The RDA Method, a Method Regarding the Variance Coefficient for Limited Access Trunk Groups

By **Ulrich Herzog** and **Alfred Lotze**

Report from the Institute for Switching and Data Technics of the University of Stuttgart

UDC 621.395.347

Introduction

If Poisson traffic A is offered to a trunk group containing $N < \infty$ lines and having an accessibility of $k \leqq N$, its probability of loss B produces the result that a quantity of traffic $R = A \cdot B$ is not carried and thus overflows. This overflow traffic R has different statistical properties from those of Poisson traffic. It has, therefore, to be characterized by another parameter, viz., by its variance V or its variance coefficient $D = (V - R)$.

For Poisson traffic $V = R$ and $D = 0$.

For overflow traffic $V > R$ and $D > 0$.

For trunk groups with full accessibility $(k = N)$ this question has been exhaustively investigated by *G. Bretschneider* [1, 2] and *R. I. Wilkinson* [3]. *Bretschneider's* variance-coefficient theory and *Wilkinson's* equivalent random theory provide the same results for V and D. Both methods simplify the calculation of a secondary line-group with full accessibility, to which overflow traffic (R, D) is offered. For special types of single-stage trunk groups with limited accessibility solutions are likewise available for calculating V or D [2, 4].

The new RDA method [5] discussed in the following paragraphs relates to single-stage trunk groups with limited accessibility $(k < N)$ and any type of grading. With overflow traffic we encounter the two following problems, which are solved by the RDA method.

a) *Calculation of the variance coefficient D of overflow traffic*

If the number of lines N_1, the accessibility k_1, the probability of blocking or of loss B_k and the overflow traffic R are known, the variance coefficient D or the variance V of this overflow traffic can be calculated in a simple manner or read off from tables [8]. For the special case of the full-accessibility group $(k_1 = N_1)$ this method gives the same values of V and D as [1, 2, 3].

b) *Calculation of secondary trunk groups*

If this overflow traffic (R, D) is offered to a secondary group with the accessibility k_2 and the specified loss B_2, its number of lines $N_2 = f\ (k_2, B_2, R, D)$ can be readily and rapidly ascertained with the help of diagrams. The loss $B_2 = f\ (k_2, N_2, R, D)$ in a secondary group having the accessibility k_2 and N_2 trunks can also be determined with the help of *RDA* tables [7] calculated for this purpose.

The relevant theory has been thoroughly discussed in [5, 6]. In the following paragraphs the application of the *RDA* method to practical cases will be described on the basis of some examples. This method is of special importance for calculating groups in networks, which have alternate routing such as, for example, of the German direct distance dialling network [10, 13].

Numerous traffic tests, which were carried out on a digital computer belonging to the German Research Society, confirm the accuracy of this method.

1. Calculation of the Variance Coefficient D

To a single-stage trunk group with limited accessibility — generally known as a grading for short — let Poisson traffic A be offered. Let the number of trunks be N_1 and the accessibility $k_1 < N_1$. Let the probability of loss B_k and thus also the overflow traffic $R = A \cdot B_k$ of the grading be known through calculations (for example, [7, 9]) or measurement. For the variance coefficient D of this overflow traffic the theory of the *RDA* method discussed in [5, 6] provides the following formulae:

$$\text{Lower limiting value: } D_{\mathrm{I}} = p \cdot R^2 \cdot \frac{k_1}{N_1}; \tag{1}$$

$$\text{Upper limiting value: } D_{\mathrm{II}} = D_{\mathrm{I}} \cdot \left\{1 + \frac{N_1 - k_1}{g \cdot k_1}\right\}; \tag{2}$$

$$\text{Arithmetic mean: } D_{\mathrm{m}} = D_{\mathrm{I}} \cdot \left\{1 + \frac{0.5}{g}(N_1/k_1 - 1)\right\}; \tag{3a}$$

$$= D_{\mathrm{I}} \cdot \left\{1 + \frac{0.5}{M_1}(1 - k_1/N_1)\right\}. \tag{3b}$$

Here the various symbols denote:

- N_1 the number of lines in the limited-access group,
- k_1 accessibility of the limited-access group,
- g number of selector-groups,
- M_1 $= g \cdot k_1/N_1$ grading ratio,
- R $= A \cdot B_k$ overflow traffic,
- p peakedness coefficient of the overflow traffic.

The "peakedness coefficient" of the overflow traffic is a function of the probability of loss B_k and the accessibility k_1. It can be read off from the diagrams in Fig. 1a, b and c. Eqn. (3) for the mean variance coefficient D_{m} is the one normally used*). It is adequate for all practical requirements with respect to accuracy.

