On Computing the Probability of Loss of Two-stage Link Systems with Preselection

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1. Introduction

For computing the probabilities of state of link systems numerical difficulties arise because of the large number of states which have to be considered in systems with a size of practical interest. Published methods (G. Basharin [1], W. Lörcher [2]) for exact calculation of the probability of loss of two-stage link systems are restricted to small systems. This is due to the fact that already small systems give rise to very large sets of linear equations of the probabilities of state.

For link systems in use methods for computing the probability of loss are mostly based on some assumptions which simplify the computation process and still describe the statistical behavior of the system sufficiently accurate. The effects of such assumptions to the calculation of the probability of loss have to be carefully checked by artificial traffic trials. A systematic analysis of methods which imply such assumptions can be found in a paper by K. Kümmerle [3].

In this paper only two-stage link systems for preselection are considered. The method to be described for computing the probability of loss uses a formerly not applied assumption. As the only assumption it is supposed that the probabilities of occupied lines of multiples of the first stage are independent from each other. One can construct special two-stage link systems for which this assumption holds. It is feasible that for other systems this assumption is a very close approximation to reality. By using this assumption the derivation of the probabilities of state is simplified considerably. From the probabilities of state the probability of loss can easily be obtained.

2. Notation and Assumptions

The considered two-stage link systems consist of g_1 multiples of the first stage and g_2 multiples of the second stage. Each multiple has k_1 outlets (first stage) or k_2 outlets (second stage). The total number of outlets of the system is $n = g_2 \cdot k_2$; *Fig. 1.*

Some further notations:

x	number of occupied outgoing trunks,
x_i	number of occupied links of multiple i stage 1,
$p_i(x_i)$	probability for x_i links of multiple i busy,
$p(x_1, x_2, \ldots, x_{g_1})$	probability of the state $\{x_1, x_2, \ldots, x_{g_1}\}$,
c_{A_i}	number of calls offered per unit time to multiple i,
$A_i = c_{A_i} \cdot t_m$	the traffic offered to multiple i.

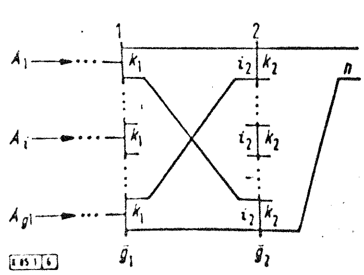

Fig. 1. Two-stage link system

For the link systems studied in this paper the following assumptions are made:

a) Calls originate from infinite sources and offer a pure chance traffic A (Poisson input) to the multiples $(i = 1, 2, \ldots, g_1)$ of the first switching stage. The traffic A is equally distributed to g_1 multiples, such that $A_i = A/g_1$.

b) Random hunting for free links in all stages $(j = 1, 2, \ldots, s)$ and for free outlets is assumed.

c) The holding times are independent from each other and exponentially distributed, $P(>t) = e^{-t/t_m}$, with the mean value of t_m.

d) The probabilities $p_1(x_1)$, $p_2(x_2)$, ..., $p_{g_1}(x_{g_1})$ of occupied lines in the multiples $i = 1, 2, \ldots, g_1$ of the first stage are to be independent from each other.

3. A new Method using Equations of State (ECPL) (Equations of State for the Calculation of two-stage Link Systems for Preselection with Loss)

In order to get the probabilities of state for a system with loss all the transition probabilities from within a state $\{x_1, x_2, \ldots, x_{g_1}\}$ into all neighbour states and the transition probabilities from these neighbour states into the state $\{x_1, x_2, \ldots, x_{g_1}\}$ have to be considered; *Fig. 2.*

3.1. *Transistions by which the State $\{x_1, x_2, \ldots, x_{g1}\}$ disappears*

(a) The transition

$$\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\} \to \{x_1, x_2, \ldots, x_i - 1, \ldots, x_{g_1}\},$$

$S(i) \to S(i-1)$.

The probability for the state $S(i)$ and the transition into the state $S(i-1)$ by termination of a busy trunk in multiple i during the time $(t, t+dt)$ is

$$p(S(i)) \cdot x_i \frac{dt}{t_m} + O(dt). \tag{1}$$

In Eqn. (1) the termination probability of an occuring call during dt is $x_i\, dt/t_m$. In the function $O(dt)$ all terms of higher order are included.

