ON THE CONGESTION IN TDM SYSTEMS

Application of the CIRB Method
and Comments on Known Formulae

by M. H u b e r
Institut für Nachrichtenvermittlung und Datenverarbeitung
Technische Hochschule Stuttgart
Federal Republic of Germany

Presented at the Fourth International Teletraffic Congress
London, July 15th to 21st, 1964

Summary

The paper investigates blocking probability in time division multiplex (tdm) switching systems with Poisson input, coincident time channels for a conversation, and random selection.

Using the method of Combined Inlet and Route Blocking (CIRB) good approximate results are obtained with a very small amount of evaluation work.

Then, the accuracy of already known loss formulae is improved for losses over 1 per cent by introducing a fictitious offered traffic which produces an Erlang distribution with the actually carried traffic as its mean.

With regard to some artificial traffic tests the CIRB method establishes an inferior limit close to reality whereas a superior limit is given by the improved formulae of other authors.

Contents

Figures 5, 6, 7, 8, and 9 are diagrams and have been inserted behind the text pages.

1. Introduction

Let us consider time-division-multiplex (tdm) switching systems which consist of time shared wires referred to as highways each providing a fixed number of time channels. Excluding extensive technical devices such as delay lines or speech storage the present paper deals with the case that a conversation can only be established if the same time channel is simultaneously free on a line of highways between two terminal points. In case of several free paths coming into question it is a basic assumption throughout this paper that one path is selected at random. Furthermore the traffic offered to the tdm switching system is assumed to be a Poissonian traffic.

2. Application of the CIRB Method to TDM Switching Systems

In space-division-multiplex (sdm) link systems, the loss probability can be calculated by using the method of Combined Inlet and Route Blocking (CIRB) as introduced by L o t z e /1/. The features of CIRB are:

a) The blocking probability B is composed of two parts, the inlet blocking probability $\lceil k_A \rceil$ in the A-stage, and the route blocking probability $\lceil p \rceil$ that occurs behind the A-stage when the state $\lceil k_A \rceil$ does not exist. Thus

$$B = \lceil k_A \rceil + \left\{ 1 - \lceil k_A \rceil \right\} \cdot \lceil p \rceil \qquad (1)$$

In case of an sdm link system $\lceil k_A \rceil$ is the probability that all k_A available outlets in a considered multiple of the A-stage are busy, whereas $\lceil p \rceil$ is the probability that in a desired route all the p lines which on the average can be hunted by the inlets of the A multiple are busy.

b) To determine p the carried traffic per multiple is taken for the average number of busy outlets in that multiple.

c) The step from p to ⌈p⌉ is performed with the Palm-Jacobaeus loss formula for limited availability taking into account, however, a useful modification by introducing an Erlang distribution that has the actually carried traffic as its mean /3/.

The evaluation of specific values with CIRB proves very easy since special tables are at the disposal of traffic engineers /1, 2/. Results for sdm link systems are sufficiently accurate. This has been checked by a great number of artificial traffic trials on an electronic computer /1/.
It will be shown that the basic ideas of CIRB also apply to tdm switching systems. Thus new approximation formulae can be given, the behaviour of which is confronted with first results of tdm traffic tests.

2.1. CIRB Approximation for Two Link TDM Systems

Let us now turn to a two link tdm switching system implying that any conversation uses two highways interconnected by gates, if internal traffic within one highway is excluded.

In many cases internal traffic amounts only to a small proportion of total traffic. The effect of additional internal traffic on the loss probability in sdm link systems has been studied by B o t s c h /4/ . An application of his findings to tdm switching systems seems to be feasible.

Figure 1 shows h_A highways in the A-stage and h_B highways in the B-stage each highway having the same frame of N time channels. Every A-highway is connected to every B-highway by a gate. There are $h_A \cdot h_B$ gates with the possibility of handling up to N conversations each.
For one conversation the same time channel must be allocated on an A-highway as well as on a B-highway.

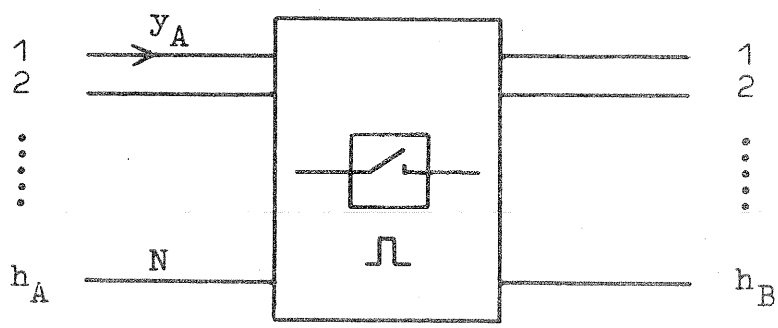

Fig. 1 Two Link TDM Switching System

It can easily be checked that this two link tdm arrangement is perfectly equivalent to a familiar two stage link system in space-division-multiplex as shown in figure 2.

