasing A_2 the probability of loss B_3 tends to $E_{1,2}(A_3)$ = 7.7 %. The choice of n_2 = 52 therefore is not favourable regarding overload.

4. CONCLUSION

The following table gives a survey of the investigated subchannel allocation principles and hunting strategies:

channel allocation principle	arbitrary subchannel allocation	regular subchannel allocation
hunting strategy	not necessary	- sequential hunting - gap hunting
equalization means for the prob. of loss	- class limitation - sum limitation	hunting limitation

Summarizing the paper, the subchannel allocation principles will be compared with the case of permanent subchannel allocation, to show the efficiency of commonly hunted subchannels.

In the following table the probabilities of loss B_1, B_2, B_3 in case of A_1 = 8, A_2 = 2 and A_3 = o.5 and n = 8o subchannels are shown as an example.

B_r	permanent	regular	arbitrary
r=1	$1.6\cdot 10^{-4}$	$3.0\cdot 10^{-5}$	$1.5\cdot 10^{-4}$
r=2	$3.4\cdot 10^{-3}$	$6.5\cdot 10^{-4}$	$7.4\cdot 10^{-4}$
r=3	$7.6\cdot 10^{-2}$	$1.9\cdot 10^{-2}$	$7.2\cdot 10^{-3}$

In case of permanent subchannel allocation the 8o subchannels have been allocated to the three bit-rate classes in such a way that the probabilities of loss have similar differences among each other as in the case of arbitrary or regular channel allocation.

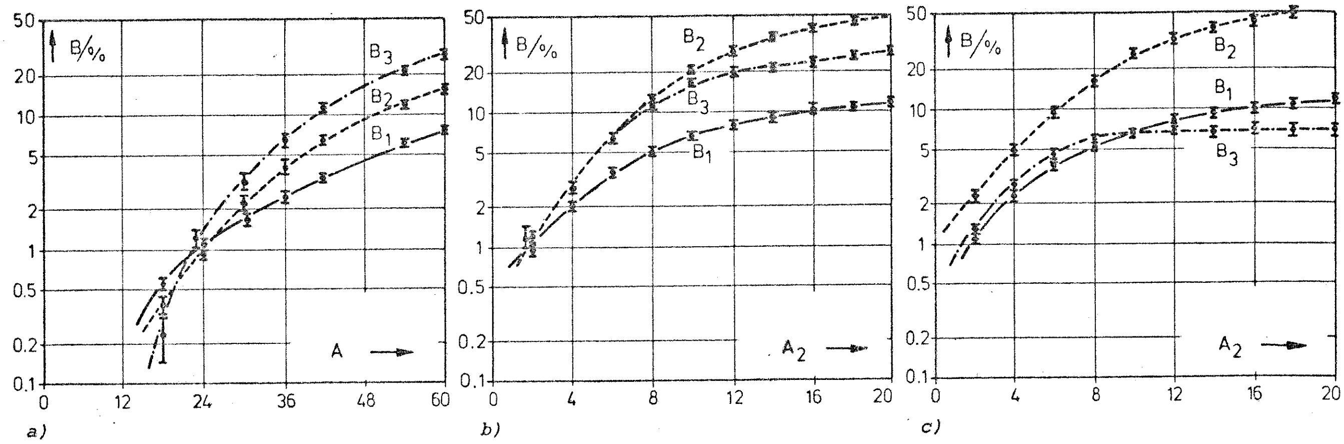

Fig. 10: Simulation results for the probabilities of loss B_{nr} for three bit-rate classes in case of limited availability as a function of

a) the offered traffic A with $A_1:A_2:A_3 = 16:4:1$ and $n_1 = 28$, $n_2 = 52$, $n_3 = n = 80$

b) the offered traffic A_2 with $A_1 = 8$, $A_3 = .5$ and $n_1 = 28$, $n_2 = 52$, $n_3 = n = 80$

c) the offered traffic A_2 with $A_1 = 8$, $A_3 = .5$ and $n_1 = 28$, $n_2 = 48$, $n_3 = n = 80$.

One can see that permanent channel allocation leads to considerably higher probablities of loss than arbitrary or regular channel allocation. The disadvantage of permanent channel allocation can be further elucidated by the following example: The three probabilities of loss B_r in the case of arbitrary subchannel allocation for $A_1 = 8$, $A_2 = 2$ and $A_3 = 0.5$ are given. Now the number of data channels for each bit-rate class is chosen such that the three probabilities of loss have (approximately) the same values for permanent and arbitrary subchannel allocation. As the number of data channels is integer, that value $E_{1,nr}(A_r) \geq B_r$ is considered, which is nearest by the given value B_r.

