ABOUT MULTI-STAGE LINK SYSTEMS WITH QUEUING

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1. Summary and Introduction

For economical reasons modern switching networks for telephone arrangement are often built up as multi-stage arrays. Most of them are controlled by conjugate selection i.e. links are only occupied if they can reach a free outlet. Such systems are called link systems.

For link systems with loss a large number of procedures is known to calculate time congestion and call congestion. An analysis of these methods can be found in a paper by K. Kümmerle [1].

Very little is known about multistage link systems with waiting. So far only some studies of waiting problems in two-stage link systems have been made by E. Gambe [3]. This study deals with link systems where incoming calls can wait in front of the multiples of the first stage. The number of calls in these queues is assumed to be not limited.

In delay systems the most important quantities for characterising traffic problems is the probability of delay and the mean waiting time. These quantities can be obtained from the probabilities of state of such systems.

In this paper a number of methods is presented to calculate the probability of delay and the mean waiting time for link systems with waiting consisting of two and more stages. It will be shown how methods for calculating call congestion of link systems with loss can be generalised and applied to link systems with waiting.

Artificial traffic trials are used to prove the formulae derived.

2. Assumptions

For the link systems studied in this paper the following assumptions are made:

1. Calls originate from infinite sources and offer a pure chance traffic A (Poisson input) to the multiple $(i=1,2,\dots,g_1)$ of the first switching stage. The traffic A is equally distributed to g_1 multiples, such that $A_i=A/g_1$.
2. In case of internal blocking or all outlets busy an incoming call enters the queue in front of the corresponding multiple of the first stage.
3. As soon as one of the occupied outlets becomes free one of the waiting calls without internal blocking to a free outlet is selected at random and is set up.
4. Random hunting for free links in all stages $(j=1,2,\dots,s)$ and for free outlets is assumed.
5. All waiting calls remain in the queue until they are served. The number of queuing calls is not limited.
6. The holding times are independent from each other and exponentially distributed, $P(>t)=e^{-t/t_m}$, with the mean value of t_m.

3. Two-stage Link Systems for Preselection

In this section a method will be shown which can be applied to two-stage link systems with preselection and loss. Later, this method is generalised such that it can also be applied to system with waiting. There are special structures of two-stage systems for which this method yields exact solutions. For other practical systems extremely close solutions are obtained as compar with the results of artificial traffic trials.

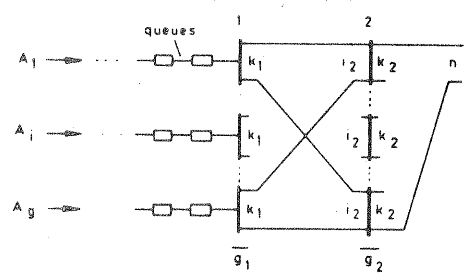


Fig. 1 Two-stage link system

Apart from the assumptions about the traffic and the serving disciplin only the following restriction is made:
The probabilities $p_1(x_1), p_2(x_2), \dots, p_{g_1}(x_{g_1})$ of occupied lines in the multiples $i=1,2,\dots,g_1$ of the first stage are to be independent from each other.

Some further notations

x	number of occupied outgoing trunks
x_i	number of occupied links of multiple i stage 1
$p_i(x_i)$	probability for x_i links of multiple i busy
$p(x_1,x_2,\dots,x_{g_1})$	probability of the state $\{x_1,x_2,\dots,x_{g_1}\}$
c_{A_i}	number of calls offered per unit time to multiple i
$A_i=c_{A_i}\cdot t_m$	the traffic offered to multiple i.

3.1 Link Systems with Loss

3.11 A new Method using Equations of State (ECPL) (Equations of State for the Calculation of two-stage Link Systems for Preselection with Loss)

In order to get the probabilities of state for a system with loss all the transition probabilitie from within a state $\{x_1,x_2,\dots,x_{g_1}\}$ into all neighbour states and the transition probabilitie from these neighbour states into the state $\{x_1,x_2,\dots,x_{g_1}\}$ have to be considered.

