ANALYSIS OF LINK SYSTEMS WITH DELAY

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ABSTRACT

Multi-stage connecting arrays with conjugated selection (link systems) are used, among others, in modern telephone exchanges for traffic concentration. An approximate grade-of-service calculation method is presented for such link systems having an arbitrary number of stages and unlimited queuing (delay systems) or limited queuing (combined delay-loss-systems). The calls of a finite or infinite number of sources are operated in point-to-group selection mode (one outgoing group only). Holding times are distributed negative exponentially or constant. For delay systems the distr. function of waiting times is derived. The calculation results are checked by simulation.

1. INTRODUCTION

In many switching exchanges the information flow to centralized control devices is switched via special link systems. The accessibility to the outlets of these link systems is full or limited. The same holds for link systems connecting and concentrating the traffic from subscriber lines to the inlets of traffic distribution link systems. These subscriber link systems are often operated as delay systems despite the fact that the approximate dimensioning is done as if they were operated as loss systems.

As to single-stage arrays, reliable methods have been developed for the calculation of the grade-of-service in case of systems without queuing (loss systems, /13/) or systems with queuing (delay systems, combined delay-loss-systems, /6,8,9/). Formulae for link systems without queuing are also available /1,2,10,11, 12/, but there is little known about link systems with queuing up to now /4,5/.

In Chapter 2 a detailed description of the investigated link systems is given. In Chapter 3 the basic ideas of the approximate calculation method are outlined. Chapter 4 shows comparisons of calculated results with simulations.

2. LINK SYSTEM PARAMETERS

2.1 Structure: Fig.1 shows the structural parameters of a traffic concentrating link system with queuing. The link system consists of $S \geqslant 2$ stages. In a considered stage No.j all g_j multiples have i_j inlets and k_j outlets. In front of each first-stage multiple an unlimited (delay system) or limited (combined delay-loss-system) number of waiting places s is provided. The outlets of a multiple are wired to the multiples of the succeeding stage in a sequential or cyclic way /1,2/. As a rule, link systems are subdivided into modules (link blocks, cf.Fig.1). The n outlets of the last stage belong to one outgoing group.

2.2 Operating mode: Whenever a call arrives at an inlet of a first-stage multiple (start-multiple at the expanded side of the link system), the link system control tries to find an idle path from this fixed start-multiple to an arbitrary idle outlet of the link system, using the point-to-group selection mode. The outlets of a multiple are hunted sequentially from home position. If an arriving call cannot be served immediately, this blocked call occupies an idle waiting place in front of its start-multiple. In case of limited queuing a blocked call is lost if the available waiting places are fully occupied.

Waiting calls are served acc.to the interqueue discipline RANDOM (selection of one out of the g_1 queues, cf.Fig.1), and within each queue acc. to the queue discipline FIFO. A waiting call occupies both a waiting place and an inlet of its start-multiple. During the service time, a call occupies a start-multiple inlet, corresponding links and one outlet of the link system, but no waiting place.

2.3 Arrival and service processes: Two types of arrival processes are considered.

- POISSON input: An infinite number q of sources generates the offered traffic A/g_1 (mean) at each of the g_1 first-stage multiples. The total call rate λ is constant and independent of the number of busy sources. The interarrival times are distributed negative exponentially.
- BERNOULLI input: A finite number q of sources generates the offered traffic at each of the g_1 first-stage multiples. Each idle source has a constant call rate α. The idle times of an individual source are distributed negative exponentially.

The independent sources start their calls at random. For all link systems studied in this paper holds the model

$$q = i_1 \quad (1)$$

The service process is characterized by negative exponentially distributed holding times (mean h) or constant holding times h, resp.

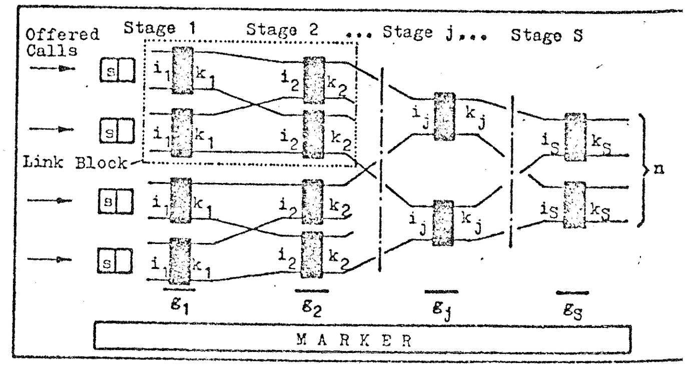

Fig.1: General structure of an S-stage link system for traffic concentration with queuing.