B_r	arbitrary	$E_{1,nr}(A_r)$	n_r	$m_r n_r$
r=1	$1.5\ 10^{-4}$	$1.6\ 10^{-4}$	20	20
r=2	$7.4\ 10^{-4}$	$8.6\ 10^{-4}$	8	32
r=3	$7.2\ 10^{-3}$	$12.7\ 10^{-3}$	3	48
			n	= 100

The example shows that 100 subchannels instead of 80 are required to obtain similar probabilities of loss in the case of permanent channel allocation.

Furthermore, arbitrary channel allocation always leads to smaller probabilities of loss B_3 than regular subchannel allocation. As this allocation principle requires additional buffer memories, its application depends on the ratio of the costs of buffer memories to the costs of the subchannels.

4. APPENDIX : APPROXIMATE CALCULATION OF THE PROBABILITIES OF LOSS IN THE CASE OF SEQUENTIAL HUNTING .

For practical applications the probabilities of loss should not exceed some percent. The offered traffics have to be chosen such that the bit-rate class with the highest transmission speed should obtain a probability of loss in this range. Then the probabilities of loss for the lower classes will be far below the value of 1%. The carried traffic of the lower classes therefore is only insignificantly influenced by the traffics of the higher classes.
An approximate calculation method for the three bit-rate classes will be shown for the interesting case of $m_1 = 1$, $m_2 = 4$, $m_3 = 16$ and $n = 80$.

Before presenting the detailed way of solution a brief survey is given.

1. In the first step the carried traffics A_1 and A_2 are assumed not to be influenced by the offered traffic A_3. With this assumption the probability is calculated that traffic A_1 and A_2 together block 1,2..5 data channels of class 3. From this follows the probability w_{k3} that k_3 data channels remain available for class 3.
2. To obtain the probability of loss B_3, the probability of loss $E_{1,k3}(A_3)$ for a full available trunkgroup having k_3 data channels is weighted with the probability w_{k3} and summed up.
3. Analogously to the calculation of the probability of loss B_3 we find the probability $p(x_3)$ that x_3 data channels are occupied by class 3 by summing up the weighted probabilities of state for a full available trunkgroup having k_3 data channels.
4. If traffic A_3 occupies x_3 data channels, $(80 - m_3x_3)/m_2$ data channels remain available for class 1 and 2 with the probability $p(x_3)$. To obtain the probabilitiy of loss B_2, the probability of loss $E_{1,k2}(A_2)$ is weighted with $p(x_3)$ and summed up. k_2 is the number of data channels only available for class 2 and has the value $k_2 = \text{entier}((80 - m_3x_3 - b_1)/m_2)$ where b_1 is the average number of blocked subchannels by class 1.
5. In a similar way as shown in point 3 we get the probability of state $p(x_2,x_3)$ that x_2 and x_3 data channels are occupied by class 2 and 3, resp.
6. To obtain finally the probability of loss B_1 the probability of loss $E_{1,k1}(A_1)$ is weighted with $p(x_2,x_3)$ and summed up for each possible set (x_2,x_3), with $k_1 = 80 - m_3x_3 - m_2x_2$.

In the following this way of solution is shown in detail:

1. We consider $n/m_2 = 20$ data channels with the offered traffic

$$A_2^* = A_2 + A_1/m_2 = A_2 + A_1/4$$

The carried traffic of the i - th data channel is given by

$$y_{2,i} = A_2^* \cdot (E_{1,i-1}(A_2^*) - E_{1,i}(A_2^*)) \qquad (7)$$

where

$$i = 1,2...20$$

and

$$E_{1,i}(A) = \frac{A^i/i!}{\sum_{j=0}^{i} A^j/j!} \qquad (8)$$

The probability to find the i - th data channel occupied by calls of class 1 and 2 is therefore equal to $y_{2,i}$.
A data channel of class 3 is idle, if its four equivalent data channels (see Fig.5) of class 2 are idle. The probability v_i of the i - th channel of class 3 to be occupied by classes 1 and 2 will be approximately defined as

$$v_i = \prod_{j=4(i-1)+1}^{4i} y^*_{2,j} \qquad (9)$$

Thus we can calculate the probability w_{k3} that k_3 = o,1,2..5 channels for class 3 are available:

$$w_5 = \prod_{j=1}^{5} (1 - v_j)$$
$$w_4 = \sum_{l=1}^{5} v_l \prod_{\substack{j=1 \\ j \neq l}}^{5} (1 - v_j) \qquad (1o)$$
$$\vdots$$
$$w_o = \prod_{j=1}^{5} v_j$$