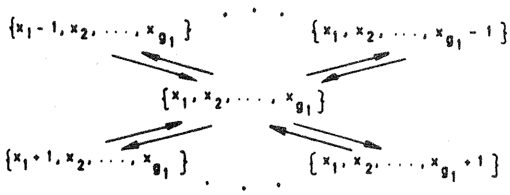


Fig. 2 The neighbour states of $\{x_1, x_2, \ldots, x_{g_1}\}$

Transitions by which the state $\{x_1, x_2, \ldots, x_{g_1}\}$ disappears

(a) The transition

$\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\} \rightarrow \{x_1, x_2, \ldots, x_i - 1, \ldots, x_{g_1}\}$

The probability for the state $\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$ and the transition into the state $\{x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}\}$ by termination of a busy trunk in multiple i during the time (t,t+dt) is

$$p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1})\, x_i \frac{dt}{t_m} + o(dt) \qquad (1)$$

In eq. (1) the termination probability of an occuring call during dt is $x_i \frac{dt}{t_m}$. In the function o(dt) all terms of higher order are included.

(b) The transition

$\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\} \rightarrow \{x_1, x_2, \ldots, x_i+1, \ldots, x_{g_1}\}$

The state $\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$ will change into the state $\{x_1, x_2, \ldots, x_i+1, \ldots, x_{g_1}\}$ if an offered call to multiple i can find a free trunk. The transition probability during (t,t+dt) is given by the following expression:

$$p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot \mu_1(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot c_{A_i} \cdot dt + o(dt) \qquad (2)$$

The function $\mu_1(x_1, x_2, \ldots, x_i, \ldots, x_{g_1})$ is defined as the passage probability that an incoming call of multiple i in the state $\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$ can find a free trunk. If the call is successfull $\mu_1=1$ otherwise $\mu_1=0$.

Transitions by which the state $\{x_1, x_2, \ldots, x_{g_1}\}$ originates

(a) The transition

$\{x_1, x_2, \ldots, x_i+1, \ldots, x_{g_1}\} \rightarrow \{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$

A link of multiple i becomes free by a termination of a busy trunk during (t,t+dt). The transition probability can be derived similarly to eq(1).

$$p(x_1, x_2, \ldots, x_i+1, \ldots, x_{g_1})(x_i+1) \cdot \frac{dt}{t_m} + o(dt) \qquad (3)$$

(b) The transition

$\{x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}\} \rightarrow \{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$

An offered call to multiple i will find a free trunk with the probability $\mu_1(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1})$. Similar to eq. (2) the transition probability becomes

$$p(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}) \cdot \mu_1(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}) \cdot c_{A_i} \cdot dt + o(dt) \qquad (4)$$

The Equations of State

If the system is in the state of statistical equilibrium the sum of the transition probabilities from within the state $\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$ is equal to the sum of the transition probabilities into the state $\{x_1, x_2, \ldots, x_i, \ldots, x_{g_1}\}$.

In eq.(1),(2),(3),(4) only multiple i was considered. To get all the transitions of the neighbour states the transition probabilities of eq.(1),(2),(3),(4) have to be summed up from i=1 to $i=g_1$.

$$\sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot x_i \cdot \frac{dt}{t_m} \qquad (5)$$

$$+ \sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot \mu_1(x_1, x_2, \ldots, x_i, \ldots, x_{g_1})\, c_{A_i}\, dt + o(dt)$$

$$= \sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i+1, \ldots, x_{g_1}) \cdot (x_i+1) \cdot \frac{dt}{t_m} + o(dt)$$

$$+ \sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}) \cdot \mu_1(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1})\, c_{A_i} \cdot dt$$

With the already made assumption of independence of all $p_i(x_i)$ the following equation holds

$$p(x_1, x_2, \ldots, x_{g_1}) = p_1(x_1) \cdot p_2(x_2) \ldots p_{g_1}(x_{g_1}) = \prod_{i=1}^{g_1} p_i(x_i) \qquad (6)$$

For infinite small time intervalls dt → o the function o(dt) → o. If eq.(6) is applied to eq.(5) and with $A_i = c_{A_i} \cdot t_m$ one can show that eq.(5) can be split into two parts

$$\sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot x_i \qquad (7a)$$

$$= \sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}) \cdot \mu_1(x_1, x_2, \ldots, x_i-1, \ldots, x_{g_1}) \cdot A_i$$

$$\sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i+1, \ldots, x_{g_1})(x_i+1) \qquad (7b)$$

$$= \sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot \mu_1(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot A_i$$

From eq.(7a) it may be observed that the transition probabilities of eq.(1) and eq.(4) are equal. The same applies to eq.(2) and (3).