Example 1

a) Given a limited-access trunk-group containing $N_1 = 30$ trunks and having the accessibility $k_1 = 6$ and the grading ratio $M = 2.5$. If Poisson traffic of 17.62 Erl is offered, then from the known loss tables we get $B_k = f\ (A, N, k)$ — [7, 9] — a probability of loss of $B_k = 5\,\%$. The overflow traffic, therefore, amounts to $R = 0.881$ Erl.

b) With $k_1 = 6$ and $B_k = 5\,\%$ we get from Fig. 1a the peakedness coefficient $p = 3.75$. Eqn. (1) for the lower limit of D thus gives the value

$$D_{\mathrm{I}} = p \cdot R^2 \cdot k_1/N_1 = 3.75 \cdot (0.881)^2 \cdot 6/30 = 0.582.$$

From eqn. (3) we get

$$D_{\mathrm{m}} = D_{\mathrm{I}} \cdot \left\{1 + \frac{0.5}{2.5} \cdot \left(1 - \frac{6}{30}\right)\right\} = 0.675.$$

Even simpler is the determination of the variance coefficient with the help of the D-tables ([8], see extract in Fig. 2):

For given values of A, k_1 and N_1 we can directly read off the overflow traffic R and its variance coefficient D_{I}. Eqn. (3a) then gives the required variance coefficient D_{m}

Example 2: Given a grading having the data:

$N_1 = 40$, $A = 40.16$ Erl,
$k_1 = 10$, $M = 2$.

*) If the number g of the selector-groups of a grading and thus their grading ratio M_1 are still unknown at the moment of calculating D, we use for eqn. (3b) the minimum grading ratio $M_{1\min}$, which is permissible for the relevant accessibility according to the guiding principles of the Federal German Post Office (see, for example, the table in [7]). As is shown by eqn. (3b), the D_{m} value thus calculated is "on the safe side".

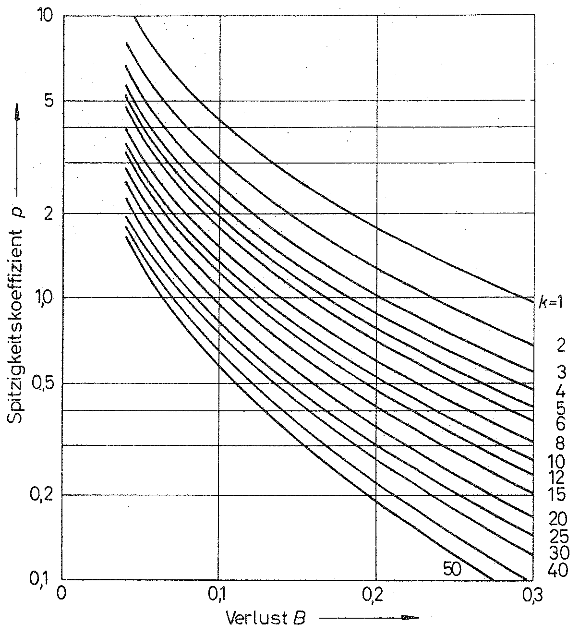


Fig. 1a. Peakedness coefficient of the overflow traffic.
Verlust = loss. Spitzigkeitskoeffizient = Peakedness coefficient.

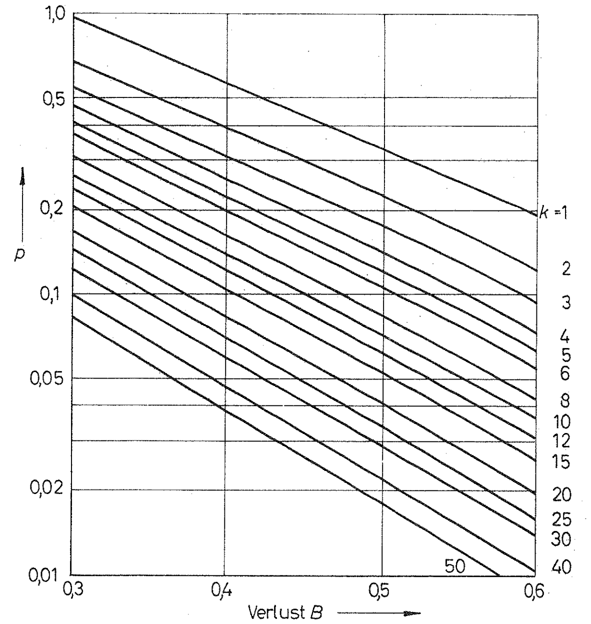


Fig. 1b. Peakedness coefficient of the overflow traffic.
Verlust = loss.