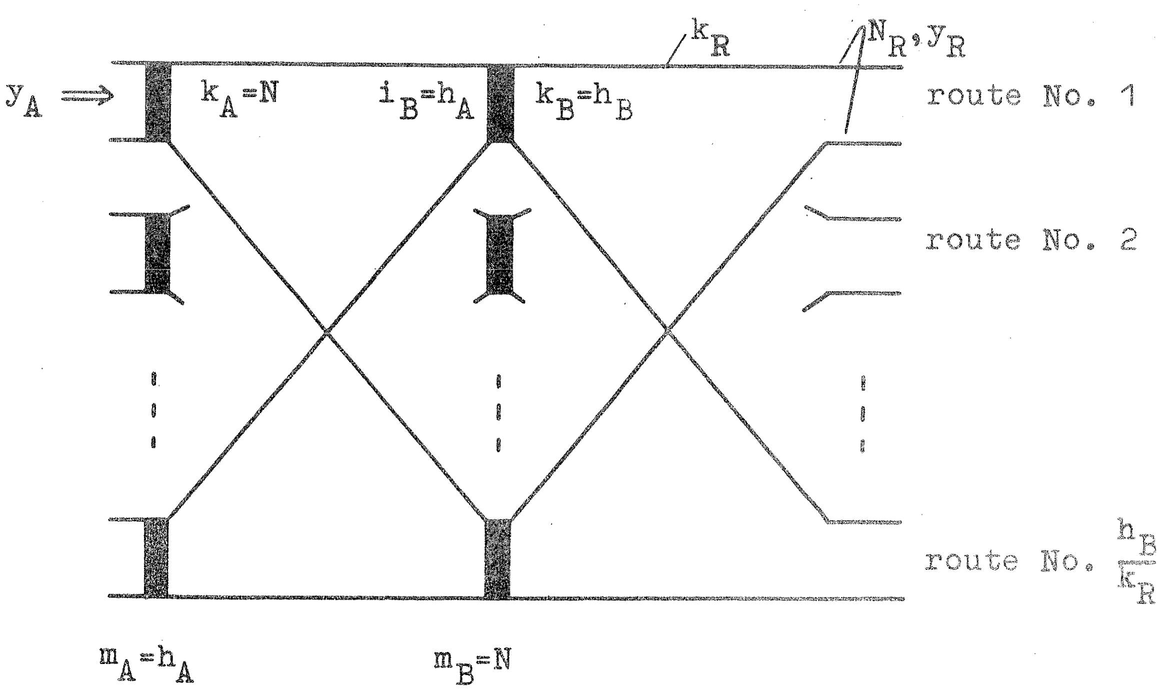

Fig. 2 Two Stage SDM Link System

The specifications of the derived sdm system with regard to the original tdm system are the following ones:

a) number m_A of multiples in A-stage equal to number h_A of highways in A-stage,

b) number m_B of multiples in B-stage equal to number N of time channels per highway,

c) availability k_A in A-stage equal to number N of time channels per highway,

d) number i_B of inlets to a multiple in B-stage equal to number h_A of highways in A-stage,

e) availability k_B in B-stage equal to number h_B of highways in B-stage,

f) availability k_R in B-stage with regard to a certain route equal to number of those highways in B-stage which lead to the same point of destination.

   In practice the case $k_R > 1$ is given when a specific subscriber is available in more than one highway of the last stage (here B-stage),

g) number N_R of lines in a route equal to $k_R \cdot N$,

h) number of routes equal to h_B/k_R.

Now the CIRB method can be applied to the obtained sdm system. Starting with traffic y_A handled by one A-multiple (one A-highway respectively) the inlet blocking probability $[k_A]$ that all $k_A = N$ outlets of an A-multiple are busy is according to Erlang

$$[k_A] = [N] = E_N (A_o) \qquad (2)$$

$$\text{with} \quad A_o = y_A / (1 - E_N (A_o)), \qquad (3)$$

A_o meaning the fictitious offered traffic producing an Erlang distribution with the actually offered traffic y_A as its mean.

Next the average number p of lines which in a certain route can be hunted busy or free by the inlets of the A-multiple is given by

$$p_2 = (N - y_A) \cdot k_R + \eta_R \cdot y_A \qquad (4)$$

where $(N - y_A)$ represents the average number of free outlets behind the considered A-multiple and k_R the number of lines leading to a route once a certain B multiple is reached. The term $\eta_R \cdot y_A$ with $0 < \eta_R \leq 1$ stands for the existing part of traffic between the A-multiple and the considered route and can be interpreted as the average number of lines which contribute to p in addition to the $(N - y_A) \cdot k_R$ lines. *)

Finally, the probability $[p]$ that all these p lines in the route of N_R lines are busy leads to the problem of limited availability and is solved with the modified Palm-Jacobaeus formula:

$$[p] = \frac{E_{N_R}(A_o^*)}{E_{N_R-p}(A_o^*)} \qquad (5)$$

with

$$A_o^* = \frac{y_R}{1 - E_{N_R}(A_o^*)} \qquad (6)$$

The term y_R represents the total traffic carried by the considered route.

Inserting equations (2) and (5) in (1) the blocking probability becomes

$$B = E_N(A_o) + \left\{1 - E_N(A_o)\right\} \cdot \frac{E_{N_R}(A_o^*)}{E_{N_R-p}(A_o^*)} \qquad (7)$$

with A_o, p, and A_o^* according to (3), (4), and (6).

---

*) The idea behind equation (4) has been used for the first time in [5] .

2.2. CIRB Approximation for Multi-Link TDM Systems

The number of interhighway gates can be reduced by passing over from a two link tdm system to tdm systems with more than two stages referred to as multi-link tdm systems. Figure 3 shows a three link tdm configuration which is achieved by forming supergroups of h_A and h_C highways respectively. Any pair of supergroups belonging to different stages is connected by a set of f_B common interlink highways B having gate access to all highways of both supergroups in consideration. Again internal traffic is excluded (see section 2.1.). There is no blocking as long as the same time channel can be found idle on three highways since an interlink highway now has also to be taken into account.

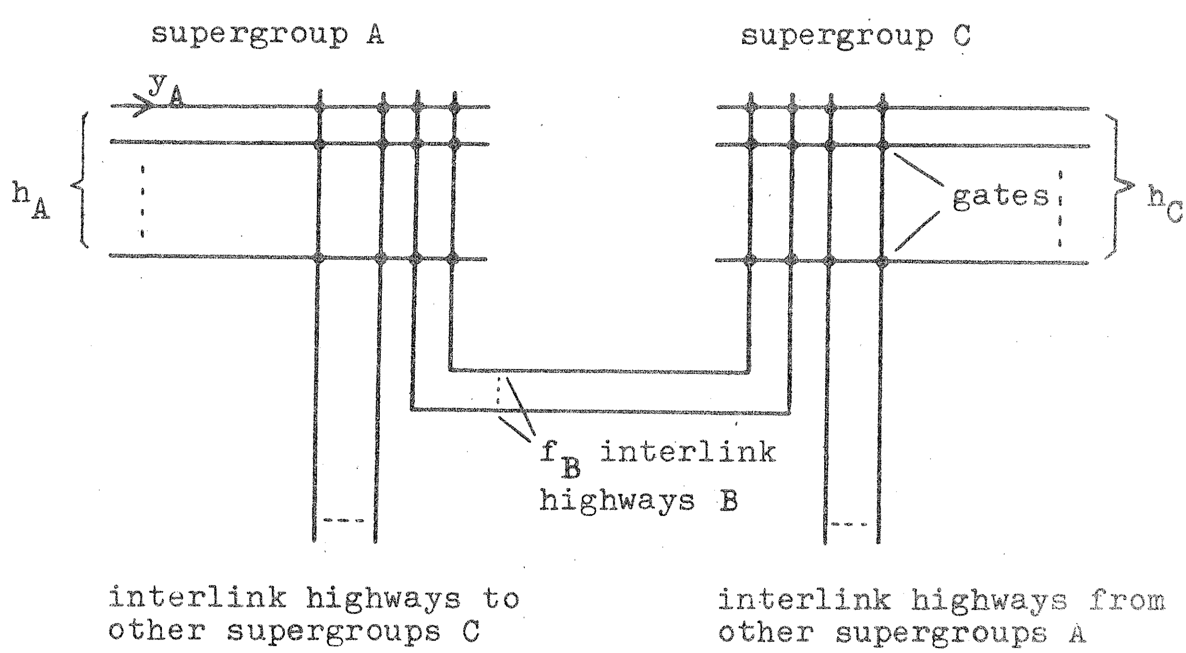

Fig. 3 Three Link TDM System

The equivalent three stage sdm link system in figure 4 comes into being by taking the above mentioned two stage sdm link system for the A- and C-stage and inserting a B-stage with m_B = N multiples, $i_B = h_A$, and $k_B = f_B$.