Both eq.(7a) and (7b) give a set of linear equations from each of which the probabilities of state can be calculated. The number of unknowns can be considerably reduced by using conditions of symmetry together with the effect of eq.(6).

Knowing the probabilities of state $p(x_1, x_2, \ldots, x_{g_1})$ the call congestion can be calculated by

$$B = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}) \cdot (1 - \mu_1(x_1, x_2, \ldots, x_i, \ldots, x_{g_1}))/g_1 \qquad (8)$$

together with the limiting condition

$$\sum_{i=1}^{g_1} x_i = x \leq n \qquad (9)$$

3.12 A new iterative Method (ICPL)

(Iteration Method for the Calculation of two-stage Link Systems for Preselection with Loss)

In this section an iterative method will be shown for calculating the probabilities of state and the call congestion. For a system with n outgoing trunks in this method only (n+1) unknowns occur. From (7a) or (7b) the probability for $x = \sum_{i=1}^{g_1} x_i$ outgoing trunks busy, p(x), may be obtained from

$$p(x) = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} p(x_1, x_2, \ldots, x_{g_1}) \qquad (10)$$

Again eq.(9) has to be considered. If the summation of eq.(10) is applied to eq.(7a) one gets

$$p(x)\cdot x = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i-1,\ldots,x_{g_1})\cdot \mu_1(x_1,x_2,\ldots,x_i-1,\ldots,x_{g_1}) A_i \qquad (11)$$

Now, equation (11) is extended on the right hand side by $A \cdot p(x-1)$ and the global passage probability is introduced by

$$\mu(x) = \frac{\sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\cdot \mu_1(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\cdot A_i}{A \cdot \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} p(x_1, x_2, \ldots, x_{g_1})} \qquad (12)$$

so that one gets

$$p(x)\cdot x = A \cdot p(x-1) \cdot \mu(x-1) \qquad (13)$$

Recurrence relation (13) has the same form as the relation for statistical equilibrium of a single-stage system.

From eq.(6) the following series can be derived

$$p_1(o) = \sqrt[g_1]{p(o)}$$

$$p_1(1) = \frac{1}{g_1 \cdot p_1(o)^{g_1-1}} \cdot p(1) \qquad (14)$$

$$p_1(2) = \frac{1}{g_1 \cdot p_1(o)^{g_1-1}} \cdot \left\{ p(2) - \frac{g_1!}{(g_1-2)!\,2!} \cdot p_1(o)^{g_1-2} \cdot p_1(1)^2 \right\}$$

$$\vdots$$

With the algorithm given below an iteration cycle is started off with an approximate solution for the probabilities of state. They are improved successively until a given limit of change between two iteration cycles is reached.

(1) The starting distribution of $p(x)$ is assumed

(2) Using eq.(14) all $p_1(x_1)$ are calculated

(3) With $p_1(x_i)$ and eq.(6) the passage probabilitiy $\mu(x)$ can be evaluated.

(4) Recurrence relation (13) allows to calculate all $p(x)$.

(5) The steps (2)-(4) are executed until in the v-th cycle a given limit ε e.g.

$$|p_v(o) - p_{v-1}(o)| < \varepsilon$$

holds.

Having got approximate values for $p(x)$ and $\mu(x)$ the call congestion can be evaluated.

$$B = \sum_{x=k_1}^{n} p(x)\cdot(1-\mu(x)) \qquad (15)$$

3.2 Systems with Waiting

3.21 A new Method using Equations of State (ECPW) (Equations for the Calculation of two-stage Link Systems for Preselection with Waiting)

In a similar way as in section 3.11 the probabilities of state are found by considering all transitions from and to a state $\{x_1, x_2, \ldots, x_{g_1}, z\}$.