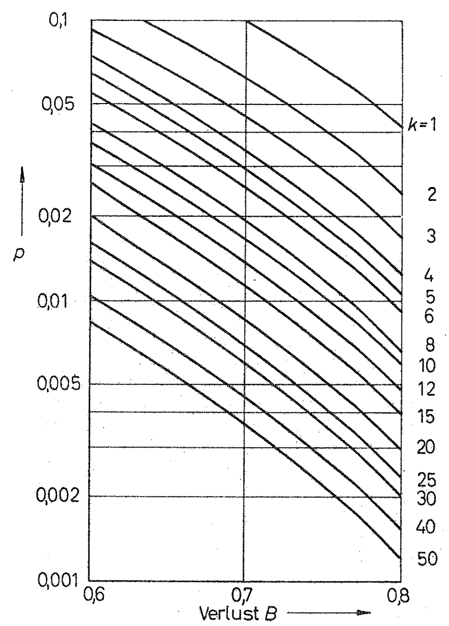


Fig. 1c. Peakedness coefficient of the overflow traffic.
Verlust = loss.

A		N = 10, k = 10	N = 11, k = 10	N = 40, k = 10	N = 200, k = 10
5.0	$\bar{R}$	0.09		0.00	0.00
	$\bar{D}$	0.07		0.00	0.00
35.0	$\bar{R}$	25.36		4.64	0.00
	$\bar{D}$	8.38		5.16	0.00
40.0	$\bar{R}$	30.31		7.92	0.00
	$\bar{D}$	8.64		8.56	0.00
45.0	$\bar{R}$	35.27		11.66	0.00
	$\bar{D}$	8.83		11.91	0.00
100.0	$\bar{R}$	90.11		62.20	0.12
	$\bar{D}$	9.55		29.88	0.08

Fig. 2. Extract from the D tables [8].

By means of linear interpolation between the tabulated values in Fig. 2 for A = 40 Erl and A = 45 Erl we get:

Overflow traffic R = 8.03 Erl,
Variance coefficient D_I = 8.66,
And thus, variance coefficient Dm = 10.3.

2. Designing Secondary Gradings

2.1. *Determination of the number of lines N_2 for a given loss B_2 with the help of diagrams*

If several — statistically independent — overflow traffics (R_i, D_{mi}) are offered jointly to a succeeding secondary route (a "secondary grading"), this total offer is described by the pair of values $(\bar{R}, \bar{D})$:

$$\bar{R} = \sum^{i} R_i \text{ and } \bar{D}_m = \sum^{i} D_{m\,i}$$

or the relative variance coefficient $\bar{D}_m/\bar{R}$.

Let the accessibility k_2 be given and also the probability of loss in the secondary grading

$$B_2 = R_2/\bar{R}.$$

Required, the number of lines N_2. This number of lines in the secondary grading is greater by a number ΔN of lines than the number of lines N_0 in the case of offered Poisson traffic with the same mean value $\bar{R}$ (nevertheless variance coefficient $\bar{D} = 0$), i. e.,

$$N_2\,(\bar{R}, \bar{D}, B_2, k_2) = N_0\,(\bar{R}, B_2, k_2) + \Delta N.$$

For $k_2 = 10$ and $B_2 = 1\,\%$ the additional line requirement ΔN is plotted in Fig. 3 as a function of R and D/R.

The broken-line curve shows the accuracy of the following linear approximation:

$$\Delta N = \frac{\bar{D}_m}{\bar{R}} \cdot \left\{C_1 \cdot (\bar{R} - 20) + C_2\right\} \tag{8}$$

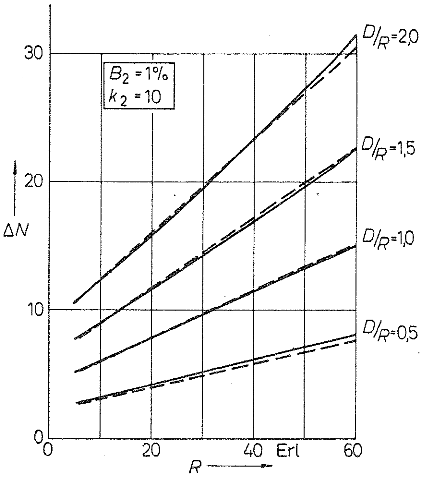


Fig. 3. Additional trunk requirement ΔN in a limited-access secondary group ($k_2 = 10$, $B_2 = 1$ %).