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2.4 Characteristic traffic parameters: In link systems with queuing, the grade-of-service is characterized by the offered traffic (A), carried traffic (Y), prob.of delay (W), prob.of loss (B), prob.of call congestion (C), mean queue length (Ω), mean waiting time of offered calls (w) or of waiting calls (t_W), and the distr.function of waiting times of offered calls $W(>t)$. It holds

$$C = W + B \qquad (2)$$

3. APPROXIMATE CALCULATION

3.1 General: In this Chapter the new approximate calculation method is presented. The basic idea (introduction of an effective accessibility k_{eff}) is well known from its successful application to link systems without queuing /1,2/ and from an earlier approach for link systems with delay /5/. Here for the first time the CLIGS-A formula of k_{eff} /1,2/ is applied to link systems for traffic concentration and queuing, with

- finite or infinite number of sources q
- limited or unlim. number of waiting places s per first-stage multiple /7/.

The approximate calculation method is based on the following considerations: Upon its arrival at a first-stage multiple (start-multiple) a call is blocked

- in the first stage, if all k_1 outlets of the call's start-multiple are busy;
- in an intermediate stage No.j $(1<j<S)$, if there exists at least one idle path from the arriving call's start-multiple to at least one multiple of stage No.j, WHEREAS the call cannot find an idle link from this stage No.j to stage No.j+1;
- in the last stage, if there exists at least one idle path from the arriving call's start-multiple to at least one multiple of this last stage, WHEREAS the call cannot find an idle trunk of the outgoing group.

Call congestion in any stage No.j $(1 \leq j \leq S)$ occurs only, if

$$i_j > k_j \qquad (3)$$

i.e. if the traffic is concentrated in that stage.

Call congestion is assumed to occur statistically independently in any of the link system stages No.j. First, the calculation of the congestion probability is done separately with regard to each link system stage No.j by means of a corresponding "subsystem" T_j. From these subsystems results then the link system results are composed. In Sections 3.2 to 3.5 this method is explained in detail.

3.2 STEP 1: Decomposition. The subsystems $T_1, T_2 .. T_S$ are part of the original S-stage link system. To get the subsystem structure, a connection graph is drawn from a first-stage multiple (start-multiple) to the link system outlets. This graph contains those multiples, links and outlets of the link system, to which a call might have access, when arriving at the considered start-multiple.

In stage No.j the connection graph consists of $\bar{g}_j$ multiples with a total of outlets

$$\bar{n}_j = \bar{g}_j \cdot k_j \qquad (4)$$

In the small example of Fig.2 the connection graph is marked by heavy lines:

Stage 1: $\bar{g}_1 = 1$ (multiple 1a) , $\bar{n}_1 = 4$

Stage 2: $\bar{g}_2 = 2$ (mult.2a,2b) , $\bar{n}_2 = 6$

Stage 3: $\bar{g}_3 = 3$ (mult.3a,3b,3c), $\bar{n}_3 = 12$

Stage 4: $\bar{g}_4 = 2$ (mult.4a,4b) , $\bar{n}_4 = 10$

By means of the connection graph the structures of the subsystems $T_1, T_2 .. T_S$ are evaluated from the link system structure:

Subsystem T_1 has a single-stage structure and is identical to the start-multiple of the connection graph (mult.1a in Fig.2).

Subsystems $T_2, T_3 .. T_S$ have a multi-stage structure. Subsystem T_j consists of j stages with $g_{v,Tj}$ multiples in stage No.v. The last stage of subsystem T_j consists of the $\bar{g}_j$ connection graph multiples in stage No.j:

$$g_{j,Tj} = \bar{g}_j \qquad (5)$$

Thus, subsystem T_j has a total of $\bar{n}_j$ outlets, cf.Eq.(4). The preceeding stages j-1, j-2, .., 1 of subsystem T_j consist of those link system multiples, which carry traffic to the $\bar{n}_j$ outlets of subsystem T_j.