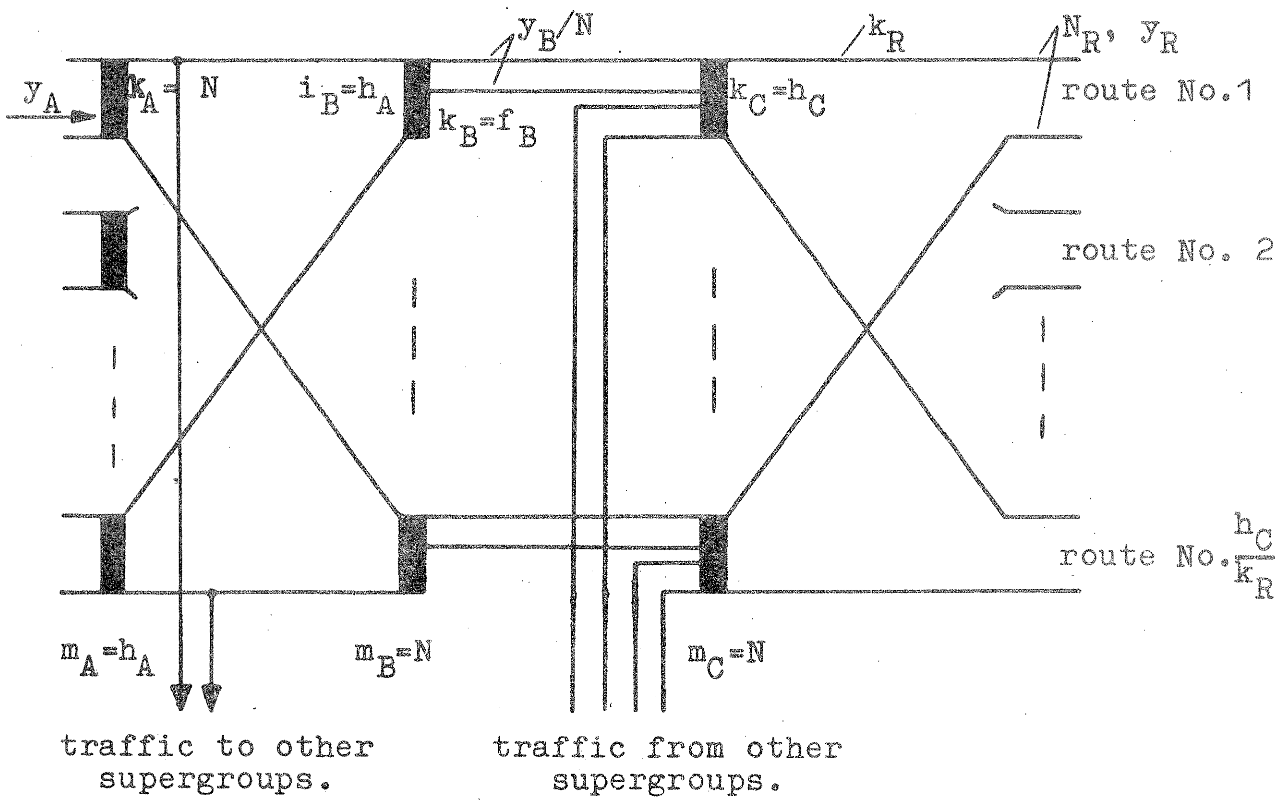

Fig. 4 Three Stage SDM Link System

Again the blocking probability can be derived with CIRB. Equations (3), (6), and (7) apply directly whereas the change of model figures only in an altered expression of p, that is equation (4) has to be replaced by

$$p_3 = (N - y_A) \cdot (f_B - y_B/N) \cdot k_R + \eta_R \cdot y_A \qquad (8)$$

The new factor $(f_B - y_B/N)$ represents the average number of free lines behind a B-multiple. The traffic y_B is the total load carried on the set of f_B interlink highways between two supergroups.

The step to n-link tdm systems with $n > 3$ can easily be taken. In general, when an n-link tdm system is chosen the B-stage in figure 4 has obviously (n - 2) times to be inserted accordingly into the two link sdm system of figure 2 in order to obtain the equivalent sdm link system. Let

$$(f_B, y_B) , (f_C, y_C) , . . . (f_q, y_q)$$

be the pairs determining number of interlink highways and total traffic in the second, third, . . . , q th stage with regard to one set of interlinks.

The formula for p then extends to

$$p_n = (N-y_A)\ (f_B-y_B/N)\ (f_C-y_C/N)\ \ldots\ (f_q-y_q/N) \cdot k_R + \eta_R \cdot y_A \qquad (9)$$

Thus by equations (3), (6), (7), and (9) n-link tdm systems are described according to the CIRB method.

2.3. Confrontation of CIRB with Artificial Traffic Tests

A first series of tests with artificial traffic has been carried out on an electronic computer to check the accuracy of the CIRB approximations for tdm systems. Results for five different models are shown in the diagrams of figures 5, 6, 7, 8 and 9 (see test points with 95 percent confidence intervals and curves number 1). The numerical test results are compiled in a table behind figure 9. In all cases the number of time channels per highway is 25, the offered traffic Poissonian, and the selection of paths at random. The models are:

a) Two link tdm system (figure 5).

   10 highways in the A-stage are connected by gates to ten highways in the B-stage. CIRB-parameters: $N = 25$, $k_R = 1$, $\eta_R = 0 . 1$, $y_A = y_R$.

b) Three link tdm system with the same traffic on all highways (figure 6).

5 supergroups with 5 highways each in the A-stage are connected by 25 interlink highways to 5 supergroups with 5 highways each in the C-stage. Each highway carries y_A Erlang. CIRB-parameters: $N = 25$, $k_R = 1$, $\eta_R = 1/25$, $f_B = 1$, $y_A = y_B = y_R$.

c) Three link tdm system with small load on interlink highways (figure 7).

