CALCULATION OF TWO-WAY TRUNK ARRANGEMENTS WITH DIFFERENT TYPES OF TRAFFIC INPUT

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ABSTRACT

This paper deals with the calculation of two-way trunk (TWT) arrangements between two private branch exchanges (PBX's) or between one PBX and the public telephone network, respectively.

For these cases, the number of private subscribers is about three up to ten times larger than the number of trunks. Therefore, private telephone traffic is adequate to "pure chance traffic no 2" (definition cf. chapter 1).

Furthermore, it must be taken into account, that sometimes internal and external traffics (definitions cf. lit |1| and sec. 2.2.1) are considerably correlated.

Setting up the equations of state, the probabilities of loss have been calculated for the different groups of subscribers. Calculations has been done with or without respect to the event "the called subscriber is busy".

1. TRAFFIC MODEL

Two types of offered traffic are considered:

I Pure Chance Traffic of Type 1 (PCT)

An infinite number of sources ($q \to \infty$) produces the offered traffic with the mean value A. Each source has an infinitely small call rate ($\alpha \to 0$) such, that the total call rate is constant ($0 < \lambda = \alpha \cdot q < \infty$).

II Pure Chance Traffic of Type 2 (PCT 2)

A finite number of sources ($q < \infty$) produces the offered traffic A, each idle source having the call rate.

In both cases, the sources are supposed to be independent from each other. Idle sources start calls at random. This inplies a negative exponential distribution of idle times of each source.

The distribution of holding times is also assumed in both cases to be negative exponential with the mean value h (termination rate: $\varepsilon = 1/h$).

Therefore, the offered traffic is given by the following equations:

PCT 1 : $A = \lambda \cdot h = \lambda/\varepsilon$

PCT 2 : $A = (q - Y) \cdot \alpha/\varepsilon$

where Y is the traffic carried on the trunk group.

Furthermore, stationarity of all traffics is assumed, i.e. the system is assumed to be in the "statistical equilibrium". Therefore, all state probabilities are independent of time.

2. CALCULATION OF THE CHARACTERISTIC TRAFFIC PARAMETERS

2.2 PRINCIPLES

Dimensioning TWT arrangements with a finite number of subscribers (PCT 2) it must be taken into account, that each successful call "binds" two traffic sources: the calling and the called source. Therefore, the call intensity of both, the group of the calling source and that one of the called source, is reduced. This fact leads to complex equations, especially in case of correlated internal and external traffics.

2.2 CALCULATION OF TWT-GROUPS WITH DIFFERENT TYPES OF TRAFFIC INPUT AND WITHOUT INTERNAL TRAFFIC

2.2.1 The System

Be given two different sets of subscribers A and B, connected by a two way trunk arrangement having n full available trunks (cf.fig.1).

Internal traffic, i.e. traffic between two subscribers of the same set (A or B) is assumed to be zero. The calculation will be done for systems with or without change-over to the switchboard position in case of the event "the called subscriber is busy".

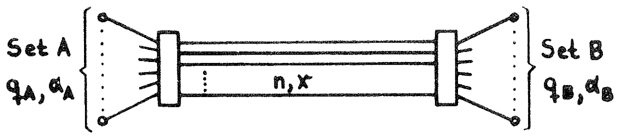


Fig.1: The system.

2.2.2 Abbreviations

q_A, q_B : number of subscribers of set A or B, respectively.

α_A, α_B : call rate of an idle source in set A or B, respectively.

λ_A : uniform call rate of set A, if the offered traffic will be PCT 1.

ε : termination rate

n : number of trunks of the TWT-group.

x : instantaneous number of occupied trunks.

p(x): state-probability, i.e. probability " x trunks are busy".

$s_A(x)$,$s_B(x)$: probability, that the called subscriber of set A or B is busy.

