A GENERAL VARIANCE THEORY

APPLIED TO

LINK SYSTEMS WITH ALTERNATE ROUTING

by

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Summary

The Traffic Variance Method for Gradings of Arbitrary Type by A.LOTZE - presented at the 4th ITC London - is extended in such a way, that multistage link systems with alternate routing can now also be treated. The special cases - calculation of the variance of overflow traffic behind a link system, when pure chance traffic is offered and the design of link systems with an offered overflow traffic - are also investigated.

1.General Remarks

Modern telephone systems usually have a hierarchical structure and the possibility of alternate routing as shown in figure 1:

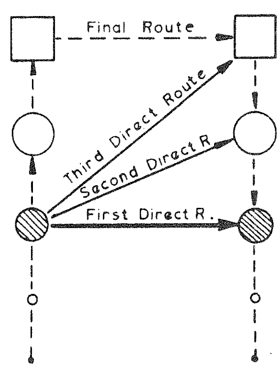


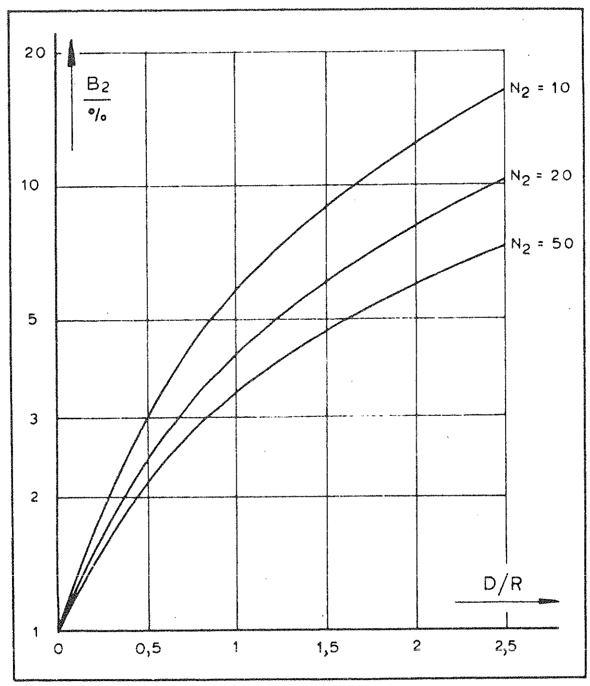
Fig.1: Example for a Hierarchical Network with Alternate Routing.

At first, the traffic is offered to a high-usage "primary trunk group", the so-called "direct route". If there is blocking, all traffic will be lead to a "secondary trunk group" (second direct route or final route).

Telephone networks of this type are, with regard to economy and reliability, superior to systems without alternate routing. Therefore national subscriber trunk dialling in many countries as well as the planned world wide subscriber dialling system are based on these principles.

As a good approximation, telephone traffic offered to the primary trunk group, can be considered as pure chance traffic with Poisson input. The calculation of both call congestion and number of lines has been investigated and solved in a great number of papers.

Overflow traffic, i.e. telephone traffic rejected by one or several primary trunk groups , has other statistical properties than pure chance traffic. Generally, its



Diagr.1: Rise of Congestion with Increasing Relative Variance Coefficient D/R for Full Available Secondary Routes (Mean Value R is constant).

distribution of call arrival times cannot be calculated exactly. Therefore, in addition to the mean value R of this traffic, one needs a second characteristic, which is

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the variance V or the variance coefficient $D = V - R$ respectively (for pure chance traffic with mean value A holds variance $V = A$ and hence $D = 0$).

The design of such secondary trunk groups was formerly performed without considering these special properties by simply assuming pure chance traffic. Diagram 1 shows some examples of incorrect planning caused by this neglection: For the same value of mean R, congestion B_2 increases with the relative variance coefficient D/R (D/R=0 for pure chance traffic).

In practice the relative variance coefficient D/R very often takes values from 1 up to 2 (see also lit./13/). Therefore, if a full available secondary group*) is designed for pure chance traffic, call congestion may effectively amount to $B_2(\text{eff}) = 6,2$ per cent compared with a nominal value of $B_2(\text{nom}) = 1$ per cent. For values of $B_2(\text{nom})$ less than one per cent as well as for secondary gradings, the nominal and effective call congestion may diverge even more distinctly.

This single example shows clearly, that the special statistical properties of overflow traffic (R,D) must be taken into account for exact planning.

R.I.WILKINSON /20,21/, G.BRETSCHNEIDER /3/ and others have studied this problem in detail with regard to full available groups.

In 1964, the so-called RDA-method by A.LOTZE /12,6,7,8/ was presented. This method extends the regard of special statistical properties of overflow traffic to gradings of arbitrary type. The RDA-method allows both calculation of variance V (or variance coefficient D) of overflow traffic, and design of secondary gradings, to which overflow traffic (R,D) is offered. Evaluation work is facilitated by tables and diagrams (short outline see section II).

In the following sections, the extension of this RDA-method to multistage link systems is presented.

Section III.1 explains how to calculate variance V or variance coefficient D of overflow traffic, if pure chance traffic is offered to link systems with preselection or group selection. Section III.2 deals with link systems, if an overflow traffic (R,D) is offered. Finally, in section III.3 a method is given for designing multistage link systems with group selection and alternate routing.