5 supergroups with 3 highways each in the A-stage are connected by 25 interlink highways to 5 supergroups with 3 highways each in the C-stage. Each highway in the A- and C-stage carries traffic y_A whereas traffic on one interlink highway is 0.6 y_A. CIRB-parameters: $N = 25$, $k_R = 1$, $\eta_R = 1/15$, $f_B = 1$, $y_A = \frac{5}{3} y_B = y_R$.

d) Three link tdm system with high load on interlink highways (figure 8).

3 supergroups with 5 highways each in the A-stage are connected by 9 interlink highways to 3 supergroups with 5 highways each in the C-stage. Traffic on a highway in the A- and C-stage is 0.6 times the traffic carried on an interlink highway. CIRB-parameters: $N = 25$, $k_R = 1$, $\eta_R = 1/15$, $f_B = 1$, $y_A = \frac{3}{5} y_B = y_R$.

e) Three link tdm system with interlink highways doubled (figure 9).

The same model as under d) but each interlink highway is doubled. CIRB-parameters: $N = 25$, $k_R = 1$, $\eta_R = 1/15$, $f_B = 2$, $y_A = \frac{3}{5} y_B = y_R$ (y_B = total traffic carried on $f_B = 2$ highways).

In spite of the extremely small scope of evaluation which is necessary for CIRB, the calculated values approximate the tested points in all considered cases. The method seems to underestimate the loss for small traffic values. Probably this effect arises because the average number of busy lines has been used instead of taking account of its statistical character that comes to bear in particular for small loads.

Compared to known methods which will be discussed in the next section and the results of which are also shown in fig. 5 to 7 the advantage of CIRB is evident with regard to necessary computation work. Nevertheless the accuracy of the CIRB method is in the investigated cases at least as satisfactory as that of other methods.

3. Comments on Known Approximation Formulae for TDM Systems

TDM switching systems with Poisson input, necessity of coincident time channels for a conversation, and random selection of a path have been studied by various authors. In many cases approximation formulae for the blocking probability have been derived under the assumption of Erlang distributions. Since the CIRB method described in section 2 makes use of Erlang distributions, too, but in a modified version it is worthwhile considering the subsequent effect of this modification on known results.

As a matter of fact the original assumption of an Erlang distribution caused by the actually offered traffic A and yet having as its mean the measurable carried traffic y holds only if there is no internal blocking. For, $E_N(A)$ denoting Erlang's blocking probability for a full available group with N lines and B standing for the real overall blocking probability, the assumption leads to a carried traffic

$$y^* = A(1 - E_N(A)) \qquad (10)$$

which is opposed to the actually carried traffic

$$y = A(1 - B) \qquad (11)$$

unless B is very close to $E_N(A)$.

The idea of the modification is to avoid computing a loss $B > E_N(A)$ by implying a carried load $y^* > y$. For this purpose equation (10) is replaced by

$$y = A_o \left(1 - E_N (A_o)\right) \qquad (12)$$

meaning that a fictitious offered traffic A_o is chosen such that the wanted Erlang distribution has the given mean y while the fundamental equation (11) still holds. The modified approach extends the validity of loss formulae to the region of high losses as a few examples will show.

3.1. Two Link TDM Systems

For two stage sdm link systems with N outlets per multiple a formula has been derived /6, 7/ and tabulated /7/ that holds for equal traffic in both stages when Poisson input, independence of traffic, and random selection are assumed. Blocking probability B is then given by

$$B = (N + 1-A)\, E_N (A) + A \cdot E_N^2 (A) \qquad (13)$$

$E_N(A)$ being Erlang's loss when traffic A is offered to N full available devices.

It has been pointed out /8/ that the result applies likewise to two link tdm systems, a fact which is illustrated by the explicit transformation between two stage tdm and sdm systems as shown in section 2 of this paper. Looking up specific values in the tables of van den Bossche and Knight curve number 2 in figure 5 can be traced. Obviously blocking probability B is too pessimistic with regard to test points at high load. An improved formula is obtained by the following approach.

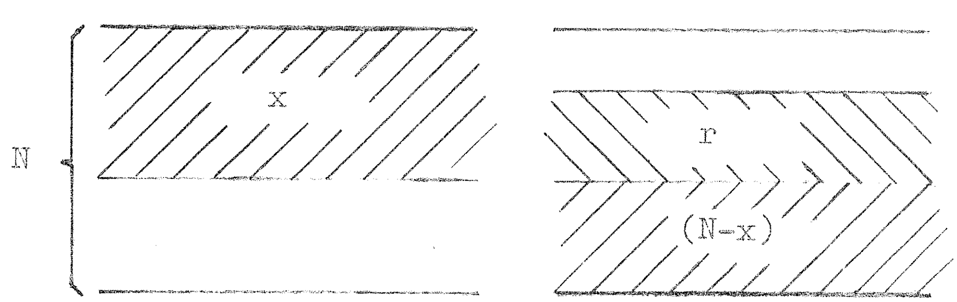

Fig. 10 Blocking State in Two Link TDM System

When the blocking state of figure 10 arises with x out of N time channels busy on the left highway and the remaining (N - x) time channels busy on the right highway, no time channel being simultaneously free on both highways can be found. The distribution for x time channels busy out of N may be written

$$p(x) = \frac{A_o^{\,x}}{x! \sum_{\mu=o}^{N} \frac{A_o^{\mu}}{\mu!}} \qquad (14)$$

with

$$A_o = \frac{y}{1 - E_N(A_o)} \qquad (15)$$

N being the total number of time channels per highway and A_o a fictitious offered traffic causing the actually carried traffic y. On the right highway (N - x) time channels are busy but also r time channels out of those which are already busy in the left highway. Thus with $0 \leq r \leq x$ and $0 \leq x \leq N$ all blocking states are taken into account. The distribution of (N - x + r) busy time channels be

$$p\,(N-x+r) = \frac{A_o^{N-x+r}}{(N-x+r)! \sum_{\mu=o}^{N} \frac{A_o^{\mu}}{\mu!}} \;. \qquad (16)$$

Here again the fictitious offered traffic A_o of equation (15) produces the carried traffic y which is assumed to be the same one on both highways. For fixed parameters x and r only $\binom{x}{r}$ patterns out of $\binom{N}{N-x+r}$ are possible on the right highway and the probability p', that one of these patterns occurs is

$$p' = \frac{\binom{x}{r}}{\binom{N}{N-x+r}} \cdot p\,(N-x+r) \;. \qquad (17)$$

If the traffic handled between the two considered highways can be neglected with regard to traffics from and to other highways the blocking probability B becomes

$$B = \sum_{x=0}^{N} p(x) \sum_{r=0}^{x} \frac{\binom{x}{r}}{\binom{N}{N-x+r}} p(N-x+r) \tag{18}$$

Inserting (14) and (16) in (18) one obtains

$$B = E_N (A_o) \sum_{x=0}^{N} \sum_{r=0}^{x} p(r) \tag{19}$$

Taking into account that

$$\sum_{i=0}^{N} p(i) = 1 \qquad \text{and} \qquad \sum_{i=0}^{N} i \cdot p(i) = y \tag{20}$$

the double sum in (19) can be transformed to $(N + 1 - y)$ and the final result is

$$B = (N + 1 - y) \cdot E_N (A_o) \tag{21}$$

with A_o according to equation (12) whereas the actually offered traffic A follows from equation (11). B tends to unity for increasing values of the offered traffic, since $y \rightarrow N$ and $E_N (A_o) \rightarrow 1$. Starting with y the value of $E_N(A_o)$ can directly be drawn from the MPJ tables [1]. In figure 5 curve number 3 shows in contrast to curve number 2 the achieved improvement towards the test points.