2. To calculate the probability of loss B_3 we assume that an incoming call of class 3 finds with the probability w_{k3} a trunkgroup with k_3 available data channels. Then B_3 is obtained by summing up the weighted probabilties of loss $E_{1,k3}(A_3)$

$$B_3 = \sum_{k3=o}^{5} w_{k3} E_{1,k3}(A_3) \qquad (11)$$

3. We find the probability of state $p(x_3)$ that x_3 data channels are occupied by class 3 and not available for classes 1 and 2

$$p(x_3) = \sum_{k3=x3}^{5} w_{k3} p_{k3}(x_3) \qquad (12)$$

with

$$p_{k3}(A_3) = \frac{A^{x3}/x_3!}{\sum_{j=o}^{k3} A_3^j/j!}$$

(state probability of a full available trunkgroup with offered traffic A_3 and k_3 trunks)

4. In the further steps the mutual influence of calls of class 3 to calls of classes 1 and 2 will be taken into account.
Regarding class 2 we obtain the probability of loss B_2 by assuming that with the probability $p(x_3)$ a number of 8o - m_3x_3 subchannels are available for the classes 1 and 2. The influence of class 1 will now be globally taken into account by reducing 8o - m_3x_3 by the number b_1 of blocked subchannels acc. to eq.(17).
Therefore the number k_2 of available channels for class 2 will be

$$k_2 = \text{entier}((8o - b_1 - m_3x_3)/m_2) \qquad (13)$$

As k_2 is rounded down to the next unit, the following calcualtions have to be done twice, for k_2 and $k_2 + 1$. The approximate probability of loss is determined afterwards by linear interpolation.
The probability of loss B_2 is obtained by summing up the probability $E_{1,k2}(A_2)$ that is weighted with $p(x_3)$ for all values of x_3.

$$B_2 = \sum_{x3=o}^{5} p(x_3)\ E_{1,k2}(A_2) \qquad (14)$$

Calculation of the average number b_1 of blocked subchannels by class 1:

Calls of class 1 occupy on the average $Y_1 = (1 - B_1)$ subchannels. But the calls do not occupy subchannles according to Fig.5, from left to right without any gaps (these are caused by terminating calls). So, on the average, more subchannels are blocked for calls of classes 2 and 3 than the value Y_1/m_2 or Y_1/m_3, resp. indicates. This average number of b_1 blocked subchannels can be calcualted approximately as follows:
The carried traffic of class 1 in the i - th subchannel is

$$y_{1,i} = A_1(E_{1,i-1}(A_1) - E_{1,i}(A_1)) \qquad (15)$$

The probability to find the i - th subchannel busy is therefore equal to $y_{1,i}$. The probability $q_{1,i}$ that subchannel i is occupied, but subchannels i+1, i+2..n are idle is given approximately by

$$q_{1,i} = y_{1,i} \prod_{j=i+1}^{n} (1 - y_{1,j}) \qquad (16)$$

The last call of class 1 with regard to the hunting direction will be found on the average on subchannel number b_1.
We obtain by summing up eq.(16) from 1 to 8o

$$b_1 = \sum_{i=o}^{8o} i \cdot q_{1,i} \qquad (17)$$

5. The probability of state $p(x_2,x_3)$ that x_2 data channels of class 2 and x_3 data channels of class 3 are occupied is given by

$$p(x_2,x_3) = p_{k2}(x_2) \cdot p(x_3) \qquad (18)$$

with

$$p_{k2}(x_2) = \frac{A_2^{x2}/x_2!}{\sum_{j=o}^{k2} A_2^j/j!}$$

$p_{k2}(k_2)$ is the conditional probability that k_2 subchannels are occupied by class 2 if a trunkgroup of k_2 = entier((8o-b_1-m_3k_3)/m_2) data channels remains available for class 2.

6. The probability of loss B_1 is obtained finally

$$B_1 = \sum_{x3=o}^{5} \sum_{x2=o}^{k2} p(x_2,x_3)\ E_{1,k1}(A_1) \qquad (19)$$

with $k_1 = 8o - m_3x_3 - m_2x_2$

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REFERENCES

/1/ CCITT- Study group VII: Contribution Nr.271, Recommendation X.5o, Fundamental Parameters of a multiplexing scheme for the international interface between synchronous data networks.

/2/ Gallenkamp,W., Kern,K.: Transmission system for the new integrated telegraph and data network of the Deutsche Bundespost. World Telecomm. Forum, 6.-8.1o.75, paper 2.3.6.1/7.

/3/ Enomoto,O., Miyamoto,H.: An analysis of mixtures of multiple bandwidth traffic on time division switching networks. Prebook, 7th ITC Stockholm, 1973, 635/1 - 635/8.

/4/ Frederikson,G.: Analysis of channel utilisation in traffic concentrators. IEEE trans.com. 22(1974) 8, 1122 - 1129.