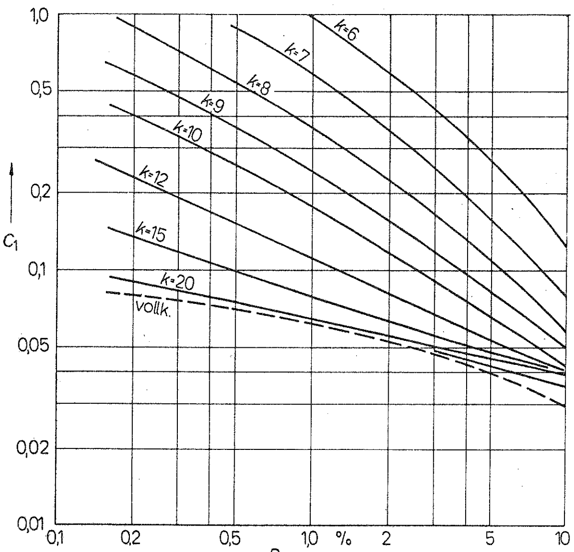


Fig. 4. Diagram of coefficient C_1 for calculating ΔN.

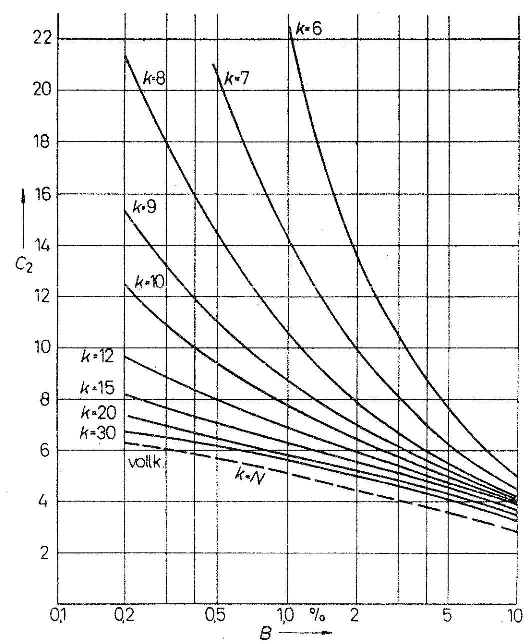


Fig. 5. Diagram of coefficient C_2 for calculating ΔN.

The coefficients C_1 and C_2 calculated by *R. Schehrer* [10] are on their part functions of k_2 and of B_2. They are depicted in Figs. 4 and 5. Tables for the direct reading off of $N_2 = f(k_2, B_2, R_1, D_1/R_1)$ are at present being calculated at the Institute and can be obtained from there.

Example 3: To a secondary grading with accessibility $k_2 = 10$ let there be offered overflow traffic $(\bar{R}, \bar{D})$ with the data

$$\bar{R} = 50 \text{ Erl}, \bar{D}_m = 50, \bar{D}_m/\bar{R} = 1.$$

The loss of the secondary grading is given as $B_2 = 0.01 \mathrel{\hat{=}} 1$ %. The diagram in Fig. 3 gives an additional trunk requirement of $\Delta N = 13$ lines. The number of trunks N_0 required for offered Poisson traffic would be $N_0 = f(A = \bar{R}, k_2, B_2) = f(50/10/0.01) = 83$ trunks (it can, for example, be read off from the tables for the MPJ loss formula [7, 9]). The number of trunks N_2 actually required is therefore:

$$N_2 = f(\bar{R}, \bar{D}, B_2, k_2) = N_0 + \Delta N = 83 + 13 = 96 \text{ trunks.}$$

Example 4: To a full-access secondary-group let overflow traffic ($\bar{R} = 30$ Erl, $\bar{D} = 30$) be offered. Let the probability of loss be given as $B_2 = 0.01 \mathrel{\hat{=}} 1$ %.

From Figs. 4 and 5 we get $C_1 = 0.062$ and $C_2 = 5.0$. Thus, the additional trunk requirement becomes

$$\Delta N = 1 \cdot \{(30 - 20) \cdot 0.062 + 5.0\} = 5.62 \text{ trunks.}$$

The number of trunks N_0 required for offered Poisson traffic would be $N_0 = f(A, B_2) = f(30/0.01) \approx 41$ trunks (it can, for example, be read off from the Palm tables [11] or from the tables for the MPJ loss formula [7, 9]).

The number of trunks actually required in the full-access secondary-group amounts to

$$N_2 = f(\bar{R}, \bar{D}, B_2) = N_0 + \Delta N = 46.62 \approx 47 \text{ lines.}$$

Note: The variance coefficient method described in [2] for full-access groups gives exactly the same result.

Example 5: To a limited-access secondary group with the accessibility $k_2 = 8$ let overflow traffic with the data

$$\bar{R} = 50 \text{ Erl}, \bar{D}_m = 25, \bar{D}_m/\bar{R} = 0.5$$

be offered. Required, the number of trunks N_2 for a specified loss of $B_2 = 0.005 \mathrel{\hat{=}} 5$ ‰.