3.2. Three Link TDM Systems

The extension of the unimproved formula (13) to three link tdm systems has been derived under the same assumptions. For the general case that the traffic on the interlink highways in the B-stage is different from the traffic on the highways in the A- and C-stage a diagram has been published [8] out of which the curves number 2 in fig. 6 and 7 have been extracted.

In principle the idea of a fictitious traffic A_o can be introduced here, too, although extensive re-computation of the triple sum in the original formula would be necessary.

Instead, a new interpretation of parameters is proposed to achieve better results.

In the diagram /8, fig.2/ A, B, and C stand for the traffic offered to a highway in the A-, B-, and C-stage respectively. Let us now interpret these parameters as the fictitious offered traffics A_o, B_o, and C_o. Then the traffics carried in each stage per highway can be determined according to equation (12). In a second step the actually offered new traffics A', B', and C' follow with equation (11). Thus in fig. 6 and 7 the curves number 3 have been established. The effect would be even more evident for higher loads for which the original curves number 2, however, were not available. Here, computation based on fictitious traffics would be useful to get results for high loads.

3.3. General TDM Systems

A valuable contribution has been made to the traffic problem in tdm systems by proposing a recursive method for loss computation /9/. Its special advantage is that a great variety of network configurations can be treated. Since this method, too, implies the assumption of Erlang distributions it is suggested to start with the really carried traffic y and to look for an offered traffic A_o that produces y. With A_o the recursive mechanism of the method remains unaffected in its structure. Finally, after the determination of B, the necessary offered traffic A follows from equation (11). The effect of the modification is that especially for high loads the computed blocking probabilities will be reduced thus fitting better reality.

4. Conclusions.

When a familiar distribution is assumed for the calculation of blocking probabilities it is generally recommended to introduce a fictitious offered traffic producing that distribution with the actually carried traffic as its mean. Once the blocking probability is known the really offered traffic can easily be evaluated in a second step . Accuracy of formulae is thus extended to loss values well over 1 percent as it has been demonstrated for various tdm systems.

Results for small loss values remain, however, somewhat pessimistic and establish a superior limit in all considered cases.

On the other hand the Combined Inlet and Route Blocking (CIRB) method can be applied to tdm systems, yielding good approximate results with a small amount of evaluation work. It has been found that the method underestimates the loss probability for small loads. Therefore, in addition to the just mentioned superior limit an inferior limit can also be given, real values of tested tdm systems lying in between.

Bibliography

/1/ A. Lotze, W. Wagner — Table of the Modified Palm-Jacobaeus Loss Formula. Institut für Nachrichtenvermittlung und Datenverarbeitung der Technischen Hochschule Stuttgart, 1962

/2/ A. Lotze — Tables for Overflow Variance Coefficient and Loss of Gradings and Full Available Groups. Institut für Nachrichtenvermittlung und Datenverarbeitung der Technischen Hochschule Stuttgart, 1964

/3/ A. Lotze — Loss Formula, Artificial Traffic Checks and Quality Standards for Characterizing One Stage Gradings. ITC Paris, Sept.1961, Doc. No. 28

/4/ D. Botsch — Internal Traffic and its Effect on the Congestion in Switching Systems. ITC London, July 1964

/5/ N. Bininda, A. Wendt — Die effektive Erreichbarkeit für Abnehmerbündel hinter Zwischenleitungsanordnungen. Nachrichtentechnische Zeitschrift (NTZ) Nr. 11, 1959

/6/ C. Jacobaeus — A Study on Congestion in Link Systems. Ericsson Technics, No. 48, 1950

/7/ M.Van den Bossche, R.G. Knight — Traffic and Blocking Problems in a Three Link Switching System. ITC The Hague, July 1958, Doc. No.2

/8/ H.H. Adelaar, D.G.N. Hunter — Use of "Stantec Zebra" to Calculate a Traffic Table for a Three-Link Time-Division-Multiplex Telephone Exchange. Electrical Communication, Vol.36, No. 3, 1960

/9/ J.G. Van Bosse — Some Recursive Aids for Switching Network Blocking Computations. ITC London, July 1964, Doc. No.21