In addition to section 3.11 a new dimension, the number of waiting calls z, has to be included. With the assumption, that the probabilities $p_1(x_1), p_2(x_2) \ldots$ are independent from each other the equations of state can be simplified similarly to section 3.11. This implies that in the state of statistical equilibrium the transition probability caused by termination of a busy trunk out of the state $\{x_1, x_2, \ldots, x_{g_1}, z\}$ is equal to the transition probability into the state $\{x_1, x_2, \ldots, x_{g_1}, z\}$ due to an offered call.

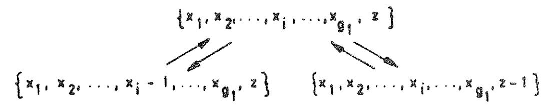


Fig. 3 The equally probable transitions

From fig. 3 the equations of state are derived similarly to section 3.11 (cf. L. Hieber [6])

$$\sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1},z)\cdot x_i \qquad (16)$$

$$= \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1},z-1)\,\sigma_1(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\,A_i$$

$$+ \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i-1,\ldots,x_{g_1},z)\,\mu_1(x_1,x_2,\ldots,x_i-1,\ldots,x_{g_1})\,A_i$$

where $\sigma_1(x_1,x_2,\ldots,x_{g_1}) = 1 - \mu_1(x_1,x_2,\ldots,x_{g_1})$

For this recurrence relation the following boundary conditions are valid

$$x_i = 1,2,\ldots,k_1 \qquad \sum_{i=1}^{g_1} x_i \le n$$

$$z = o,1,\ldots \qquad (17)$$

One gets the probability $p(x_1, x_2, \ldots, x_{g_1})$ by the following summation

$$p(x_1,x_2,\ldots,x_{g_1}) = \sum_{z=o}^{\infty} p(x_1,x_2,\ldots,x_{g_1},z) \qquad (18)$$

If eq.(16) is summed up in the same way as eq.(18) one obtains with eq.(17) the following recurrence relation

$$\sum_{i=1}^{g_1} x_i\, p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1}) \qquad (19)$$

$$= \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\,\sigma_1(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\cdot A_i$$

$$+ \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\cdot \mu_1(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\cdot A_i$$

From eq.(19) a set of linear equations can be obtained for the unknown probabilities $p(x_1, x_2, \ldots, x_{g_1})$. The number of unknowns can be drastically reduced by using conditions of symmetry.

With $p(x_1, x_2, \ldots, x_{g_1})$ and $\sigma_1(x_1, x_2, \ldots, x_{g_1})$ the probability of delay can be evaluated by

$$W = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{i=1}^{g_1} p(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})\cdot\sigma_1(x_1,x_2,\ldots,x_i,\ldots,x_{g_1})/g_1 \qquad (20)$$

For calculating the mean waiting time the waiting traffic Ω has to be known. The waiting traffic Ω is defined as the average number of calls waiting simultaneously.

$$\Omega = \sum_{x_1=0}^{k_1} \sum_{x_2=0}^{k_1} \cdots \sum_{x_{g_1}=0}^{k_1} \sum_{z=1}^{\infty} z \cdot p(x_1,x_2,\ldots,x_{g_1},z) \qquad (21)$$

If eq.(19) is multiplied by z and the abbreviation

$$\omega(x_1,x_2,\dots,x_{g_1}) = \sum_{z=1}^{\infty} z \cdot p(x_1,x_2,\dots,x_{g_1},z)$$

s used, one obtains

$$\omega(x_1,x_2,\dots,x_{g_1}) \tag{22}$$

$$= \frac{1}{\sum_{i=1}^{g_1}(x_i-\sigma_i(x_1,x_2,\dots,x_i,\dots,x_{g_1}))}\Big\{\sum_{i=1}^{g_1}\omega(x_1,x_2,\dots,x_i-1,\dots,x_{g_1})\mu_i(x_1,x_2,\dots,x_i-1,\dots,x_{g_1})$$

$$+p(x_1,x_2,\dots,x_{g_1})\sum_{i=1}^{g_1}\sigma_i(x_1,x_2,\dots,x_i,\dots,x_{g_1})\Big\}$$

he waiting traffic can now be calculated

$$\Omega = \sum_{x_1=0}^{k_1}\sum_{x_2=0}^{k_1}\cdots\sum_{x_{g_1}=0}^{k_1}\omega(x_1,x_2,\dots,x_{g_1}) \tag{23}$$

or eq.(23) the condition $n \geq x = \sum_{i=1}^{g_1} x_i$ is valid.

