TDM LINK-SYSTEMS

WITH BY-PATHS FOR NON-COINCIDENT SWITCHING

by

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Abstract

This paper investigates into two-stage tdm link-systems with by-paths by means of which two non-coincident time-channels may work together, if a first attempt of setting up a call fails to find two idle coincident time-channels. Thus, according to the number of by-paths, the blocking probability can be reduced with respect to tdm link-systems having but coincident switching facilities. An inferior limit of the blocking probability is obtained in a tdm link-system with so many by-paths that a call may only fail for lack of free time-channels.

The blocking probability in tdm link-systems with primarily coincident, and if necessary non-coincident switching attempts is calculated for a given number of by-paths. The variance of traffic rejected by the first coincident switching attempt is taken into account.

Results of artificial traffic tests are throughout in good accordance with theory. This is true not only for the blocking probability as the final result but also for intermediate steps. In particular, the assumed distributions of busy time-channels and busy by-paths are verified by checks sampling the actual distributions as obtained by test runs.

It is found that a relatively small number of by-paths is already sufficient to get closely at the inferior limit of blocking probability.

Introduction

Growing importance is attached to integrated systems using the principle of time-division-multiplex (tdm) for both switching and transmission. The limited number of time-channels per highway, especially in case of pulse-code-modulation (pcm), makes it worthwhile examining how to obtain multistage tdm link-systems with a reduced blocking probability. Coincident switching is the most adequate way of handling economically a great amount of traffic in tdm link-systems. In this case, the number of necessary gates is considerably smaller than the number of cross-points in comparable systems with space-division-multiplex (sdm). Typically, coincident switching implies that the selected free time-channels on the highways must have the same time-position.

Using but coincident switching technics, the setting up of a call fails if the only free time-channels do not fulfill the requirement of time alignment. To redress this restriction, further means can be provided for an interconnection of non-coincident time-channels. Obviously, the necessary shifts of time-position need stores bypassing the gates for mere coincident switching.

TDM Link-System with By-Paths

Connecting Arrangement

Figure 1 shows the investigated connecting arrangement where all highways, being represented by thick lines, have the same number of N synchronized time-channels.

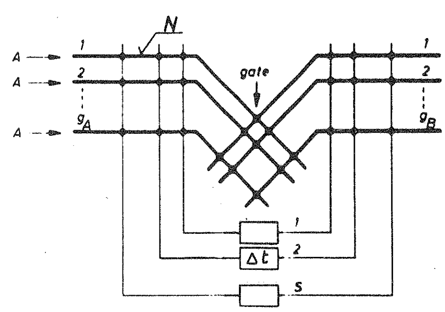

Fig. 1. Connecting Arrangement

There are g_A highways in the A-stage (A-highways) and g_B highways in the B-stage (B-highways). Each A-highway is offered a random traffic A which aims symmetrically at each B-highway.

A matrix of $g_A \cdot g_B$ gates at the intersections of the highways is provided for coincident switching. By applying a pulse to one of these gates, a pair of coincident

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time-channels on the corresponding A- and B- highways can be connected. Significantly, up to N pairs of coincident time-channels may be switched through one single gate in this matrix.

In addition to this wellknown two-stage tdm link-system with coincident switching, S by-paths are inserted in figure 1, each having full access to all A- and B-highways by individual gates. A by-path is able to pick up, by one of its gates, the periodic impulses of a time-channel on a certain A-highway, to store the information for a suitable time Δt, which is in any case shorter than the tdm sampling period, and to release the information by another gate to a non-coincident time-channel on a B-highway. Thus, a pair of idle time-channels that are non-coincident may be connected by use of one idle by-path.

Obviously, it does not make sense to add more than the maximum number of $S_{max} = g_o \cdot N$ by-paths, where g_o stands for the smaller value of g_A and g_B.

The additional non-coincident switching facilities will certainly reduce the blocking probability, i.e. the probability of failing to set up a call that arrives at random and demands a connection between a certain A-highway and a certain B-highway.