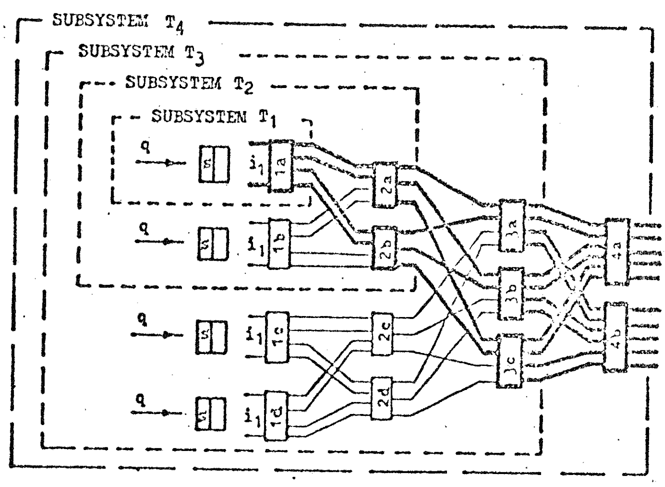


Fig.2: Example of STEP 1 (Decomposition into subsystems). ▬▬ Connection graph

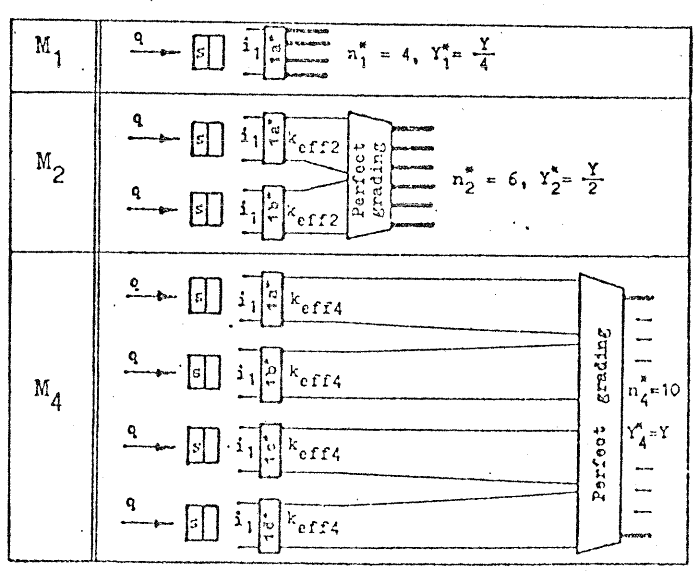


Fig.3: Example of STEP 2 (Mapping each subsystem on an equivalent single-stage system).

For the link system in Fig.2 holds

Subsystem T_1 (single-stage structure):
$g_{1,T1} = \bar{g}_1 = 1$ (multiple 1a)

Subsystem T_2 (two-stage structure):
$g_{1,T2} = 2$ (m.1a, 1b), $g_{2,T2} = \bar{g}_2 = 2$ (m.2a, 2b)

Subsystem T_3 (three-stage structure):
$g_{1,T3} = 4$ (m.1a,1b,1c,1d), $g_{2,T3} = 4$ (m.2a,2b,2c,2d)
$g_{3,T3} = \bar{g}_3 = 3$ (m.3a,3b,3c)

Subsystem T_4 (four-stage structure):
$g_{1,T4} = 4$ (m.1a,1b,1c,1d), $g_{2,T4} = 4$ (m.2a,2b,2c,2d)
$g_{3,T4} = 3$ (m.3a,3b,3c), $g_{4,T4} = \bar{g}_4 = 2$ (m.4a, 4b)

The carried traffic Y_{Tj}, which flows through the subsystem T_j, is calculated as a part of the total carried traffic Y flowing through the link system:

$$Y_{Tj} = Y \cdot g_{j,Tj} / g_j \qquad (6)$$

In Eq.(6) a balanced traffic distribution is assumed among the multiples in each link system stage.

3.3 STEP 2 : Mapping. The equivalence of the single-stage system M_j with respect to the corresponding mapped subsystem T_j is such that both are characterized approximately by the same value for congestion probability C_j at a prescribed carried traffic on their outlets. If in a considered stage No.j the multiples do not concentrate the traffic, no call congestion can occur in that stage, cf.Eq.(3) :

$$C_j = 0 \text{ , if } i_j \leq k_j \qquad (7)$$

In this case the corresponding equivalent single-stage system M_j is out of interest.