he waiting traffic Ω can also be defined by

$$\Omega = c_W \cdot t_W \tag{24}$$

here t_W is the mean waiting time of delayed calls nd c_W is the average number of delayed calls ffered per unit time. The standardised mean aiting time of delayed calls is

$$\tau_W = \frac{t_W}{t_m} = \frac{\Omega}{A \cdot W} \tag{25}$$

.22 A new iterative Method (ICPW)

Iteration Method for the Calculation of two-tage Link Systems for Preselection and aiting)

t first some transformations on the equations of tate (cf.eq.(19)) are made such that one obaines a similar form as for the state of tatistical equilibrium of a single-stage system ith unlimited waiting.

gain the global passage probability, already efined by eq.(12) and the global congestion robability $\sigma(x)=1-\mu(x)$, will be used.

t is already known that the probability of x utgoing trunks busy can be obtained by

$$p(x) = \sum_{x_1=0}^{k_1}\sum_{x_2=0}^{k_1}\cdots\sum_{x_{g_1}=0}^{k_1} p(x_1,x_2,\dots,x_{g_1}) \tag{26}$$

ith the condition

$$n \geq x = \sum_{i=1}^{g_1} x_i$$

The summation of eq.(26) is applied to the recurrence relation (19). With eq.(26) one obtains

$$x \cdot p(x) = \sum_{x_1=0}^{k_1}\sum_{x_2=0}^{k_1}\cdots\sum_{x_{g_1}=0}^{k_1}\sum_{i=1}^{g_1}\big(p(x_1,x_2,\dots,x_i,\dots,x_{g_1})\cdot\sigma_i(x_1,x_2,\dots,x_i,\dots,x_{g_1})\cdot A_i \tag{27}$$

$$+\, p(x_1,x_2,\dots,x_i-1,\dots,x_{g_1})\cdot\mu_i(x_1,x_2,\dots,x_i-1,\dots,x_{g_1})\cdot A_i\big)$$

The right hand side of eq.(27) is multiplied by $A \cdot p(x-1)$. Using $\sigma(x)$ and $\mu(x)$ eq.(27) becomes

$$p(x)\,\{x - \sigma(x)\,A\} = A \cdot p(x-1)\cdot\mu(x-1) \tag{28}$$

Eq.(28) has in fact the same form as the recurrence relation for single-stage delay systems with limited accessibility derived by M. Thierer [4].

With eq.(12),(14) and (28) a similar iteration algorithm as shown in section 3.11 can be derived. This iteration algorithm leads to an approximate solution for the probabilities of state. The probability of delay W and the mean waiting time τ_W can then be evaluated according to eq.(20) and (25).

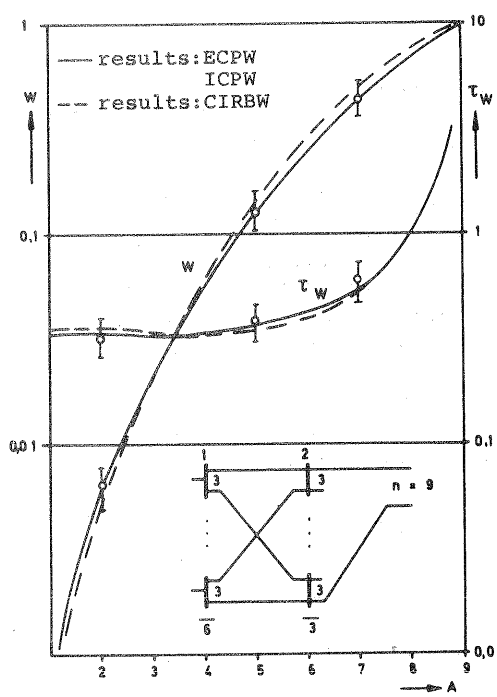
The global passage probability $\mu(x)$ according to eq.(12) does not depend on the number of waiting calls in the system.