Traffic Handling

The separate steps of the hunting procedure are now analyzed in more detail. In a first attempt of setting up a call between a given A- and B-highway, a pair of idle coincident time-channels must be hunted for. To make this step clear, the N time-channels of an A- and B-highway are spatially shown in figure 2. Time-channels having the same time-position are considered to be placed on the same level. Therefore, if the hatched parts signify schematically distinct patterns of busy time-channels, the shown horizontal arrow indicates a pair of idle coincident time-channels that may be connected by a gate. The probability of failing to find such a pair of simultaneously free time-channels is denoted by Coincident Blocking Probability B_C.

In the event occurring with probability B_C, a second attempt of setting up a call follows suit. To begin with, a pair of non-coincident free time-channels is indispensable. An arrow, which is no longer a straight one, must find its way through both highways as also indicated in figure 2. The probability of not finding such a pair of non-coincident time-channels is called Non-Coincident Blocking Probability B_N. Obviously, B_N implies B_C.

Finally, to complete the second switching attempt successfully, an idle by-path has to be hunted in the trunk group of S by-paths. The probability, that inspite of a pair of free non-coincident time-channels the non-coincident switching attempt will fail for lack of a free by-path, is designed By-Path Loss Probability B_P.

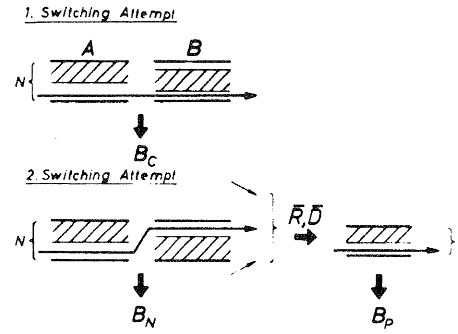

Fig. 2. Hunting Procedures and Blocking Probabilities

The fundamental relation between the Total Blocking Probability B, i.e. the probability that both switching attempts are unsuccessful, and the introduced probabilities B_C, B_N, and B_P can readily be seen.

One term of B is certainly the non-coincident blocking probability B_N. The second term to be added is the probability $(B_C - B_N) \cdot B_P$, where $(B_C - B_N)$ is the probability that a call, failing at the first coincident switching attempt, finds a pair of free non-coincident time-channels; however, the multiplication by the probability B_P means, that in this case an idle by-path is not available. The complete equation

$$B = B_N + (B_C - B_N) \cdot B_P \qquad (1)$$

implies two important limits. If there is no by-path at all, the by-path loss probability B_P is 1. In equation (1), the total blocking probability B is then as expected equal to the coincident blocking probability B_C. Conversely, for the maximum number S_{max} of by-paths, the by-path loss probability B_P is equal to zero. The total blocking probability B then coincides with the non-coincident blocking probability B_N, as can also be seen from equation (1).

Outline of Solution

In the present paper the total blocking probability B will be determined as a function of the offered traffic per time-channel A/N, the number S of added by-paths being the parameter.

The proposed method of solution sets out from the given carried traffics y_A and y_B on the A- and B-highways respectively. This approach is very convenient for practical traffic engineering, since the actually carried traffics are normally given. Besides, fixed values between 0 and 1 are prescribed for the by-path loss probability B_P in the beginning of the calculation.

In a first step, approximations are given for calculating the coincident and non-coincident blocking probabilities B_C and B_N. Then, after having obtained the total blocking probability B according to equation (1), the offered traffic A per A-highway follows immediately, since the traffics carried by the highways are known.

In a second step, a total overflow traffic characterized by mean $\bar{R}$, variance $\bar{V}$, and variance coefficient $\bar{D}=\bar{V}-\bar{R}$, is calculated. As shown in figure 2, this total overflow traffic $(\bar{R},\bar{D})$, being caused by the entire tdm link-system in the first switching attempt, is already reduced by that traffic which is lost due to the non-coincident blocking probability B_N. Thus by definition, the total overflow traffic $(\bar{R},\bar{D})$ in any case disposes of two non-coincident idle time-channels and its chance of not being placed in the tdm link-system is given by the probability B_P of not finding a free by-path.