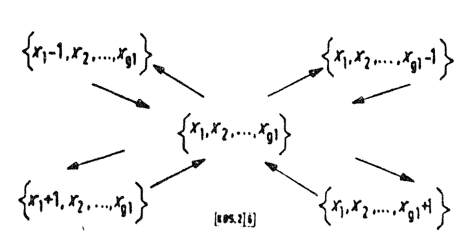

Fig. 2. The neighbour states of $\{x_1, x_2, \ldots, x_{g1}\}$.

(b) The transition

$\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\} \to \{x_1, x_2, \ldots, x_i + 1, \ldots, x_{g_1}\}$,

$S(i) \to S(i+1)$.

The state S(i) will change into the state S(i+1) if an offered call to multiple i can find a free trunk. The transition probability during $(t, t+dt)$ is given by the following expression:

$$p(S(i)) \cdot \mu_1(S(i)) \cdot c_{A_i} \cdot dt + O(dt)\,. \qquad (2)$$

The function $\mu_1(S(i))$ is defined as the passage probability that an incoming call of multiple i in the state S(i) can find a free trunk. If the call is successfull $\mu_1 = 1$ otherwise $\mu_1 = 0$.

3.2. *Transitions by which the State $\{x_1, x_2, \ldots, x_{g1}\}$ originates*

(a) The transition

$\{x_1, x_2, \ldots, x_i + 1, \ldots, x_{g_1}\} \to \{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$,

$S(i+1) \to S(i)$.

A link of multiple i becomes free by a termination of a busy trunk during $(t, t+dt)$. The transition probability can be derived similarly to Eqn. (1):

$$p(S(i+1)) \cdot (x_i + 1) \cdot \frac{dt}{t_m} + O(dt)\,. \qquad (3)$$

(b) The transition

$\{x_1, x_2, \ldots, x_i - 1, \ldots, x_{g_1}\} \to \{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$,

$S(i-1) \to S(i)$.

An offered call to multiple i will find a free trunk with the probability $\mu_1(S(i-1))$. Similar to Eqn. (2) the transition probability becomes

$$p(S(i-1)) \cdot \mu_1(S(i-1)) \cdot c_{A_i} \cdot dt + O(dt)\,. \qquad (4)$$

3.3. *The Equations of State*

If the system is in the state of statistical equilibrium the sum of the transition probabilities from within the state S(i) is equal to the sum of the transition probabilities into the state S(i).

In Eqns. (1, 2, 3, 4) only multiple i was considered. To get all the transitions of the neighbour states the transition probabilities of Eqns. (1, 2, 3, 4) have to be summed up from $i=1$ to $i=g_1$.

$$\begin{aligned}&\sum_{i=1}^{g_1} p(S(i)) \cdot \frac{dt}{t_m} \\ &+ \sum_{i=1}^{g_1} p(S(i)) \cdot \mu_1(S(i)) \cdot c_{A_i}\, dt + O(dt) \\ &= \sum_{i=1}^{g_1} p(S(i+1)\,(x_i+1) \cdot \frac{dt}{t_m} + O(dt) \qquad (5) \\ &+ \sum_{i=1}^{g_1} p(S(i-1)) \cdot \mu_1(S(i-1)) \cdot c_{A_i} \cdot dt.\end{aligned}$$

With the already made assumption of independence of all $p_i(x_i)$ the following equation holds

$$p(x_1, x_2, \ldots, x_{g_1}) = p_1(x_1) \cdot p_2(x_2) \ldots p_{g_1}(x_{g_1}) = \prod_{i=1}^{g_1} p_1(x_i)\,. \qquad (6)$$

For infinite small time intervalls $dt \to o$ the function $O(dt) \to o$. If Eqn. (6) is applied to Eqn. (5) and with $A_i = c_{A_i} \cdot t_m$ one can show that Eqn. (5) can be split into two parts

$$\sum_{i=1}^{g_1} p(S(i)) \cdot x_i = \sum_{i=1}^{g_1} p(S(i-1)) \cdot \mu_1(S(i-1)) \cdot A_i; \qquad (7a)$$

$$\sum_{i=1}^{g_1} p(S(i+1)) \cdot (x_i + 1) = \sum_{i=1}^{g_1} p(S(i)) \cdot \mu_1(S(i)) \cdot A_i. \qquad (7b)$$

From Eqn. (7a) it may be observed that the transition probabilities of Eqn. (1) and Eqn. (4) are equal. The same applies to Eqn. (2) and (3).