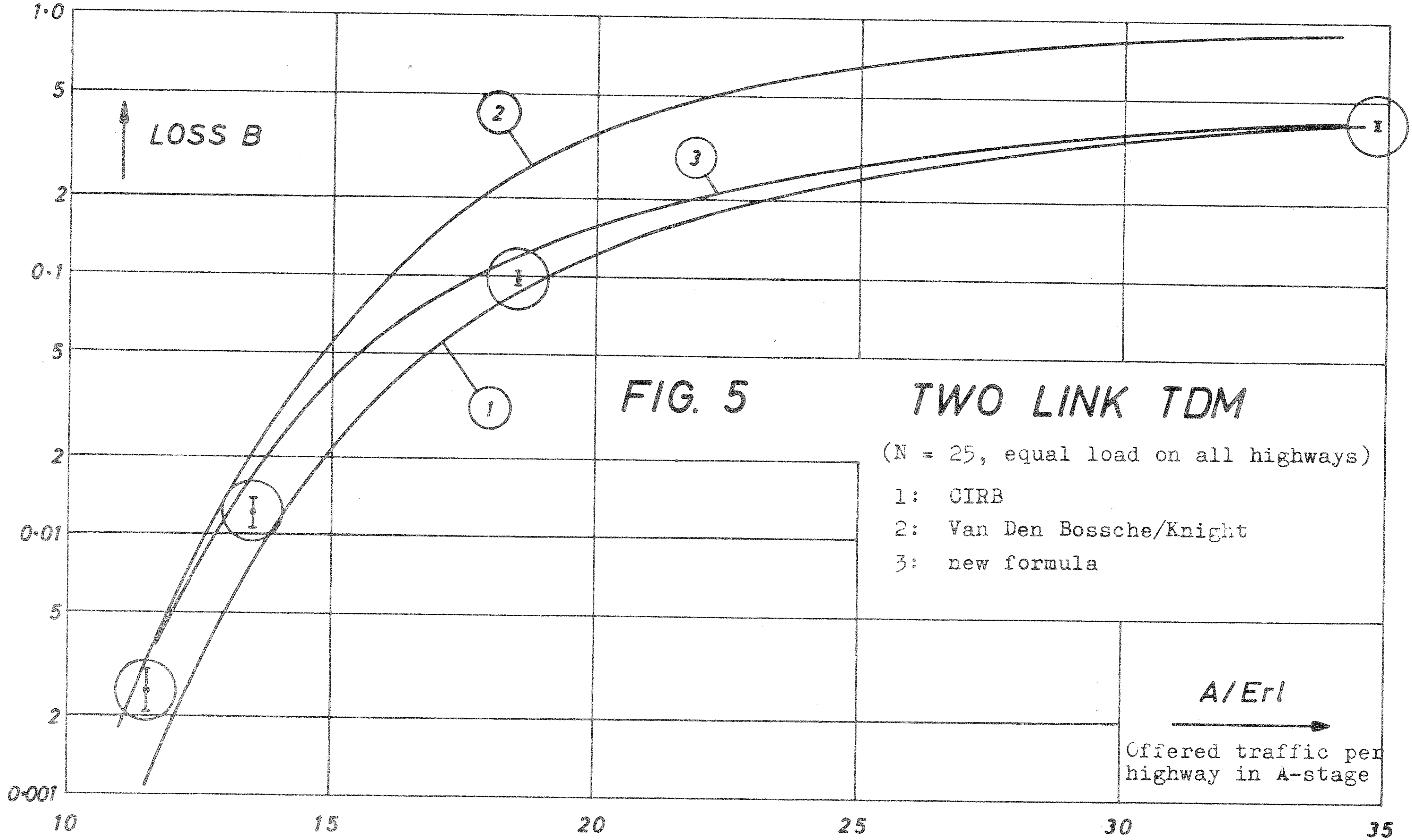
1·0
5
2
0·1
5
2
0·01
5
2
0·001
LOSS B
10
15
20
25
30
35
TWO LINK TDM
(N = 25, equal load on all highways)
1: CIRB
2: Van Den Bossche/Knight
3: new formula
A/Erl
Offered traffic per highway in A-stage

FIG. 5

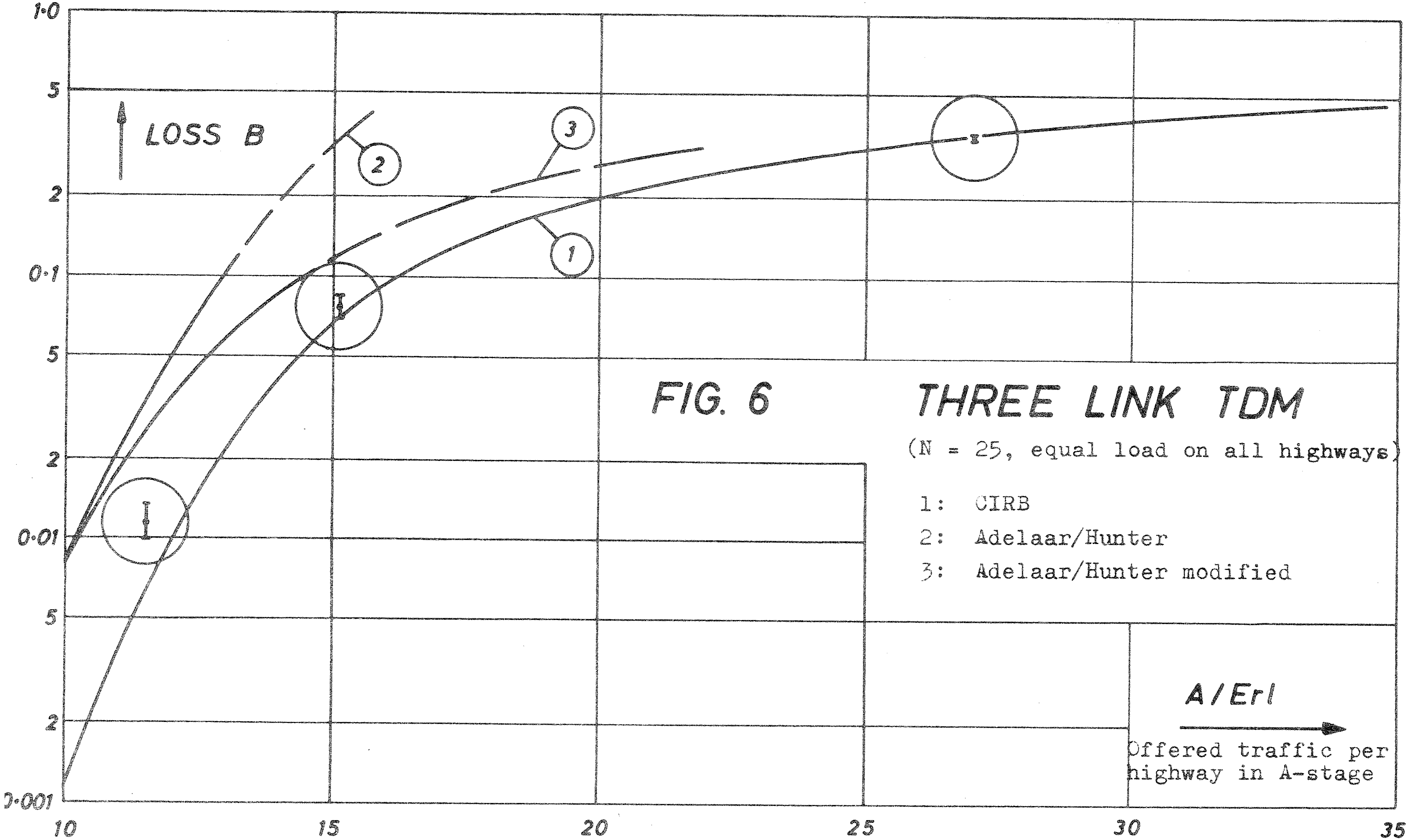

FIG. 6

THREE LINK TDM

(N = 25, equal load on all highways)