Considering the known overflow $(\bar{R},\bar{D})$ and its loss probability B_P in the secondary route of S by-paths, the unknown number S can be determined. Normally, the values obtained for S will not be integers. For a given integer of S by-paths the total blocking probability B as a function of A/N may be finally found by interpolation.

Blocking Probabilities

The general solution as outlined above has the advantage that tdm link-systems with more than two stages can be dealt with, too, provided the by-paths are connecting the first and last stage, whereas B_C will stand for the coincident blocking probability as obtained for coincident switching through more than two stages. The results of the present paper are limited to two-stage tdm link-systems to show the basic effects of by-paths. The investigations are based on a traffic model with a Poisson input and negative exponential distribution of holding times. It is assumed that the distributions of busy time-channels and the distribution of busy by-paths are independent from one another.

Coincident Blocking Probability

As shown in an earlier paper /1/, tdm link-systems with coincident switching can be transformed into link-systems with space-division-multiplex offering identical connection facilities. Thus, all methods of calculating blocking probabilities in sdm link-systems apply likewise to the corresponding tdm link-systems. The same paper contains the application of the method of combined inlet and route blocking (CIRB) to tdm link-systems with random hunting. For small loads slightly too optimistic values have come out as compared with results of artificial traffic tests.

In the meantime, however, it was found that in the whole considered range the CIRB formula represents a remarkably good approximation to test results with sequential hunting, since the tendency of CIRB is compensated by the more efficient sequential hunting procedure.

Using the CIRB formula /2/, one gets for the coincident blocking probability

$$B_C = E_N(A_A) + \{1 - E_N(A_A)\} \cdot \left\{ \frac{E_N(A_B)}{E_{N-k}(A_B)} \right\} \quad (2)$$

A_A and A_B are fictitious traffics generating in a full available route of N lines the given carried traffics y_A and y_B respectively according to the following expressions, written with Erlang's blocking probability E_N

$$A_A = y_A / \{1 - E_N(A_A)\} , \quad (3)$$

$$A_B = y_B / \{1 - E_N(A_B)\} . \quad (4)$$

In the present application, the coincident blocking probability B_C will take higher values with increasing proportion of non-coincidently handled traffic, since this traffic induces a distribution of busy time-channels where high numbers of simultaneously busy time-channels become more probable. The resulting increase of B_C can be fairly well approximated for tdm link-systems with about g_B =10 B-highways by interpolating between two limits of parameter k in equation (2).

For tdm link-systems with mere coincident switching (i.e. by-path loss probability B_P =1) the expression

$$k_1 = N - y_A + \frac{y_A}{g_B} \quad (5)$$

is taken according to Bininda and Wendt /3/, whereas Kharkevich's more pessimistic formula /4/

$$k_2 = N - y_A \quad (6)$$

is used for tdm link-systems with S_{max} by-paths (i.e. by-path loss probability B_P =0). The interpolation may be performed as a function of the by-path loss probability $0 \leq B_P \leq 1$ being prescribed in the beginning:

$$k = N - y_A + \frac{y_A}{g_B} \cdot B_P . \quad (7)$$

Non-Coincident Blocking Probability

The non-coincident blocking probability B_N can be written as the probability that the considered A- and B- highways have not simultaneously at least one free time-channel each. With $p_A(N)$ and $p_B(N)$ denoting the probabilities of finding all N time-channels busy on the A- and B-highway respectively, one gets

$$B_N = 1 - \{1 - p_A(N)\} \cdot \{1 - p_B(N)\} . \quad (8)$$

For a given traffic carried on a highway, the distribution of busy time-channels changes with the proportion of non-coincidently handled traffic and so do in particular the probabilities $p_A(N)$ and $p_B(N)$.