Subsystem T_1 always has a single-stage structure (one multiple only); therefore the single-stage system M_1 is identical to subsystem T_1. Regarding the multi-stage subsystems $T_2..T_S$, the following mapping rules are observed:

- systems M_j and T_j have the same number of outlets
$$n_j^* = \bar{n}_j \qquad (8)$$
- These n_j^* outlets of single-stage system M_j carry the same amount of traffic as in subsystem T_j
$$Y_j^* = Y_{Tj} \qquad (9)$$
- The equivalent single-stage system M_j consists of g_j^* multiples which equals the number of multiples in the first stage of the corresponding subsystem T_j
$$g_j^* = g_{1,Tj} \qquad (10)$$
- Each of these g_j^* multiples has k_{effj} outlets. The effective accessibility k_{effj} is derived from structure and carried traffic of subsystem T_j acc. to the method CLIGS-A as explained in /1,2/, cf.also Annex of this paper.
- The k_{effj} outlets of the g_j^* multiples are assumed to be graded by a high-efficiency grading ("Perfect grading") to the n_j^* outlets of system M_j.

If these rules are applied to the four-stage link system in Fig.2, then n_j^*, Y_j^* and g_j^* are obtained acc. to Fig.4. In Fig.3 the corresponding single-stage systems M_1, M_2 and M_4 are shown. Single-stage system M_3 is out of interest, as $i_3 = k_3$, cf.Eq.(7).

Single-stage system	Number of outlets Eq.(8)	Carried traffic Eq.(9)	Number of multiples Eq.(10)
M_1	$n_1^* = 4\ (= k_1)$	$Y_1^* = Y \cdot \frac{1}{4}$	$g_1^* = g_{1,T1} = 1$
M_2	$n_2^* = 6$	$Y_2^* = Y \cdot \frac{1}{2}$	$g_2^* = g_{1,T2} = 2$
M_3	No mapping, as $i_3 = k_3 = 4$, cf.Eq.(7)		
M_4	$n_4^* = 10\ (= n)$	$Y_4^* = Y \cdot 1$	$g_4^* = g_{1,T4} = 4$

Fig.4: Mapping rules applied to the four-stage link system in Fig.4 .

3.4 STEP 3 : Calculation of single-stage systems. Acc.to Sect.3.3 the structure and the carried traffic of each equivalent single-stage system M_j ($1 \leq j \leq S$) can be determined. From the original link system further parameters of the single-stage systems are known:

q...number of sources per multiple
s...number of waiting places in front of each multiple
i_1..number of inlets per multiple ($i_1 = q$).

Thus the grade-of-service can be calculated for each single-stage system M_j, based on the prescribed carried traffic Y_j^*. There exist well known calculation methods /6,9/ for

W_j...probability of delay

B_j...probability of loss

C_j...prob.of call congestion, $C_j = W_j + B_j$ (11)

w_j...mean waiting time of offered calls

$W_j(>\tau)$...prob.that an offered call has to wait longer than τ, with normalized time t acc.to $\tau = t/h$ (12)

Fig.5 shows, that in some cases these values can also be looked up in tables.

Type of single-stage queuing model		DELAY SYSTEM PCT1	DELAY SYSTEM PCT2	DELAY SYSTEM PCT3	DELAY-LOSS SYSTEM PCT1	DELAY-LOSS SYSTEM PCT2
M_1 : single-queue ($g_1^* = 1$)	full access	/8/ /3/ /14,15/ /16,17/	/8/ /3/	/8/	/8/	5.2
$M_2..M_S$: multi-queue ($g_j^* > 1$)	full access				/8/	5.4
	limited access	/8/	4.2	/8/	5.5	5.6

Fig.5: Single-stage queuing models.
/../: Reference on a table book.
x.y : Formulae given in Section x.y of /6/.
PCT1: POISSON-input, neg.exp.distrib. holding times
PCT2: BERNOULLI-input, neg.exp.distrib. holding times
PCT3: POISSON-input, const.holding times

3.5 STEP 4 : Composition of link system results. The call congestion C of a link system is assumed to occur statistically independently in the S stages of the link system. Therefore, the value of C can be composed by the call congestions $C_1, C_2..C_S$ calculated for the equivalent single-stage systems $M_1, M_2..M_S$, cf.Eq.(13).