1: CIRB

2: Adelaar/Hunter

3: Adelaar/Hunter modified

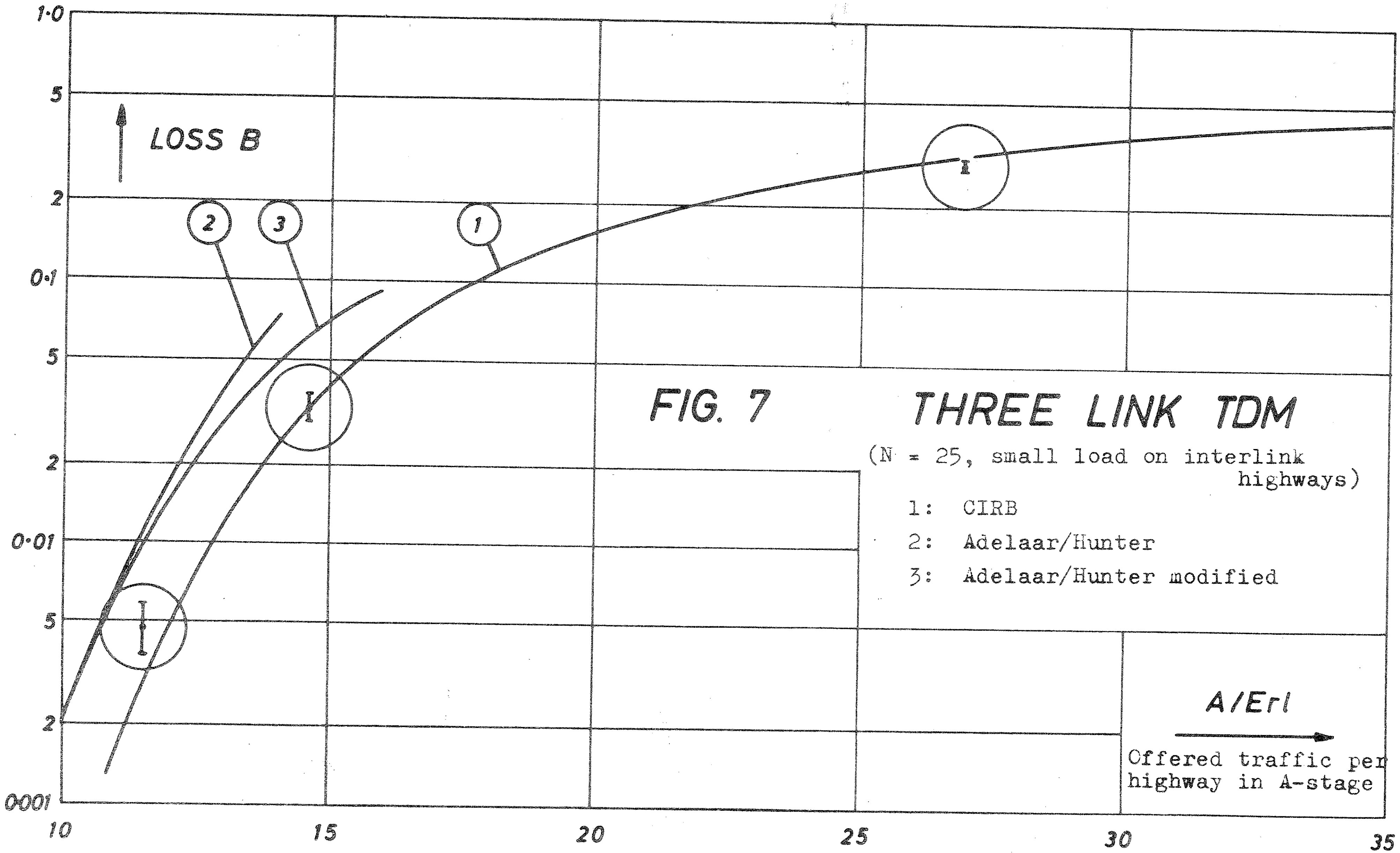
LOSS B
1·0
5
2
0·1
5
2
0·01
5
2
0·001
10
15
20
25
30
35
1
2
3
THREE LINK TDM
(N = 25, small load on interlink highways)
1: CIRB
2: Adelaar/Hunter
3: Adelaar/Hunter modified
A/Erl
Offered traffic per highway in A-stage