The distribution of Erlang's interconnection formula (EIF) may serve as a means of calculating approximate values of $p_A(N)$ and $p_B(N)$. For example, considering an A-highway with N time-channels carrying a traffic y_A, the EIF distribution of busy time-channels

$$p_{A,EIF}(x)=f(N,A_{A,EIF},\tilde{k},x) \qquad (9)$$

can be evaluated with a suitable offered traffic $A_{A,EIF}$ and an approximate availability $\tilde{k}$. Having determined $p_{A,EIF}(x)$, the EIF blocking probability $B_{A,EIF}$ follows in a second step.

As the EIF method is based on an offered traffic, the additional relation

$$A_{A,EIF}=y_A/(1-B_{A,EIF}) \qquad (10)$$

has to be observed the value of y_A being prescribed. For the availability $\tilde{k}$ the approximation

$$\tilde{k}=N-(y_A-y_A/g_B)\cdot B_p \qquad (11)$$

has been chosen. Equation (11) coincides with the Bininda-Wendt equation (5) for $B_p=1$ (no by-paths). For $B_p=0$ one gets with equation (11) $\tilde{k}=N$. In this case the EIF distribution passes into Erlang's distribution for full availability. Actually, the condition of full availability is attained with S_{max} by-paths which have full access to all highways, the by-path loss probability B_p being zero.

The wanted probability $p_A(N)$ is finally given by

$$p_A(N)=p_{A,EIF}(x=N). \qquad (12)$$

Analogously to the equations (9,10,11,12), $p_B(N)$ can also be determined and inserted in equation (8).

Total Blocking Probability

Having calculated B_C and B_N as described above, the total blocking probability B follows for a prescribed by-path loss probability B_p according to equation (1). For an infinite number of traffic sources B is also the probability of loss.

The actual traffic being offered to an A-highway follows with

$$A=y_A/(1-B). \qquad (13)$$

Further, the proportion of traffic that is handled coincidently on an A-highway will be

$$y_{AC}=A\cdot(1-B_C), \qquad (14)$$

whereas the proportion of non-coincidently handled traffic per A-highway is given by

$$y_{AN}=y_A-y_{AC}. \qquad (15)$$

Total Overflow Traffic

The mean $\bar{R}$ of the total overflow traffic $(\bar{R},\bar{D})$ shown in figure 2 is

$$\bar{R}=g_A\cdot A\cdot(B_C-B_N). \qquad (16)$$

As to the variance coefficient $\bar{D}$ of this overflow, it was found that fairly accurate results can be obtained by working with a basic idea of Herzog, which as an extension to a work of Lotze /5/ is published in this issue /6/.

Number of By-Paths

A full available route with N^* lines and an offered traffic A^* can be determined such that an overflow traffic $(\bar{R},\bar{D})$ is generated. In a substitute arrangement /7,8/ this full available route (A^*,N^*) may be followed by a secondary route, the loss probability of which is equal to the by-path loss probability B_p. The number S of lines in the secondary route, i.e. the number of by-paths, can then be evaluated for a given value of B_p. One only has to determine a full available route of (N^*+S) lines in which the offered traffic A^* suffers a loss of

$$E_{N^*+S}(A^*)=\frac{\bar{R}\cdot B_p}{A^*} \qquad (17)$$

It should be noted, however, that for $B_p=0$ the number S in the substitute arrangement tends to infinity. In tdm link-systems the number of highways is finite, and so is the maximum number of by-paths for a by-path loss probability B_p equal to zero. Yet in view of the good final accordance of theory and test in the interesting range of low by-path numbers (see also overflow distribution in the following section), no modification of the substitute arrangement has been thought necessary so far.