Call congestion in stage 1	$C = C_1$ (13a)
Call congestion in stage 2 (simultaneously no call congestion in stage 1)	$+ (1-C_1)\cdot C_2$
Call congestion in stage 3 (simultaneously no call congestion in stages 1,2)	$+ (1-C_1)\cdot(1-C_2)\cdot C_3$
⋮	⋮
Call congestion in stage S (simultaneously no call congestion in preceeding stages)	$+ \prod_{j=1}^{S-1} (1-C_j)\cdot C_S$

From Eq.(13a) : $C = 1 - \prod_{j=1}^{S} (1-C_j)$ (13b)

The product in Eq.(13b) represents the probability that call congestion does not exist in any stage of the link system.

Further characteristic values are analogously calculated from the single-stage results:

Prob.of delay $W = W_1 + (1-C_1)W_2 + (1-C_1)\cdot(1-C_2)W_3$

$$\ldots + \prod_{j=1}^{S-1} (1-C_j)W_S \quad (14a)$$

i.e. $$W = W_1 + \sum_{j=2}^{S} W_j \cdot \prod_{i=1}^{j-1} (1-C_i) \quad (14b)$$

with W_j prob.of delay in single-stage system M_j.

For unlimited queuing holds $W = C$ (14c)

Prob.of loss $B = B_1 + (1-C_1)B_2 + (1-C_1)\cdot(1-C_2)B_3$

$$\ldots + \prod_{j=1}^{S-1} (1-C_j)B_S \quad (15a)$$

i.e. $$B = B_1 + \sum_{j=2}^{S} B_j \cdot \prod_{i=1}^{j-1} (1-C_i) \quad (15b)$$

with B_j prob.of loss in single-stage system M_j.

For unlimited queuing holds $B = 0$ (15c)

Offered traffic $A = Y/(1-B)$ (16)
with Y prescribed carried traffic of link system.

Distr.fct. of waiting times of offered calls

$$W(>\tau) = W_1(>\tau) + \sum_{j=2}^{S} W_j(>\tau)\cdot\prod_{i=1}^{j-1}(1-C_i) \quad (17)$$

with $W_j(>\tau)$ distr.function of waiting times of offered calls in single-stage system M_j.

Mean waiting time of offered calls

$$w = w_1 + \sum_{j=2}^{S} w_j \cdot \prod_{i=1}^{j-1} (1-C_i) \quad (18)$$

with w_j mean waiting time of offered calls in single-stage system M_j.