2.2.3 General equations of state, state-probabilities (PCT 2)

The recurrence formula (o = x < n)

$$(1)\quad [q_A - x]\alpha_A p(x)\cdot[1-s_B(x)] + [q_B - x]\cdot\alpha_B\cdot p(x)\,[1-s_A(x)] = [x+1]\cdot\varepsilon\cdot p(x+1)$$

yields with the normalizing condition

$$(2)\qquad \sum_{x=0}^{n} p(x) = 1$$

the following formulae for the state-probabilities

$$(3)\quad p(x) = \frac{p(0)}{x!}\prod_{i=0}^{x-1}\left\{[q_A - i]\frac{\alpha_A}{\varepsilon}[1-s_B(i)] + [q_B - i]\frac{\alpha_B}{\varepsilon}[1-s_A(i)]\right\}$$

$$(4)\quad p(0) = \left\{1 + \sum_{\xi=1}^{n}\frac{1}{\xi!}\prod_{i=0}^{\xi-1}([q_A - i]\frac{\alpha_A}{\varepsilon}[1-s_B(i)] + [q_B - i]\cdot\frac{\alpha_B}{\varepsilon}[1-s_A(i)]\right\}^{-1}$$

2.2.4 Calculation of the state-probabilities for TWT-groups between two PBX's without automatic change-over (PCT 2).

Calculating the state-probabilities, it has to be taken into consideration that the called subscriber may be busy.
If there are occupied trunks, say x, the probability "a called subscriber of set A is, already busy" will be

$$(5)\qquad s_A(x) = \frac{x}{q_A}$$

and in the case of a called subscriber of set B

$$(6)\qquad s_B(x) = \frac{x}{q_B}$$

Inserting (5,6) in (3,4), the equations for the state-probabilities take - after some simple transformations - the following form:

$$(7)\qquad p(x) = p(0)\cdot\binom{q_A}{x}\binom{q_B}{x}x!\left[\frac{\alpha_A}{q_B\cdot\varepsilon} + \frac{\alpha_B}{q_A\cdot\varepsilon}\right]^x$$

$$(8)\qquad p(0) = \left\{1 + \sum_{i=1}^{n}\binom{q_A}{i}\binom{q_B}{i}i!\left[\frac{\alpha_A}{q_B\cdot\varepsilon} + \frac{\alpha_B}{q_A\cdot\varepsilon}\right]^i\right\}^{-1}$$

2.2.5 Calculation of the state-probabilities for TWT-groups between two PBX's with automatic change-over (PCT 2).

This technique is often used in modern private branch exchanges: if the called subscriber is busy the call is directed to the switchboard position. Therefore, the probability "called subscriber is busy" vanishes:

$$(9)\qquad s_A(x) = s_B(x) = 0$$

Inserting (9) in (3, 4) we get the state-probabilities

$$(10)\qquad p(x) = \frac{p(0)}{x!}\prod_{i=0}^{x-1}\left\{(q_A - i)\frac{\alpha_A}{\varepsilon} + (q_B - i)\cdot\frac{\alpha_B}{\varepsilon}\right\}$$

$$(11)\qquad p(0) = \left\{1 + \sum_{\xi=1}^{n}\frac{1}{\xi!}\prod_{i=0}^{\xi-1}\left[(q_A - i)\frac{\alpha_A}{\varepsilon} + (q_B - i)\frac{\alpha_B}{\varepsilon}\right]\right\}^{-1}$$

2.2.6 Time- and Call Congestion for TWT-groups between two PBX's (PCT 2)-

The general formulae for time- and call congestion are the same for systems with or without automatic change-over. We have to insert only the state-probabilities according to the equations (7, 8) or (10, 11), respectively.

The time congestion E, i.e. the probability that all trunks are busy, is the same for both sets of subscribers:

$$(12)\qquad E_A = E_B = p(n) \qquad n\text{: number of trunks}$$

There are two definitions of the call congestion:

- Call congestion because of busy trunks or busy subscriber or both.
- Call congestion because of busy trunks only

Dimensioning TWT-groups, the second definition of call congestion is commonly used.
Therefore, call congestion B_A, i.e. the probability that a new call from set A to B will fail because of the event "all TWT's are occupied" is given by:

$$(13)\qquad B_A = \frac{q_A - n}{q_A - Y}\cdot p(n)$$

where Y is the traffic carried on the TWT-group:

$$(14)\qquad Y = \sum_{x=0}^{n} x\cdot p(x)$$

Accordingly, the call congestion B_B is obtained by:

$$(15)\qquad B_B = \frac{q_B - n}{q_B - Y}\cdot p(n)$$

Finally, the total call congestion is given by the following formula:

$$(16)\qquad B_{tot} = \frac{(q_A - n)\cdot\lambda_A + (q_B - n)\alpha_B}{(q_A - Y)\cdot\lambda_A + (q_B - Y)\alpha_B}\cdot p(n)$$

2.2.7 State-probabilities for TWT-groups between the public telephone network and a PBX without change-over (PCT 1 and PCT 2).