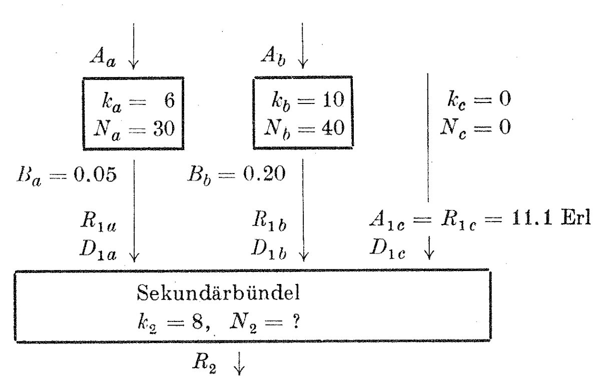
The diagrams in Figs. 4 and 5 give for this pair of values (k_2, B_2) the coefficients $C_1 = 0.54$ and $C_2 = 14.3$, i. e., the additional trunk requirement is

$$\Delta N = 0.5 \cdot \{(50 - 20) \cdot 0.54 + 14.3\} = 15.25.$$

The number of trunks required for offered Poisson traffic would be $N_0 = f(A, k_2, B_2) = f(50, 8, 0.05) = 100$ lines. Thus, the number of trunks actually required ist

$$N_2 = f(\bar{R}, \bar{D}, B_2, k_2) = N_0 + \Delta N \approx 116 \text{ lines.}$$

Example 6: Let the following arrangement be given:



(Sekundärbündel = secondary group)

Let the specified loss in the secondary group be

$$B_2 \equiv \frac{R_2}{\overline{R}} = \frac{R_2}{R_{1a} + R_{1b} + R_{1c}} = 0.02 \triangleq 2\%.$$

The number of trunks N_2 has to be calculated.

A) Calculation of the overflow-traffic data

The data for the two primary groups are already known from examples 1 and 2. Thus, we have:

Overflowing traffics $R_{1a} = 0.881$ Erl, $R_{1b} = 8.03$ Erl, $R_{1c} = 11.1$ Erl.

Variance coefficients $D_{ma} = 0.675$, $D_{mb} = 10.3$, $D_{mc} = 0$.

For the total traffic offered to the common secondary group we thus get

Overflow traffic: $\overline{R} = R_{1a} + R_{1b} + R_{1c} \approx 20$ Erl,
Variance coefficient: $\overline{D}_m = D_{ma} + D_{mb} + D_{mc} \approx 11$,
Relative variance coefficient: $\overline{D}_m/\overline{R} = 0.55$.

B) Calculation of the number of trunks N_2

The Diagram in Figs. 4 and 5 give for the pair of values ($k_2 = 8$, $B_2 = 0.02$) the coefficients $C_1 = 0.219$ and $C_2 = 7.9$. According to eqn. (8), therefore, the additional trunk requirement is

$$\Delta N = 0.55 \cdot \{C_1 \cdot (20 - 20) + 7.9\} = 4.35 \text{ trunks.}$$

N_2 \ k_2	6	8	10	12	14	16	18	20
10	3	2	1	—	—	—	—	—
20	6	4	3	3	2	2	2	1
30	9	6	4	4	3	2	2	2
40	12	8	6	5	4	3	3	3
50	14	10	7	6	5	4	4	4
60	18	12	9	8	6	5	5	4
80	20	16	12	10	9	7	6	6
100	25	18	14	12	10	9	8	7
120	—	20	18	16	14	10	9	8
140	—	—	20	18	16	12	10	10
160	—	—	—	20	18	14	12	12
180	—	—	—	—	20	16	14	12
200	—	—	—	—	—	18	16	14

Fig. 6. Table for the ratio N^*/k^*.

					$N/k = 8$
$R = AB$	N k	8 1	24 3	32 4	208 26
0.1	D	0.00	0.02	0.03	0.13
	A	0.95	5.98	9.79	147.66
20	D	2.91	7.94	10.19	41.52
	A	26.12	38.84	45.41	202.61
100	D	6.03	17.75	23.45	130.71
	A	107.45	122.36	129.83	296.19

Fig. 7. Extract from the RDA Tables [7].