The derivation of the recurrence relation (28) for systems with waiting is very similar to the derivation of eq. (13), section 3.11. Eq.(13) is the recurrence relation for two-stage systems with loss.

It is obvious that the iteration method for loss systems of section 3.11 can be adapted for the calculation of W and τ_W of delay systems just by interchanging the corresponding recurrence relation of the state of statistical equilibrium.

In the next two sections this principle will be applied to already known methods for calculating call congestion in loss systems such that they can be used for delay systems as well.

Artificial traffic trials were performed to investigate the accuracy of ECPW and ICPW. For the system of diagr.1 results of ECPW and ICPW are almost identical. The confidence intervall of all tests is 95 %.



Diagr.1 Probability of delay W and mean waiting time τ_W of a two-stage link system with waiting

4. A new Method for two-stage Link Systems with Group Selection and Waiting (ICGW)

(Iteration Method for the Calculation of 2-stage Link Systems for Group Selection and Waiting)

An iterative method for calculating W and τ_W of two-stage delay link-systems with group selection will be derived. The iteration algorithm for obtaining the probabilities of state is similar to the algorithm for loss system given by A. Elldin [2] and K. Kümmerle [1].

The state x_1 link-lines busy of a considered link-unit (cf. Fig. 4) may be denoted by $\{x_1\}$

and the state x_2 outgoing trunks occupied by $\{x_2\}$; the corresponding probabilities are $p(x_1)$ and $p(x_2)$. The probability distributions $p(x_1)$ and $p(x_2)$ are dependent on each other

$$p(x_1)=f(p(x_2))\ ,\ p(x_2)=g(p(x_1)) \qquad (29)$$

The passage probability $\mu(x_1)$ is defined as the probability that a call ariving in the state $\{x_1\}$ can be connected to one of the n_2 trunks. If the probabilities $p(x_2)$ are known then $\mu(x_1)$ can be calculated by a combinatorial statement. The probability that an offered call in the state $\{x_2\}$ can be set up is denoted by $\mu^*(x_2)$. Again $\mu^*(x_2)$ can be evaluated if the probabilities $p(x_1)$ are known.

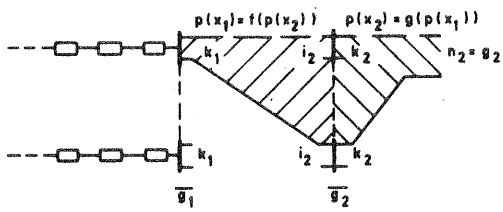


Fig. 4 Link unit

For the approximate evaluation of $p(x_1), p(x_2)$ and $\mu(x_1)$, $\mu^*(x_2)$ with a prescribed accuracy the following algorithm can be used:

(a) Starting values for $p(x_1)$ are obtained according to the formula $E_{2,k_1}(A_1)$ by A.K. Erlang.

(b) With $p(x_1)$ and a combinatorial statement the values of $\mu^*(x_2)$ are calculated.

(c) Statistical equilibrium for single-stage delay system with $\mu^*(x_2)$ and the traffic $A_2 = Y_2$ is used to get the probabilities $p(x_2)$.

(d) With $p(x_2)$ and a combinatorial statement, all $\mu(x_1)$ are evaluated.

(e) Statistical equilibrium for a single-stage delay system with $\mu(x_1)$ and the traffic $A_1=Y_1$ is used to get improved probabilities $p(x_1)$.

(f) The steps (b), (c), (d), (e) are repeated until after the v-th iteration a prescribed accuracy ε is reached.