The dimensioning of TWT-groups between one PBX (set B) and the public telephone network (set A) is also possible by means of the equations, presented above. Then, however, the traffic offered from the public telephone network will be PCT 1 with the mean value A_A:

$$(17)\qquad A_A = \lim_{\substack{q_A \to \infty \\ \alpha_A \to 0}} (q_A - x)\frac{\alpha_A}{\varepsilon} = \frac{\lambda_A}{\varepsilon} \quad \text{(cf. chapter 1)}$$

Moreover, the probability, that a called subscriber of the public network A is busy, does not depend on the instantaneous number of occupied TWT-trunks.

Therefore

$$(18)\qquad \delta_A(x) = \delta_A$$

The traffic, offered from the PBX (set B) to the TWT-group does not change its character, if PCT 1 is offered from the public telephone network (set A) and remains PCT 2. Hence we can find after some simple transformations by means of (7, 8) and (17, 18) the following expressions for the state-probabilities:

$$(19)\qquad p(x) = p(0)\binom{q_B}{x}\left\{\frac{A_A}{q_B} + \frac{\alpha_B}{\varepsilon}(1-\delta_A)\right\}$$

$$(20)\qquad p(0) = \left\{1 + \sum_{i=1}^{n}\binom{q_B}{i}\left\{\frac{A_A}{q_B} + \frac{\alpha_B}{\varepsilon}(1-\delta_A)\right\}^{i}\right\}^{-1}$$

2.2.8 State-probabilities for TWT-groups between the public telephone network and a PBX with automatic change-over.

According to section 2.2.5 we will have in this case a vanishing probability "called subscriber of the PBX is busy" ($\delta_B(x) = o$). To find a general solution, the probability "called subscriber of the public network is busy" $\delta_A(x)$ has to be taken into account with a constant value δ_A (cf. section 2.2.7). However, for practical applications this value is often neglected ($\delta_A = o$). The general solution is given by:

$$(21)\qquad p(x) = \frac{p(0)}{x!}\prod_{i=0}^{x-1}\left\{A_A + (q_B - i)\frac{\alpha_B}{\varepsilon}(1-\delta_A)\right\}$$

$$(22)\qquad p(0) = \left\{1 + \sum_{\xi=1}^{n}\frac{1}{\xi!}\prod_{i=0}^{\xi-1}\left\{A_A + (q_B - i)\frac{\alpha_B}{\varepsilon}(1-\delta_A)\right\}\right\}^{-1}$$

2.2.9 Time- and Call Congestion in case of PCT 1 and PCT 2.

The implicite formulae for time- and call congestion are independent of the chosen PBX model. We have to insert only the state-probabilities presented by the equations (19, 20) or (21, 22), respectively.

According to section 2.2.6 we then get:

Time congestion

$$(23)\qquad E_A = E_B = p(n)$$

Call congestion for traffic ($A \rightarrow B$)

$$(24)\qquad B_A = E_A = p(n)$$

Call congestion for traffic ($B \rightarrow A$)

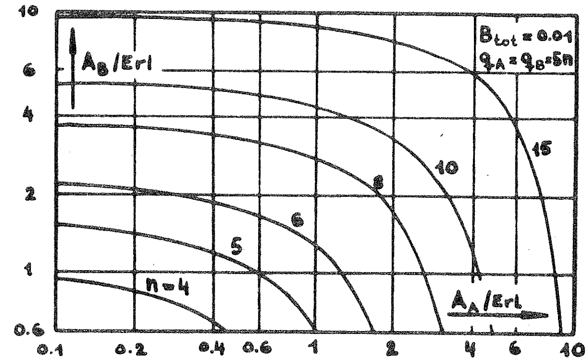
$$(25)\qquad B_B = \frac{q_B - n}{q_B - y}\cdot p(n)$$

Total call congestion

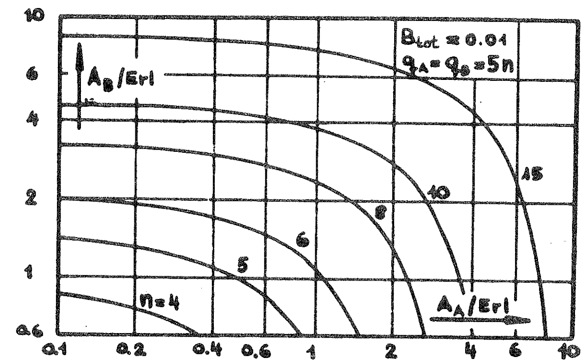
$$(26)\qquad B_{tot} = \frac{\lambda_A + \alpha_B(q_B - n)}{\lambda_A + \alpha_B(q_B - y)}\cdot p(n)$$