Both Eqns. (7a) and (7b) give a set of linear equations from each of which the probabilities of state can be calculated. The number of unknowns can be considerably reduced by using conditions of symmetry together with the effect of Eqn. (6).

Knowing the probabilities of state $p(x_1, x_2, \ldots, x_{g1})$ the probability of loss can be calculated by

$$B = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(S(i) \cdot [1 - \mu_1(S(i))]/g_1 \qquad (8)$$

together with the limiting condition

$$n \geq x = \sum_{i=1}^{g_1} x_i\,. \qquad (9)$$

4. A new Iterative Method (ICPL)

(Iteration Method for the Calculation of two-stage Link Systems for Preselection with Loss)

In this section an interative method will be shown for calculating the probabilities of state and the call congestion. For a system with n outgoing trunks in this method only $(n+1)$ unknowns occur. From Eqn. (7a) or (7b) the probability for

$$x = \sum_{i=1}^{g_1} x_i$$

outgoing trunks busy, $p(x)$, may be obtained from

$$p(x) = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} p(x_1, x_2, \ldots, x_{g_1})\,. \qquad (10)$$

Again Eqn. (9) has to be considered. If the summation of Eqn. (10) is applied to Eqn. (7a) one gets

$$p(x) \cdot x = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(S(i-1)) \times \mu_1(S(i-1)) \cdot A_i\,. \qquad (11)$$

Now, Eqn. (11) is extended on the right hand side by $A \cdot p(x-1)$ and the global passage probability is introduced by

$$\mu(x) = \frac{\sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(S(i)) \cdot \mu_1(S(i)) \cdot A_i}{A \cdot \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} p(x_1, x_2, \ldots, x_{g_1})}\,, \qquad (12)$$

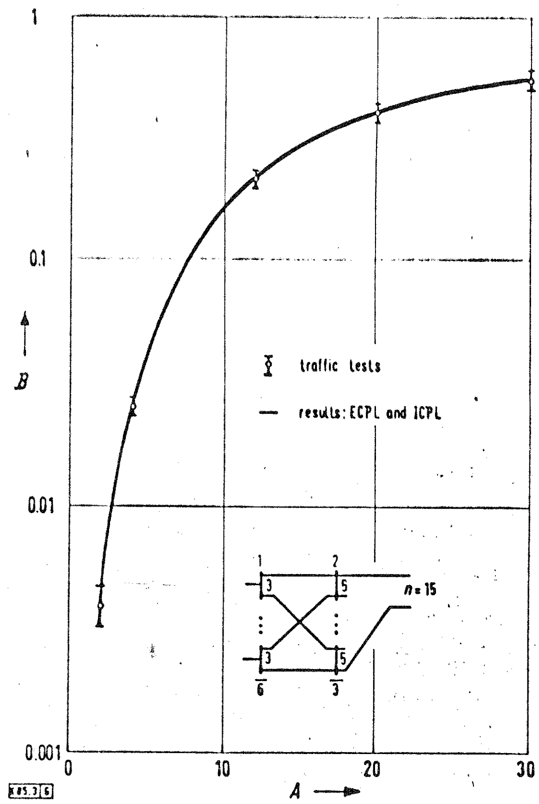

Fig. 3. Probability of loss of a two-stage link system.

so that one gets

$$p(x) \cdot x = A \cdot p(x-1) \cdot \mu(x-1). \qquad (13)$$

Recurrence relation (13) has the same form as the relation for statistical equilibrium of a single-stage system.

From Eqn. (6) the following series can be derived

$$p_1(0) = \sqrt[g_1]{p(0)},$$

$$p_1(1) = \frac{1}{g_1 \cdot p_1(0)^{g_1-1}} \cdot p(1), \qquad (14)$$

$$p_1(2) = \frac{1}{g_1 \cdot p_1(0)^{g_1-1}} \times$$

$$\vdots \qquad \times \left\{p(2) - \frac{g_1!}{(g_1-2)!\,2!} \cdot p_1(0)^{g_1-2} \cdot p_1(1)^2\right\}.$$

With the algorithm given below an iteration cycle is started off with an approximate solution for the probabilities of state. They are improved successively until a given limit of change between two iteration cycles is reached.