The number of trunks required for offered Poisson traffic ($\overline{R}$ = A) would be

$$N_0 = f(A, k_2, B_2) = f(20/8/0.02) = 35.4 \text{ trunks.}$$

For the total number of trunks N_2 required in the secondary line-group, we therefore get:

$$N_2 = N_0 + \Delta N = 35.4 + 4.35 = 39.75 \approx 40 \text{ trunks.}$$

2.2. *Determination of the loss B_2 with the help of the RDA tables*

a) The same arrangement as in example 6 is to be considered, but let the number of lines be known to be $N_2 = 40$, with the probability of loss B_2 to be calculated.

In example 6 the following values were calculated for the data of the overflow traffic offered.

Mean value: $\overline{R} = \overset{i}{\Sigma} R_{1i} = 20.0$ Erl, and

Variance coefficient: $\overline{D}_m = \overset{i}{\Sigma} D_{mi} = 11.0$.

b) Now we have to find a suitable „equivalent primary grading" (EPG), which has the accessibility k^* and the number of lines N^*. This EPG must have the following characteristics:

- Overflow traffic $\overline{R}$,
- Variance coefficient $\overline{D}_m$.
- The ratio N^*/k^* must be so chosen that the EPG is to be regarded as the section first hunted of a total grading with the following data

Offered traffic A^*,
Number of trunks $N_{total} = N^* + N_2$,
Acessibility $k_{total} = k^* + k_2$.

The *RDA* tables [7] give

$$(A^*, N_{total}, k_{total}) = f\,(R_1, N^*/k^*, \overline{D}_I).$$

Thus, N^*/k^* and $\overline{D}_I$ have still to be determined (see the following Sections c and d).

c) It is known that good gradings for sequential hunting should have an interconnecting number h, which is not uniform, but increases in the direction of hunting. Thus,

$$\frac{N^*}{k^*} > \frac{N_2}{k_2}$$

must be valid.

The appropriate N^*/k^* value can be obtained from the table given in Fig. 6. In our example, with ($k_2 = 8$, $N_2 = 40$) we get the value

$$N^*/k^* = 8.$$

d) The variance coefficient D_m known from Section a) must now be converted into the lower limiting value $\overline{D}_I$, which is independent of the grading ratio of the EPG. In all cases the following formula gives results that are sufficiently accurate for practical purposes:

$$\overline{D}_I \approx 0.8\, D_m.$$

In our example, we get therefore

$$\overline{D}_I = 0.8 \cdot 11.0 = 8.8.$$

e) If we now consult the *RDA* table for $N^*/k^* = 8$ (Fig. 7) we find with $R = 20$ Erl and $D_I = 10.19$[1]) the following triplet of values for the EPG:

Offered traffic $A^* = 45.41$ Erl,
Number of trunks $N^* = 32$,
Accessibility $k^* = 4$.

With k^* and N^* we get the overall accessibility of the total grading

$$k_{total} = k^* + k_2 = 4 + 8 = 12$$

and the total number of trunks

$$N_{total} = N^* + N_2 = 32 + 40 = 72.$$

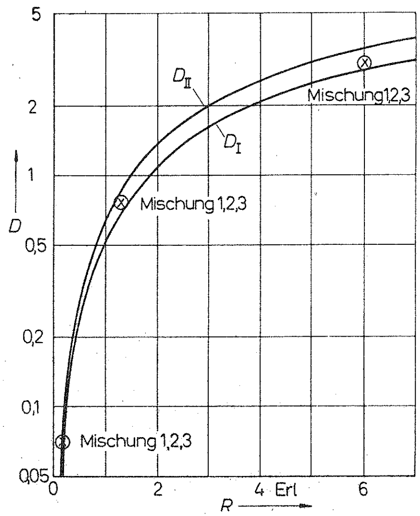


Fig. 8a. Comparison of the RDA limiting curves with precisely calculated values for small limited-access trunk-groups. Mischung = grading.

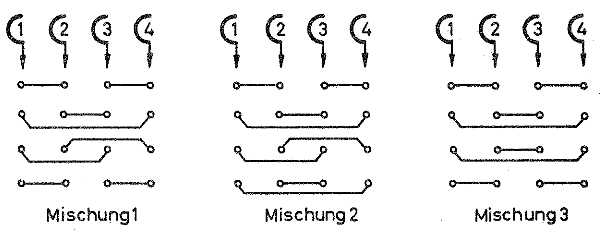


Fig. 8b. Small limited-access trunk-groups, which can be precisely calculated [12]. Mischung = grading.

The next thing to be determined is the loss B_{total} in the total grading; this is done with the help of the loss tables (e. g. [7] in Section 2.1, A).