2.2.10 Diagrams

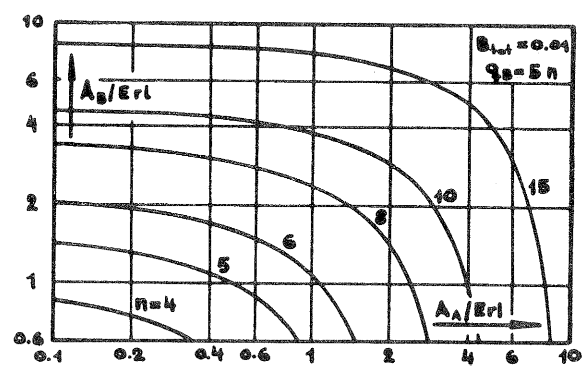
A computer program has been written to evaluate the given formulae for the call congestion. Some results are enclosed in the diagrams 1, 2, 3, and 4.



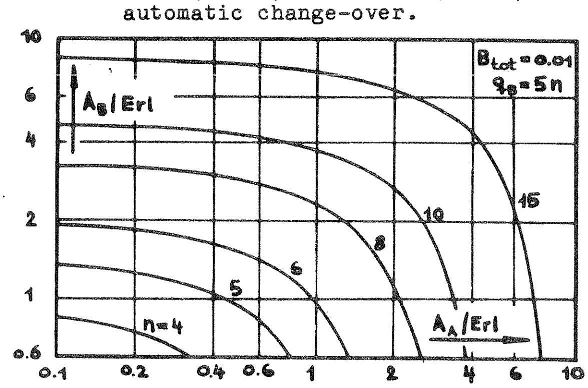
Diagr.1: TWT's between two PBX's without automatic change-over to the switchboard position.



Diagr.2: TWT's between two PBX's with automatic change-over to the switchboard positio



Diagr.3: TWT's between the public telephone network (set A) and a PBX (set B) without automatic change-over.



Diagr.4: TWT's between the public telephone network (set A) and a PBX (set B) with automatic change-over.

2.3 CALCULATION OF TWT-GROUPS BETWEEN THE PUBLIC TELEPHONE NETWORK AND PBX'S WITH REMARKABLE INTERNAL TRAFFIC (PCT 1 AND PCT 2).

Calculating subscriber lines for PBX's with remarkable internal traffic, the correlation between internal and external traffic has to be taken into consideration.

The solution is presented only for PBX's with automatic change-over to the switchboard position in the case of "called PBX-subscriber is busy" ($\lambda_B(x) = o$; cf. section 2.25).

Moreover the constant value $\lambda_A(x) = \lambda_A$ may be zero, as is commonly used in practice (cf. sec. 2.28). However, the method is also applicable for all other systems and the solution can be found analogeously.

2.3.1 The system.

Be given a TWT-group with n_e trunks, connecting the PBX-subscribers with the public telephone network as sketched in figure 2. Figure 2 shows also the three different types of carried traffic to be distinguished: external traffic "outgoing" Y_{eg}, external traffic "incoming" Y_{ec}, and the internal traffic Y_i of the PBX.

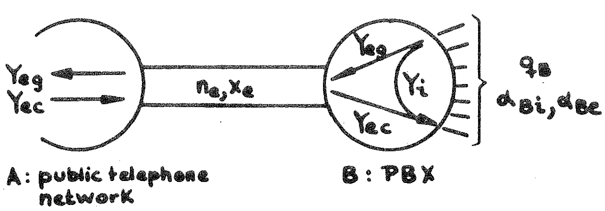


Fig.2: The system.

The number of PBX-subscribers may be finite, say q_B, and that one of the public telephone network infinite. Therefore, traffic offered by the PBX-subscribers will be PCT 2 and on the other hand PCT 1 from the public telephone network. Moreover, it is assumed that the number q_B of PBX-subscribers satisfies the following condition:

$$q_B \geq 2n_i + n_e \tag{27}$$

where n_i is the maximum number of PBX-trunks for internal traffic. This condition is satisfied for all systems, applied in practice and relieves the calculation of the state-probabilities.