FIG. 7

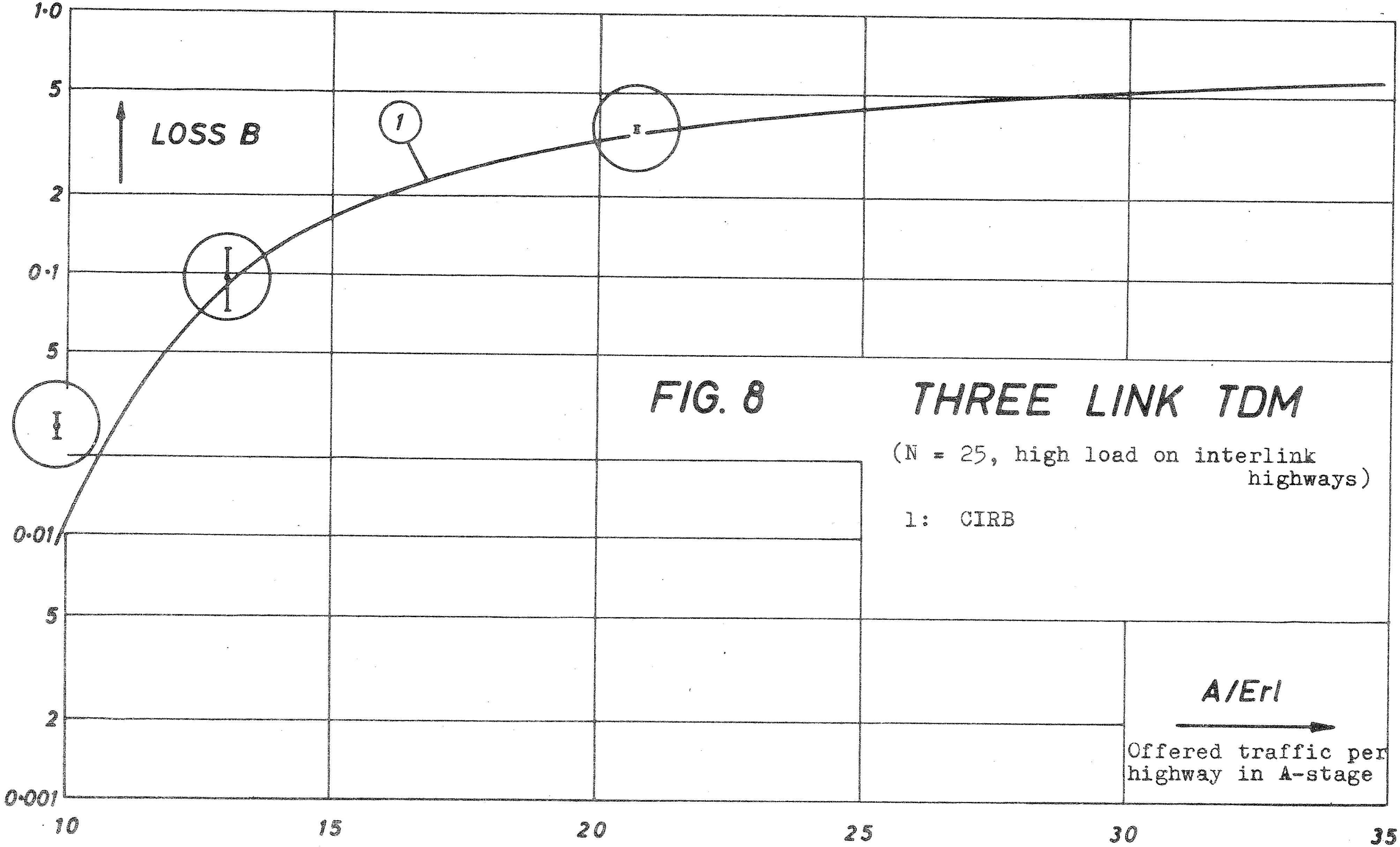

FIG. 8 THREE LINK TDM

(N = 25, high load on interlink highways)

1: CIRB

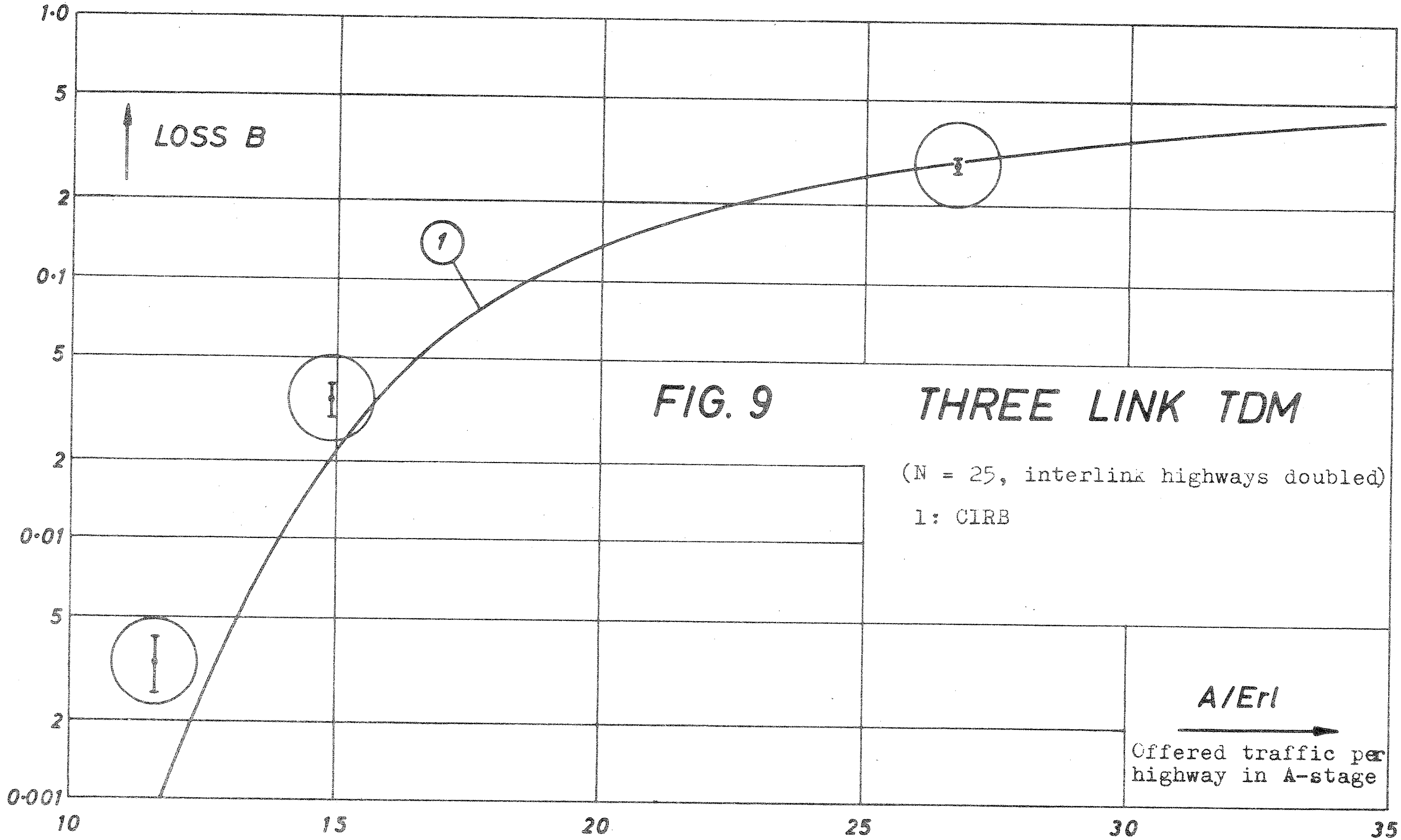

FIG. 9 THREE LINK TDM

(N = 25, interlink highways doubled)

1: CIRB

T A B L E

Numerical Results of TDM Traffic Tests

In columns 5 and 6 the digits $m_1m_2m_3m_4/e$ are an abbreviation for

$$(0\cdot m_1m_2m_3m_4)\cdot 10^{-e} .$$

For loss B the 95% confidence limits based on student's t-distribution are in absolute terms: B +/- ΔB.

1	2	3	4	5	6
No	Model see p. 11	Offered traffic A in erl/highway	Number of calls	Loss B	ΔB
1		34.8	30 000	4083/0	7637/2
2	a	18.5	50 000	9666/1	5198/2
3		13.5	80 000	1201/1	1736/2
4		11.5	160 000	2506/2	4890/3
5		27.0	30 000	3419/0	8164/2
6	b	15.1	40 000	7675/1	7944/2
7		11.5	80 000	1161/1	1774/2
8		26.9	15 000	2901/0	1272/1
9	c	14.6	50 000	3270/1	3635/2
10		11.5	70 000	4714/2	1048/2
11		20.7	20 000	3601/0	1104/1
12	d	13.0	25 000	9668/1	2554/1
13		9.8	50 000	2598/1	3038/2
14		26.7	15 000	2826/0	2013/1
15	e	14.9	30 000	3467/1	5103/2
16		11.6	50 000	3360/2	7799/3