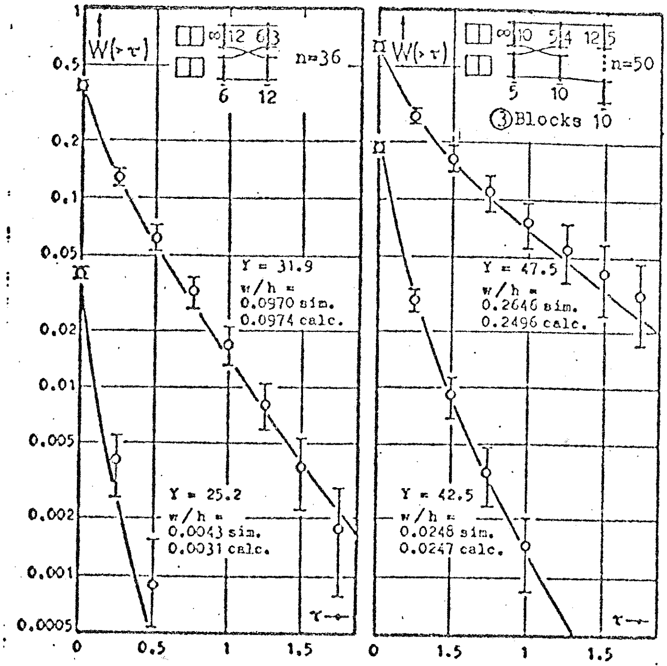
Mean waiting time of waiting calls

$$t_W = w/W \quad (19)$$

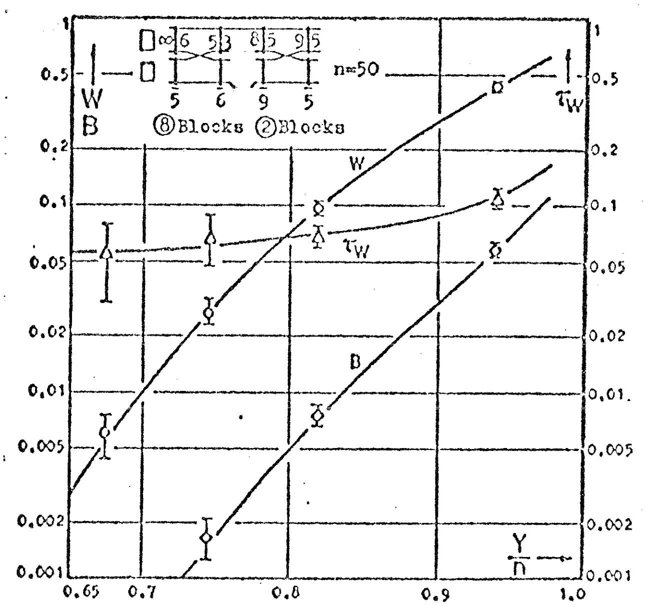
Mean queue length $\Omega = A\cdot w/h$ (20)
with h mean holding time.

4. COMPARISON WITH SIMULATION

In Fig.6 simulation and calculation results are compared for link systems with an infinite number of sources and neg.exp.distr. holding times. In Fig.6(a) the distribution of waiting times $W(>\tau)$ and also the mean waiting time w of offered calls are given for link systems with unlimited queuing. Fig.6(b) was obtained from a four-stage link system with limited queuing (s=1 waiting place per multiple). Simulation and calculation show good accordance.



(a) Distribution function of waiting times of offered calls $W(>\tau)$; delay systems.



(b) Prob.of delay W, prob.of loss B, norm.mean waiting time of waiting calls $\tau_W = t_W/h$; combined delay-loss-system.

Fig.6: Calculation and simulation results.
Y : Total carried traffic
—— Calculation (cf.Chapter 3)
Simulation (95% confid. intervals)
Hunting: Homing selectors in all stages
Wiring /1,2/: (a)sequential, (b)cyclical

5. CONCLUSION

In this report an approximate calculation method is presented for traffic-concentrating link systems with queuing, point-to-group selection and one outgoing group. Mean values as well as the distr.function of waiting times are derived. The link system results are composed by simple formulae from single-stage systems results which may be looked up in delay tables. Calculated and simulated results are in good accordance, regarding link systems with unlimited queuing (delay systems) or limited queuing (combined delay-loss systems). Considered is a finite number of sources with neg.exp.distr.holding times, or an infinite number of sources with neg.exp. distributed or constant holding times.

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ANNEX: EFFECTIVE ACCESSIBILITY

In /1,2/ a formula has been presented for the effective accessibility of link systems with point-to-group selection mode and without queuing (loss systems). Extensive studies have proved, that this method CLIGS-A can also be applied to link systems with queuing (delay systems or combined delay-loss systems).

Following the calculation concept of Chapter 3, for each multi-stage subsystem T_j an effective accessibility k_{effj} is derived, with

j......number of stages of subsystem T_j

$g_{v,Tj}$..numb.of multiples in stage No.v of T_j (serial index $v = 1,2..j$)

i_v, k_v..number of inlets,outlets per multiple in stage No.v

$\bar{n}_j$.....number of outlets of subsystem T_j

Y_{Tj}....prescribed carried traffic on $\bar{n}_j$ outlets

Y......prescribed total carried traffic on the n outlets of the link system

Acc. to method CLIGS-A /1,2/ it holds in case of one outgoing group

$$\boxed{k_{effj} = FF_j \cdot k_j + BF_j \cdot k_j \cdot (Y_{Tj}/\bar{n}_j) \cdot f_j} \quad (21)$$

with free-fan $FF_j = \prod_{v=1}^{j-1} (k_v - Y/g_v)$ (22)

$$\text{limited by } \prod_{v=1}^{i} (k_v - Y/g_v) \leq g_{i+1,Tj} \quad (23)$$

and busy-fan $BF_j = \min(g_{j,Tj}; \prod_{v=1}^{j-1} k_v) - FF_j$ (24)

and factor $f_j = 1/k_j$ (fan-out structure) (25a)

or $f_j = (i_j - Y/g_j)/k_j$ (meshed struct.) (25b)

"fan-out" stands for at most one path, and "meshed" for more than one path leading from a first-stage multiple to a certain multiple in the last stage (stage No.j) of subsystem T_j.

A subsystem T_j is characterized either by full accessibility ($k_{effj} = \bar{n}_j$) or by limited accessibility ($k_{effj} < \bar{n}_j$).

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