2.3.2 Abbreviations

The abbreviations are the same as above (sec. 3.2.2). However, to distinguish clearly internal- and external characteristics, it is convenient to introduce some new indices:

- i : internal
- e : external
- g : outgoing, i.e. traffic generated by the private branch exchange.
- c : incoming, i.e. traffic generated by the public telephone subscribers.

2.3.3 General equations of state.

The state-diagram in figure 3 demonstrates all possible state-transitions if a new call arrives (and will be successful) or if an established call ends. The state-diagram is bidimensional, because we have to distinguish between internal and external (incoming or outgoing) calls. The values x_e (or x_i) can change at random between zero and a maximum number n_e (or n_i). It is evident, that a transition to states "outside of these limits" is impossible (transition-probability will be zero).

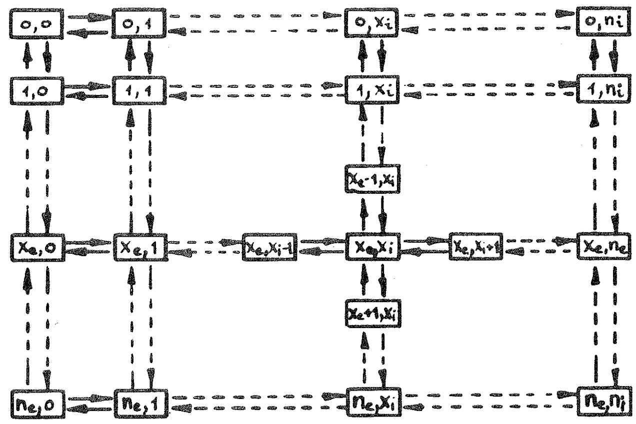


Fig.3: State-diagram.

Using the agreements above and taking into account, that each successful internal call binds two traffic sources of the PBX, the equations of state are given by the following expression: (transition-probabilities, indicated by the arrows on fig. 3):

$$(28)\begin{cases}\left.\begin{array}{l}\left\{[q_B-(x_e-1+2x_i)]\alpha_{Be}+\lambda_A\right\} dt\, p(x_e-1,x_i)\\ +\,[q_B-(x_e+2(x_i-1))]\alpha_{Bi}\,dt\cdot p(x_e,x_i-1)\\ +\,(x_e+1)\cdot\varepsilon\cdot dt\cdot p(x_e+1,x_i)\\ +\,(x_i+1)\cdot\varepsilon\cdot dt\cdot p(x_e,x_i+1)\end{array}\right\} = \\[2ex] = \left\{\begin{array}{l} x_e\cdot\varepsilon\cdot dt\cdot p(x_e,x_i)\\ +\,x_i\cdot\varepsilon\cdot dt\cdot p(x_e,x_i)\\ +\left\{[q_B-(x_e+2x_i)]\alpha_{Be}+\lambda_A\right\} dt\cdot p(x_e,x_i)\\ +\,[q_B-(x_e+2x_i)]\alpha_{Bi}\cdot dt\cdot p(x_e,x_i)\end{array}\right.\end{cases}$$

with

$$\sum_{x_e=o}^{n_e}\sum_{x_i=o}^{n_i} p(x_e,x_i) = 1 \tag{29}$$

2.3.4 Separation of the equations of state for internal and external traffic.

Transforming the equations of state (28) we can extract seperate equations for both external- and internal traffic. These new equations are the basis for the calculation of the state-probabilities and traffic parameters. The transformation can be done as follows:

Summing up the equations of state (28) over all possible values x_i for a fixed value $x_e = o$ and deviding by dt we can find directly:

$$\sum_{x_i=o}^{n_i}\left\{[q_B-2x_i]\alpha_{Be}+\lambda_A\right\}\cdot p(o,x_i) = \sum_{x_i=o}^{n_i}\varepsilon\cdot p(1,x_i) \tag{30}$$

Summing up equation (28) over all possible values x_i for a fixed value $x_e = 1$ and deviding by dt we can find by the aid of equation (30) :

$$(31)\quad \sum_{x_i=0}^{n_i}\left\{[q_B-(1+2x_i)]\alpha_{Be}+\lambda_A\right\}p(1,x_i)=\sum_{x_i=0}^{n_i}2\varepsilon\cdot p(2,x_i)$$