$$|p_v(x_1)-p_{v-1}(x_1)| < \varepsilon \quad \text{for } x_1=0,1,\ldots,k_1$$

Using $p(x_1)$ and $\mu(x_1)$ the probability of delay can be calculated

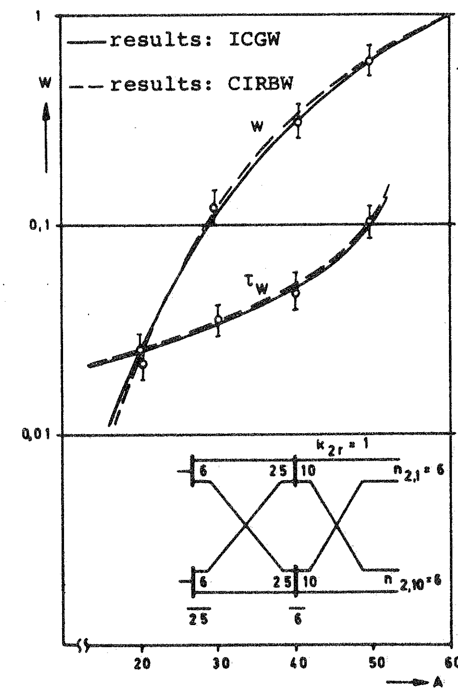
$$W = \sum_{x_1=0}^{k_1} p(x_1)(1-\mu(x_1)) \qquad (30)$$

From the values of W, A_1, k_1 one can evaluate an availability k_{eff} of a corresponding single-stage delay system with the same values W, A_1, k_1. For such a virtual system with the same tupel (A_1, k_1, k_{eff}) the mean waiting time τ_W can be calculated by using the Interconnection Delay Formula (IDF) by M. Thierer [4].

$$\tau_W=\tau_{W_{IDF}}(k_1, k_{eff}, A_1) \qquad (31)$$

The values of k_{eff} can only be integer. For values of W which give rise to an availability k_{eff} which is not of type integer, the value of τ_W has to be found by interpolation.

Diagr. 2 shows curves calculated by using the method ICGW and results of simulation.



Diagr. 2 Probability of delay and mean waiting time of a two-stage link system with group selection

5. The new Method CIRBW for multi-stage Link Systems

The method described in this section can be applied to delay systems with an arbitrary number of selector stages with preselection or group selection. Gradings are allowed between the selector stages and behind the last stage (cf. Fig.5).

A similar method for calculating call congestion and time congestion of multi-stage link systems with loss is known as the method of combined inlet-route blocking CIRB, by A. Lotze [5]. Accordingly, the method to be derived here may be called combined inlet-route blocking with waiting, CIRBW.

As the name indicates the calculation of W and τ_W is divided into two parts: inlet blocking and outlet blocking.

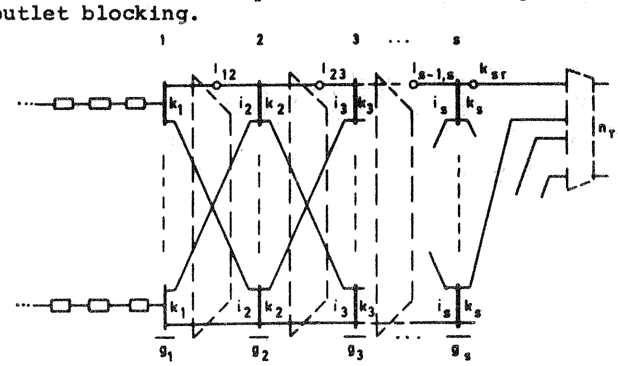


Fig.5 Multi-stage link system with grading and waiting

It is assumed that the probability distribution of a full access trunk group with waiting exists in the multiples of the first stage. It is also assumed, that for the outgoing trunk groups a probability distribution exists which is of the same type as that of a single-stage delay system with limited accessibility. Both the probability distribution of the multiples of the first stage and the distribution of the outgoing trunk groups are to be independent from each other.

Inlet blocking occurs, if an offered call finds all k_1 lines of a multiple of the first stage busy. Inlet blocking can be evaluated by Erlang's formula $E_{2,K_1}(Y_1)$. To calculate route blocking the average number m_r of those lines in the desired route r which can be hunted "free" or "busy" is used. For the simplest case that from one multiple of stage j only one link leads to each of the multiples of stage j+1, j=1,2,...,s-1, the average hunting number is

$$m_r = \sum_{j=1}^{s-1} (k_j - Y_j) \cdot k_{sr} + n_r \cdot Y_1 \qquad (32)$$

$$m_r \leq n_r$$

From m_r, the number of trunks n_r in group r and the traffic $A_r = Y_r$ the route blocking can be obtained by using the Interconnection Delay Formula (IDF).