Results of Theory and Traffic Tests

The described method is available in ALGOL version. Linear or parabolic interpolation was programmed when this was necessary. Besides, tests based on artificial traffic have been performed on a digital computer. For the comparison of theoretical and tested results, a symmetrical two-stage tdm link-system with $g_A=g_B=10$ highways was chosen. For the number N of time-channels per highway, the representative values of N=25 (technically feasible systems with pulse-code-modulation) and N=100 (technically feasible systems with pulse-amplitude-modulation) have been taken. All test points are shown with a confidence interval of 95 per cent.

1. For N=25 time-channels per highway and the limits S=0 and $S=S_{max}=g_A\cdot N=250$ by-paths, the coincident blocking probability B_C and the non-coincident blocking probability B_N are shown in the figures 3 and 4 as a function of A/N. There is good accordance between calculated and tested values.

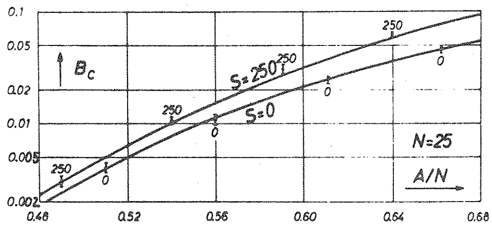

Fig. 3. Coincident Blocking Probability
(Test ; —— Theory)

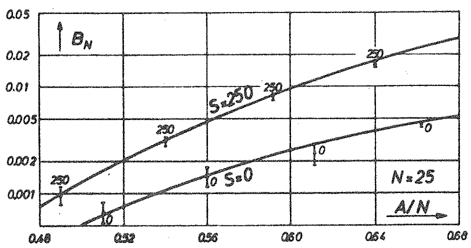

Fig. 4. Non-Coincident Blocking Probability
(Test ; —— Theory)

2. Two distributions p(x), i.e. the probabilities of finding x time-channels simultaneously busy on a highway, are given in the figures 5 and 6 for the examples S=0 and S=250 by-paths. The result is very satisfactory.

3. The overflow distribution of busy by-paths as implicitely assumed in the chosen substitute arrangement has been calculated according to Chastang's formula /9/. For the example of a tdm link-system with S=5 by-paths, figure 7 shows a confrontation of a calculated and a tested overflow distribution $p_s(x)$, i.e. the probabilities of finding x by-paths simultaneously busy in the trunk group of S by-paths.

4. The final result, the total blocking and loss probability B as a function of A/N for discrete numbers S of by-paths, is shown in figure 8 (N=25) and figure 9 (N=100). Theoretical curves are within the test intervals. For a given by-path number S between 0 and S_{max}, the curves B=f(A/N) clearly start at the inferior limit $B=B_N$. Passing through the transition area between B_N and B_C, the curves adjust themselves to the upper limit $B=B_C$ with increasing values of A/N.

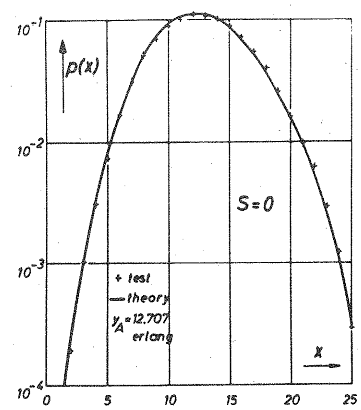

Fig. 5. Distribution of Busy Time-Channels (S=0 By-Paths)

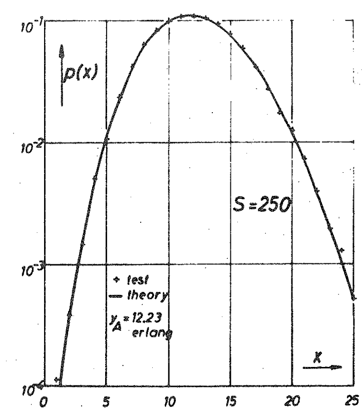

Fig. 6. Distribution of Busy Time-Channels ($S=S_{max}$=250 By-Paths)