Carrying through the procedure shown for $x_e = 0$ and $x_e = 1$ we can find by induction the general equation for an arbitrary number of external calls:

$$(32)\quad \sum_{x_i=0}^{n_i}\left\{[q_B-(x_e+2x_i)]\alpha_{Be}+\lambda_A\right\}p(x_e,x_i)=\sum_{x_i=0}^{n_i}(x_e+1)\varepsilon\, p(x_e+1,x_i)$$

A simple transformation yields the wanted separated equations for external traffic:

$$(33)\quad \left\{(q_B-2Y_i(x_e)-x_e)\frac{\alpha_{Be}}{\varepsilon}+\lambda_A\right\}p(x_e)=(x_e+1)\cdot p(x_e+1)$$

where

$$(34)\quad Y_i(x_e)=\sum_{x_i=0}^{n_i}x_i\cdot p(x_i/x_e)$$

is the expectation value of the internal carried traffic under the condition, that there are x_e external calls established at the same time (conditional carried internal traffic). Accordingly, $p(x_i/x_e)$ is the conditional probability.

Taking into account the normalizing condition, we obtain an explicit solution for the state-probabilities of the external traffic:

$$(35)\quad p(x_e)=\frac{p(x_e=0)}{x_e!}\prod_{\xi=0}^{x_e-1}\left[(q_B-2Y_i(\xi)-\xi)\frac{\alpha_{Be}}{\varepsilon}+\frac{\lambda_A}{\varepsilon}\right]$$

$$(36)\quad p(x_e=0)=\left\{1+\sum_{x_e=1}^{n_e}\frac{1}{x_e!}\prod_{\xi=0}^{x_e-1}\left[(q_B-2Y_i(\xi)-\xi)\frac{\alpha_{Be}}{\varepsilon}+\frac{\lambda_A}{\varepsilon}\right]\right\}^{-1}$$

Analogously, we can find the state-probabilities for internal traffic:

$$(37)\quad p(x_i)=p(x_i=0)\cdot\frac{\left(\frac{\alpha_{Bi}}{\varepsilon}\right)^{x_i}}{x_i!}\prod_{\xi=0}^{x_i-1}\left[q_B-Y_e(\xi)-2\xi\right]$$

$$(38)\quad p(x_i=0)=\left\{1+\sum_{x_i=1}^{n_i}\frac{\left(\frac{\alpha_{Bi}}{\varepsilon}\right)^{x_i}}{x_i!}\prod_{\xi=0}^{x_i-1}\left[q_B-Y_e(\xi)-2\xi\right]\right\}^{-1}$$

where

$$(39)\quad Y_e(\xi)=\sum_{x_e=0}^{n_e}x_e\cdot p(x_e/\xi)$$

is the internal carried traffic under the condition, that there are ξ internal calls established at the same time.

Equations (35, 36) and (37, 38) are the exact formulae for the state-probabilities of external or internal traffic, respectively. The only unknowns are the conditional carried traffics, which will be determined in the next two sections. Calculation of the time and call congestion for internal and for incoming or outgoing external traffic has to be done according to section 2.2.9.

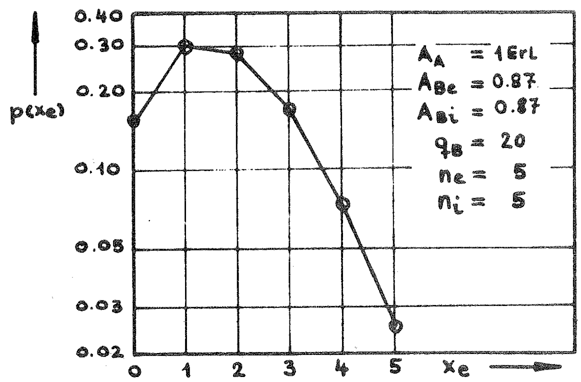
2.3.5 Method of constant source reduction (MCR).

The left hand side of equation (33) shows, that in calculating the state-probabilities for external traffic, the total number of PBX-subscribers q_B has to be reduced by the number of internal occupied PBX-subscribers (during the state x_e). This number of internal occupied subscribers is twice the conditional internal carried traffic $Y_i(x_e)$. Obviously the mean value of $Y_i(x_e)$ is the carried traffic Y_i and therefore Y_i is used as a first approximation of $Y_i(x_e)$.