With $A^* = 45.41$ Erl, $k_{\text{total}} = 12$ and $N_{\text{total}} = 72$ we get

$$B_{\text{total}} = 0.0094 \triangleq 0.94\,\%.$$

From this we obtain the required probability of loss B_2 in the secondary group as

$$B_2 = \frac{A^*}{R_1} \cdot B_{\text{total}}.$$

In our example this becomes

$$B_2 = \frac{45.41}{20.0} \cdot 0.0094 = 0.0213 \triangleq 2.13\,\%.$$

3. Comparison with Traffic Tests and Precisely Calculated Values

Precise values for the overflow traffic R and its variance coefficient D were calculated in [12] for very small limited-access groups. In addition, numerous traffic tests were carried out on an electronic computer in the Computer Centre at the University of Stuttgart. Comparison with the traffic tests and the results obtained with the computer confirmed the reliability of the method.

A few examples of the variance coefficient are shown in Figs. 8a, 9a and 10a (for the associated grading diagrams see Figs. 8b, 9b and 10b). A result for the designing of overflow groups is given in Fig. 11.

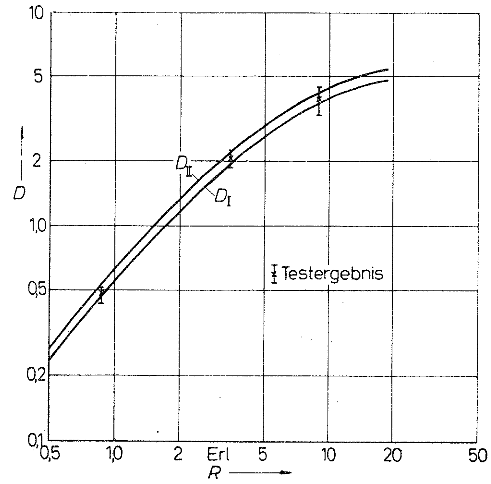


Fig. 9a. Comparison of test results and computed RDA results for grading 4. Testergebnis = test result.

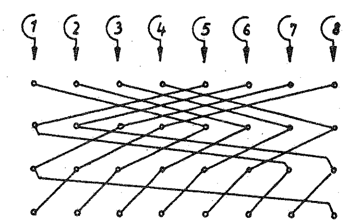


Fig. 9b. Grading 4 ($N = 8$, $k = 4$, $g = 8$).

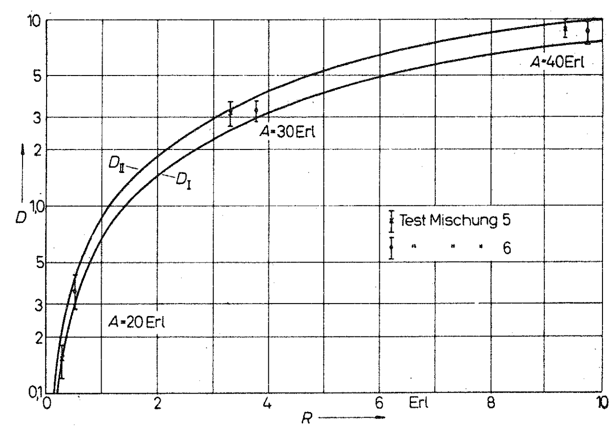


Fig. 10a. Comparison of test and RDA calculation for gradings 5 and 6. Test-Mischung = test grading.

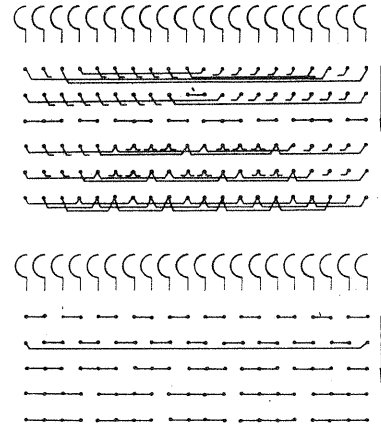


Fig. 10b. Grading 5 (at the top) and grading 6 (both gradings have $N = 40$, $k = 6$, $g = 20$).

References

[1] *Bretschneider, G.; Geigenberger, H.*: Berechnung der Leitungszahlen von Überlaufbündeln (Calculation of the numbers of lines in overflow groups). Report from Siemens & Halske AG., October, 1964.

[2] *Bretschneider, G.*: Berechnung von Leitungsgruppen für überfließenden Verkehr in Fernsprechwählanlagen (The calculation of groups for overflowing traffic in automatic telephone systems). Nachrichtentechn. Z. 9 (1956) p. 533.