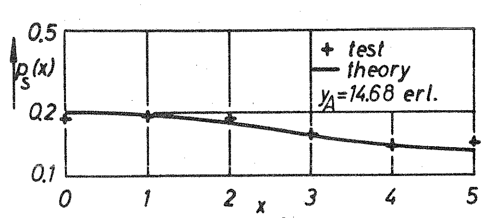

Fig. 7. Overflow Distribution of Busy By-Paths (S=5 By-Paths)

Conclusions

Optimum Number of By-Paths

As may be seen from the figures 8 and 9, one gets closely at the inferior limit $B=B_N$ by working with a number of by-paths that is considerably smaller than the maximum number S_{max}. It is convenient to define an optimum

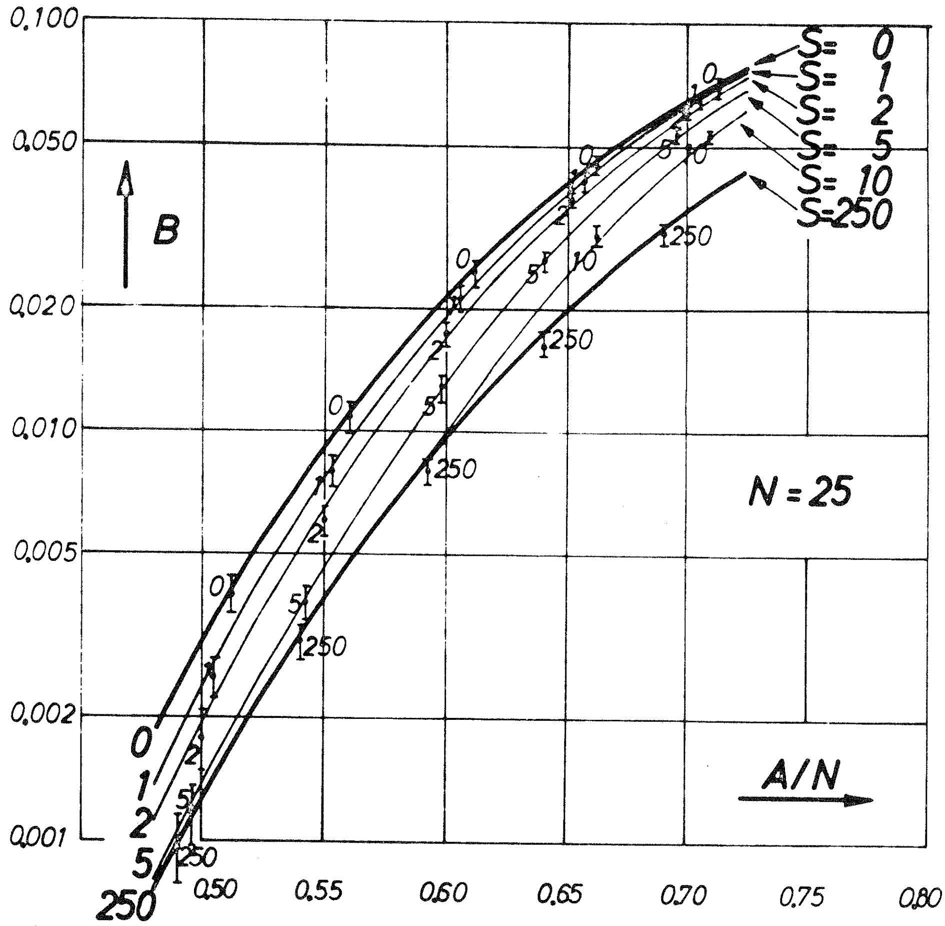

Fig. 8. Total Blocking Probability B,
N=25(I Test ; —— Theory)