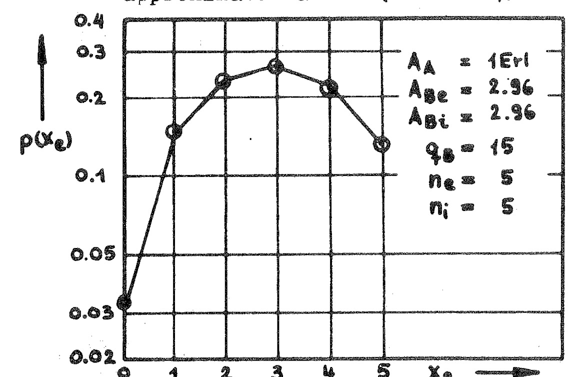
Calculating the state-probabilities for internal traffic, the corresponding solution can be found.

This approximate solution, which reduces the number of PBX-subscribers by a constant value is called the MCR (method of constant source reduction).

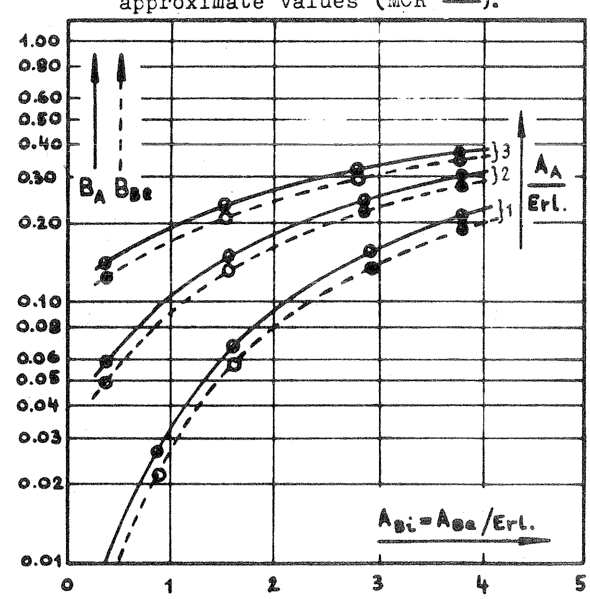
Some results compared to exact values (solution of the total system of linear equations and abbreviated LE) are shown in the diagrams 5, 6 and 7. The comparison shows, that both approximate and exact values are in good accordance.



Diagr.5: State-probabilities for the TWT-group. Comparison between exact (LE ⊙) and approximate values (MCR ——).



Diagr.6: State-probabilities for the TWT-group. Comparison between exact (LE ⊙) and approximate values (MCR ——).



Diagr.7: Call congestion for TWT-traffic. Comparison between exact (LE ⊙) and approximate values according to the MCR-method (—— and ---).

2.3.6 Method of state dependent source reduction (MSR)

All conditional carried traffics are calculated as follows: It is assumed, that stationarity of the external traffic for a fixed number of internal calls is also satisfied and vice versa. Then the conditional state-probabilities of the internal traffic are given by the following equations:

$$[(q_B - x_e) - 2x_i]\,\alpha_{Bi}\cdot p(x_i/x_e) = (x_i+1)\cdot\varepsilon\cdot p(x_i+1/x_e) \tag{40}$$

$$\sum_{x_i=0}^{n_i} p(x_i/x_e) = 1 \tag{41}$$

Combining (40) and (41) we can find the following solution for the conditional state-probabilities of internal traffic:

$$p(x_i/x_e) = p(0/x_e)\cdot\frac{\left(\frac{\alpha_{Bi}}{\varepsilon}\right)^{x_i}}{x_i!}\prod_{\xi=0}^{x_i-1}[q_A - x_e - 2\xi] \tag{42}$$

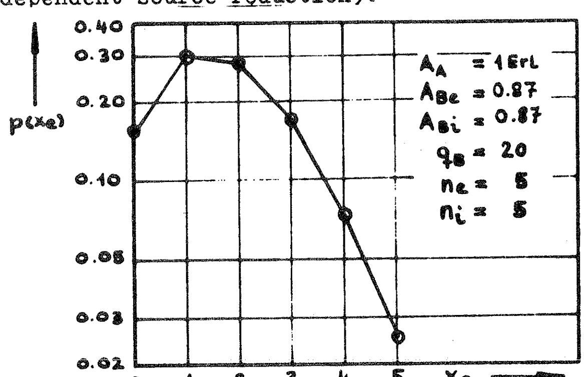
$$p(0/x_e) = \left\{1+\sum_{x_i=1}^{n_i}\frac{\left(\frac{\alpha_{Bi}}{\varepsilon}\right)^{x_i}}{x_i!}\prod_{\xi=0}^{x_i-1}[q_A - x_e - 2\xi]\right\}^{-1} \tag{43}$$

Since now all conditional state-probabilities are known, the conditional internal carried traffic $Y_i(x_e)$ can be calculated by means of equation (34).