Because inlet blocking and route blocking are to be independent the probability of delay is

$$W = E_{2,k_1}(Y_1) + \{1 - E_{2,k_1}(Y_1)\} \cdot W_{IDF}(n_r, m_r, Y_r) \qquad (33)$$

The mean waiting time of delayed calls has already been defined as

$$\tau_W = \frac{\Omega}{A \cdot W}$$

The mean waiting time of all calls divided by the mean holding time is

$$\tau_W^* = \frac{t_W \cdot W}{t_m} = \tau_W \cdot W \qquad (34)$$

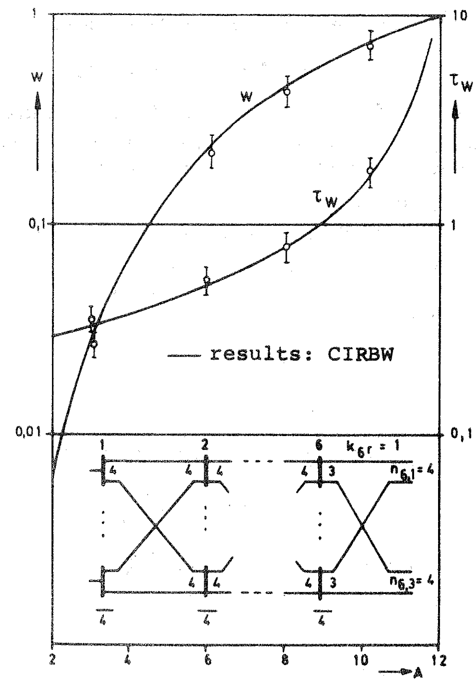
Inlet blocking and route blocking are assumed to be independent. The contribution of inlet blocking $\tau_{W_1}^*$ and route blocking $\tau_{W_2}^*$ can be added to get the τ_W^* for the whole system.

$$\tau_W^* = \tau_{W_1}^* + \tau_{W_2}^* \qquad (35)$$

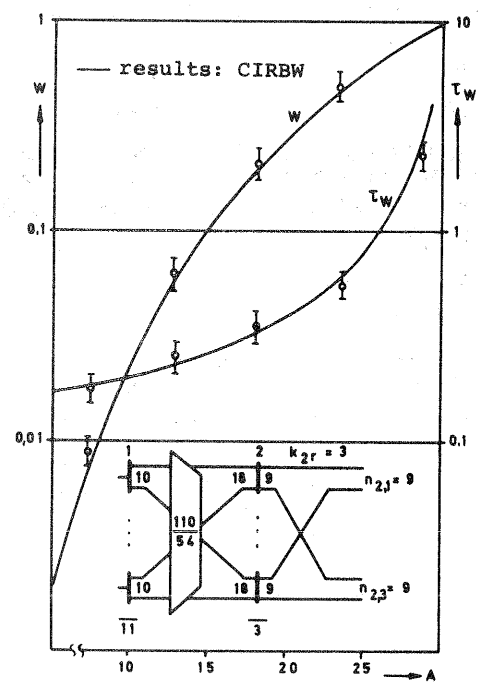
For $\tau_{W_2}^*$ the Interconnection Delay Formula is used and for $\tau_{W_1}^*$ the well known formula of fully available trunk groups with waiting is applied.

$$\tau_W = \frac{\frac{1}{k_1 - Y_1} \cdot E_{2,k_1}(Y_1) + \tau_{W_{IDF}}(n_r, m_r, Y_r) \cdot W_{IDF}(n_r, m_r, Y_r)}{E_{2,k_1}(Y_1) + \{1 - E_{2,k_1}(Y_1)\} \cdot W_{IDF}(n_r, m_r, Y_r)} \qquad (36)$$

For another two systems results of artificial traffic trials are plotted on diagr. 3 and 4. Again the method CIRBW proves to be applicable to most link systems in use.



Diagr. 3 Probability of delay W and mean waiting time τ_W of a 4-stage link system



Diagr. 4 Probability of delay and mean waiting time of a two-stage link system with grading

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