(1) The starting distribution of $p(x)$ is assumed.
(2) Using Eqn. (14) all $p_1(x_1)$ are calculated.
(3) With $p_1(x_1)$ and Eqn. (6) the passage probability $\mu(x)$ can be evaluated.
(4) Recurrence relation (13) allows to calculate all $p(x)$.
(5) The steps (2)–(4) are executed until in the v-th cycle a given limit ε e.g.

$$|p_v(0) - p_{v-1}(0)| < \varepsilon$$

holds.

Having got approximate values for $p(x)$ and $\mu(x)$ the probability of loss can be evaluated:

$$B = \sum_{x=k_1}^{n} p(x) \cdot (1 - \mu(x)). \qquad (15)$$

5. Numerical Results

The methods ECPL and ICPL were used to compute the probability of loss for a number of systems. For both methods the numerical results are almost identical. A comparison with exact solutions according to G. Basharin [1] for small systems proved agreement of three significant figures. For bigger systems results of artificial trials show unusually close agreement with values obtained by ECPL and ICPL.

As an example *Fig. 3* shows curves computed by ECPL and ICPL for a two-stage link system together with results of simulation. The confidence intervall of all tests is 95 %.

References

[1] *Basharin, G. P.*: Derivation of equations of state for two-stage telephone circuits with losses. Telecommunications (1960) pp. 79 to 90.

[2] *Lörcher, W.*: Exact calculation for the probability of loss for two-stage link systems with group selection. Arch. Elektron. Übertr. techn. 25 (1971) H. 10/11.

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Dissertationen

Zur Wellenausbreitung in Glasfaser-Lichtwellenleitern. Von O. Krumpholz. Dissertation TH Karlsruhe (20. 1. 1971). Berichter: Prof. Dr. techn. G. K. Grau; Mitberichter: Prof. Dr. rer. nat. H. Friedburg.

In der Arbeit wird gezeigt, daß bei der Herstellung von ummantelten Glasfaser-Lichtwellenleitern Kern- und Mantelglas teilweise ineinander diffundieren. Dies bewirkt eine Verbreiterung der Intensitätsverteilung des HE_{11}-Grundmodus in der Faser. Durch eine nochmalige Erwärmung der Faser bis in die Nähe des Transformationspunktes kann das Ineinanderdiffundieren der beiden Gläser weiter vorangetrieben werden, wodurch sich das Intensitätsprofil der HE_{11}-Grundwelle abermals verbreitert. Die verbreiterte Leistungsverteilung der Welle in der Faser hat, wie theoretisch und experimentell gezeigt wird, eine Verschmälerung der Strahlungskeule und eine Erhöhung des Einkoppelwirkungsgrades zur Folge.

Harmonische Balance für eine Klasse von Systemen mit örtlich verteilten Parametern. Von Dieter Franke. Dissertation Universität (TH) Karlsruhe, 1971. Berichter: Professor Dr. rer. nat. O. Föllinger; Mitberichter: Professor Dr. phil. nat. G. Schneider.

Zahlreiche Systeme mit örtlich verteilten Parametern weisen eine dem nichtlinearen Standardregelkreis mit konzentrierten Parametern ähnliche Struktur auf. Das lineare Teilsystem wird durch eine lineare partielle Differentialgleichung beschrieben, die Rückkopplung erfolgt über einen nichtlinearen Operator. Läßt sich die Greensche Funktion des linearen Teilsystems in eine genügend rasch konvergierende bilineare Reihe nach Eigenfunktionen entwickeln, so kann man die Ruhezustände des Systems näherungsweise durch Balance der ersten räumlichen Harmonischen (Eigenfunktion) ermitteln, die Schwingungszustände durch Balance der ersten zeitlichen und räumlichen Harmonischen. Die hierzu erforderlichen Gleichungen der zeit- und ortsharmonischen Balance erhält man durch eine Erweiterung des Begriffs der Beschreibungsfunktion. (1534)