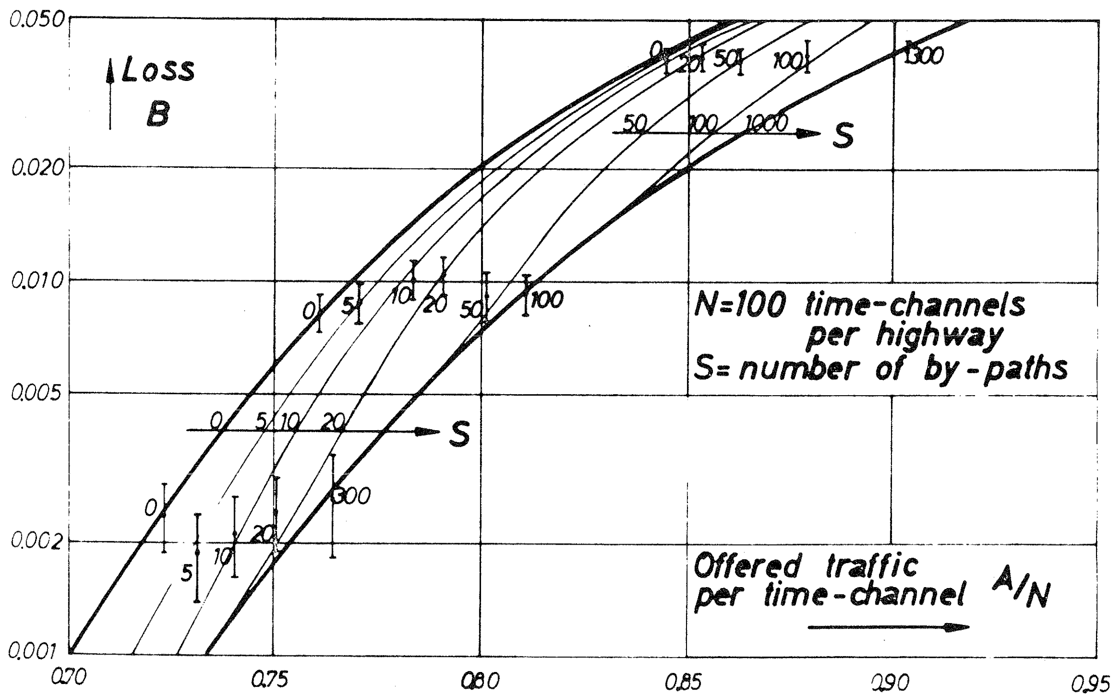

Fig. 9. Total Blocking Probability B,
N=100(I Test ; —— Theory)

number S_{opt} of by-paths yielding a total blocking probability B_{opt} which would have to be reduced by only 5 per cent to come down to the at best obtainable non-coincident blocking probability B_N. Suitably, the blocking probabilities B_{opt} and B_N may be determined for the same carried traffic.

Table 1 shows for example the necessary optimum number of by-paths for a wanted total blocking probability of B_{opt} = 1 per cent

N	$y_A = y_B$	S_{max}	S_{opt}	$\frac{S_{opt}}{S_{max}}$	$\frac{B_c}{B_{opt}}$
25	14.85	250	10	4 per cent	2.4
100	80.30	1000	60	6 per cent	3.5

Table 1. Optimum Number of By-Paths for an Optimum Total Blocking Probability of B_{opt}= 1 per cent.

It is surprising to see that in the considered cases only a very small percentage of S_{max} has to be inserted into the tdm link-system as by-paths in order to attain practically the inferior limit of the total blocking probability.

Number of Additional Gates

It can be seen from figure 1 that each by-path needs g_A+g_B individual gates. Assuming that 10·N traffic sources with one gate each will be concentrated on an A-highway, the totally necessary number of gates have been compiled in table 2.

	N=25	N=100
gates of traffic sources	2500	10000
gates for S_{opt} by-paths (table 1)	200	1200
gates for coincident switching	100	100

Table 2. Number of Necessary Gates

It can be seen that the additional amount of gates for by-paths is not at all exceedingly high, when the gates of the traffic sources are also taken into consideration.

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