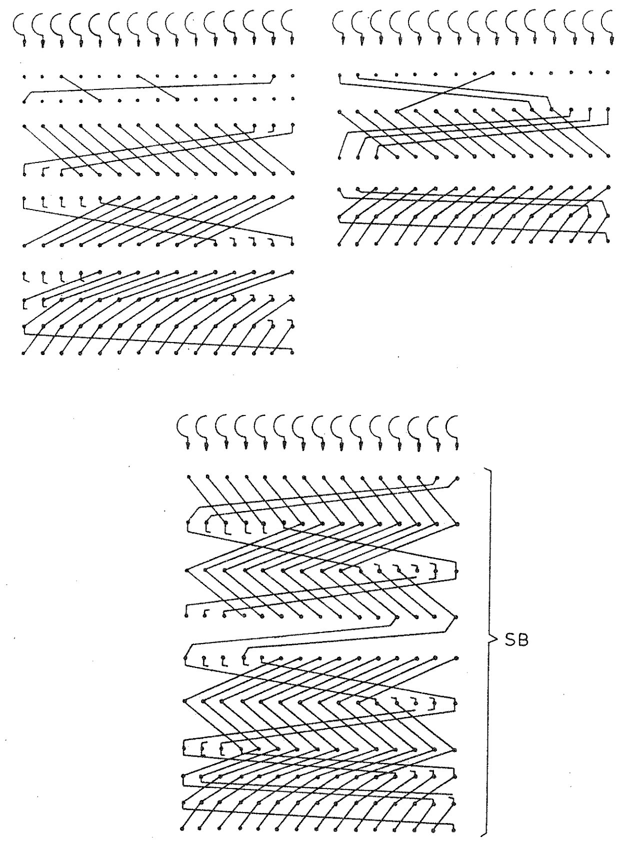


Fig. 11. Arrangement with 2 direct routes and a common secondary route (SB).

Simulated result for the secondary loss: $B_2 = 0.005798 \pm 0.002$.
Computed result by the RDA method: $B_2 = 0.00576$.

[3] *Wilkinson, R. I.; Riordan, J.*: Theories for Toll Traffic Engineering in the U.S.A. Presented at the first ITC 1955 Copenhagen and Bell System Technical Journal No. 35 (1956) p. 514.

[4] *Bretschneider, G.*: Ein verallgemeinertes Verfahren zur Näherungsberechnung der Leistungsfähigkeit von Überlaufanordnungen (A generalized method for the approximate calculation of the load capacity of overflow arrangements). Nachrichtentechn. Z. 15 (1962) p. 639.

[5] *Lotze, A.*: A Traffic Variance Method for Gradings of Arbitrary Type. a) ITC London 1964, Document No. 8/80. b) Post Off. Telecommun. J., Special Issue, Rep. of the Proc. of the Fourth Intern. Teletraffic Congr., London 1964, p. 50.

[6] *Herzog, U.*: Näherungsverfahren zur Berechnung des Streuwertes von Überlaufverkehr hinter Mischungen (Approximate Methods of Calculating the Variance Coefficient of Overflow Traffic behind gradings). Institute for Switching and Data Technics of the University Stuttgart, 1964.

[7] *Lotze, A.*: Tables for overflow variance coefficient and loss of gradings and full available groups. Issued by the Institute for Switching and Data Technics of the University Stuttgart, 1964.

[8] *Lotze, A.*: Tafeln für Streuwert D und Überlaufverkehr R von einstufigen Koppelanordnungen mit unvollkommener Erreichbarkeit. Berechnung von Sekundärbündeln für angebotenen Überlaufverkehr (R, D) (Tables for variance coefficient D and overflow traffic R of one stage gradings with limited access. Calculation of secondary routes in case of offered overflow traffic (R, D)). Issued by the Institute for Switching and Data Technices of the University Stuttgart, 1965.

[9] *Lotze, A.; Wagner, W.*: Table of the Modified Palm-Jacobaeus-Loss-Formula. Issued by the Institute for Switching and Data Technics of the University Stuttgart, 1962.

[10] *Schehrer, R.*: Die Berücksichtigung des Streuwerts bei der Bemessung von Kennzahlwegen in der Landesfernwahl (Observance of the variance coefficient for the designing of last choice routes in the national trunk-dialling network). Institute for Switching and Data Technics of the University Stuttgart, 1964.

[11] *Palm, C.*: Table of the Erlang Loss Formula. Kungl. Telestyrelsen, Stockholm 1954.

[12] *Bretschneider, G.*: Die exakte Bestimmung der Verkehrsleistung kleiner unvollkommener Fernsprechbündel (The exact determination of the traffic capacity of small limited-access groups of telephone trunks. Nachrichtentechn. Z. 16 (1963) p. 199.

[13] *Wahl, H.*: Die Anwendung des Streuwertverfahrens bei der Planung von Fernsprechanlagen (The application of the variance coefficient method in planning telephone systems). Siemens Zeitschrift 33 (1959) p. 17.