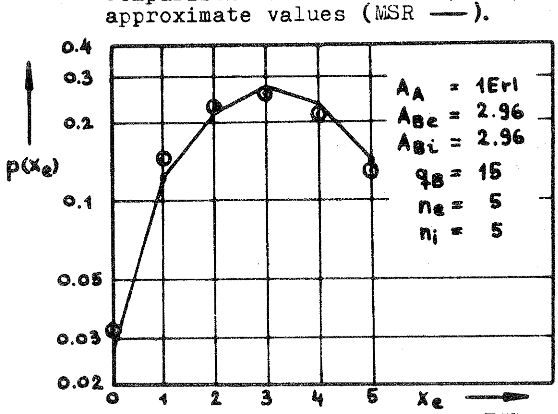
Inserting these values $Y_i(x_e)$ into the equations (35) and (36) we obtain all state-probabilities $p(x_e)$ of the external traffic. Calculation of the call- and time congestion has to be done according to section 3.2.9.

It is evident that we can find the solution for internal traffic accordingly.

This method is called the MSR (method of state-dependent source reduction).

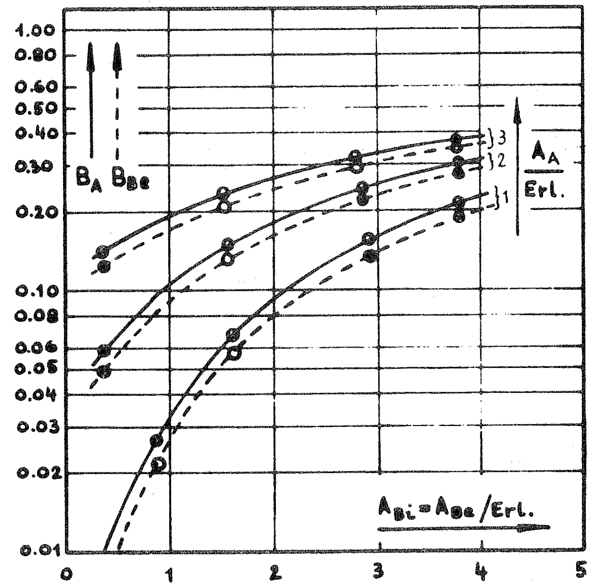


Diagr.8: State-probabilities for the TWT-group. Comparison between exact (LE ⊙) and approximate values (MSR ——).



Diagr.9: State-probabilities for the TWT-group. Comparison between exact (LE ⊙) and approximate values (MSR ——).

The diagrams 8, 9 and 10 show some results compared with values calculated by the solution of the total system of linear equations (LE).



Diagr.10: Call congestion for TWT-traffic. Comparison between exact (LE ⊙) and approximate values according to the MSR-method (—— and ---).

3. CONCLUSION

Setting up the equations of state, all characteristic traffic parameters for TWT-groups with different types of traffic input have been calculated. Exact results are obtained for all systems without internal traffic. These results are also applicable to systems with a small share of internal traffic.

Calculating two way trunk arrangements with a remarkable share of internal traffic, correlation between external and internal traffic has also been taken into account. The results obtained by the MCR- and MSR-method are compared with exact values, calculated by the solution of the total system of linear equations. The comparison shows the good accordance between exact and approximate results.

The MCR-Method (sec. 2.3.5) can be used most simple for practical applications: we only have to reduce the number of PBX-subscribers by a constant value. Then, the number of TWT's can be determined by means of the diagrams, shown in section 2.2.10.

The accurate results obtained for TWT-arrangements suggest to calculate other systems in an equivalent manner. This problem will be investigated at some future time.

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References:

|1| Botsch, D.: Die Verlustwahrscheinlichkeit einstufiger Koppelanordnungen der Vermittlungstechnik mit Intern- und Externverkehr. Doctor-thesis, University Stuttgart, 1966.

|2| Pruskil, M.-H.: Doppelt gerichtete Bündel mit Verkehrsangeboten verschiedener Intensität. Diploma thesis University Stuttgart, 1968.