Like the RDA-method for gradings, the proposed method for link systems has the significant advantage that the evaluation work can be done most easily and accurately by hand.

In the special case of full available groups, the method yields the same results as /2,3,20/.

Many tests run with artificial traffic on a digital computer of the "German Research Society" verify the accuracy of the RDA-method extended to link systems.

*) with $N_2 = 20$ lines and actually D/R= 1,5

II. Outline of the RDA-Variance Theory for Gradings/12,5-8,14-17/

II.1 Calculation of Variance Coefficient D

The overflow traffic of an exactly calculable full available group corresponds most precisely to that of a grading, if in both cases the alternation of the states "overflow" and "no overflow" is in good agreement. Comparing one selector group of the

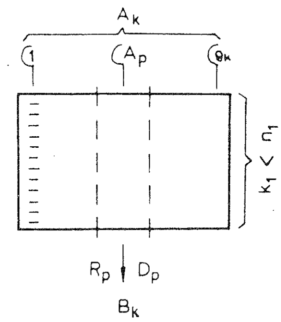


Fig.2:
Grading. Number of Selector Groups $g_k = A_k/A_p$.

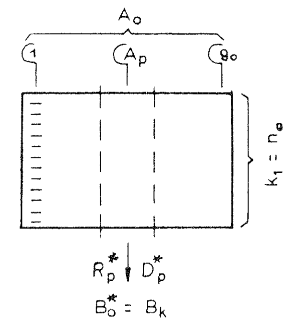


Fig.3:
Full Available Group. Number of Selector Groups $g_o = A_o/A_p$.

grading in figure 2 with the full available group in figure 3, one can see, that time congestion, probability of non-blocking and mean value of overflow traffic are the same. Furthermore, the average duration, and also the number and the distribution of blocking intervals are identical. Finally, the average duration and the number of non-blocking intervals are equal. The distribution of these non-blocking intervals can differ, because the remaining ($g_K - 1$) and ($g_o - 1$) selector groups have a different influence.

Admitting this only approximation we can equate the variance coefficients of both part-overflow traffics. Hence

$$D_p = D_p^* = R_p^2 \cdot \left[\frac{1}{B_k \cdot (k_1 + 1 - A_o \cdot (1 - B_k)} - 1 \right]$$

$$D_p \approx D_p^* = R_p^2 \cdot p$$

The peakednesscoefficient p can easily be computed or looked up in diagram 2 as a function of availability k and loss B.

The relation between partial and total overflow for full available groups yields:

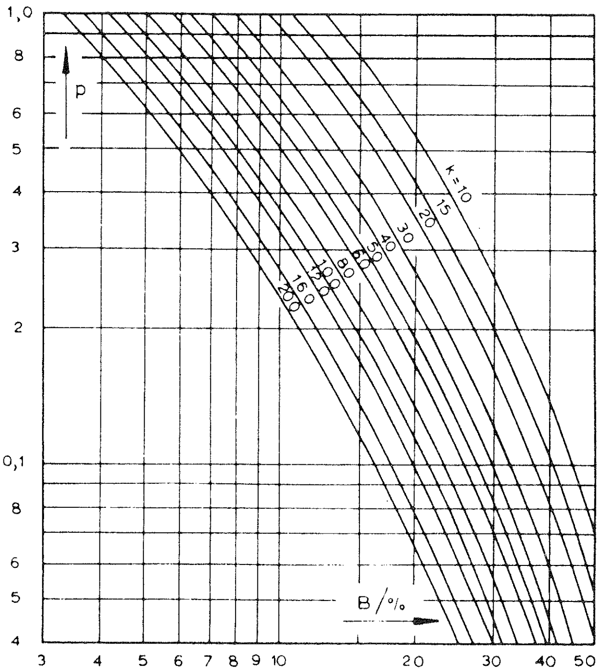
$$D_{tot}^* = g_o^2 \cdot D_p^*$$

Taking into account the correlation between different selector groups, one obtains the relation for gradings. The theory, fully described in /12/, yields

$$\boxed{D_I = p \cdot R^2 \cdot k_1/n_1} \qquad (1)$$

$$\boxed{D_{II} = D_I \cdot \left[1 + \frac{1}{g_k} (n_1/k_1 - 1) \right]} \qquad (2)$$

The value D_I is an inferior limit whereas D_{II} is a superior limit, which is true for inhomogenous, suitable balanced gradings with skipping. Sufficient accuracy for all practical purposes is obtained by the arithmetic mean of D_I and D_{II}.



Diagr.2: Coefficient of Peakedness p.

II.2 Design of Secondary Gradings

Both the RDA-method for secondary grading and the method for full available groups start from the same fundamental idea: One has to determine a fictitious primary grading ($A_k^*, n_1^*, k_1^* \leq n_1^*$), generating the actual offered overflow traffic (R,D) for the secondary grading (cf. fig.4). Furthermore, it must be taken into account that this substitude primary grading is the first hunted part of an inhomogenous total grading ($n_1^*+n_2$, $k_1^*+k_2$), which is sequentially hunted from home position. Then the call congestion of this total grading can be calculated/9-11/:

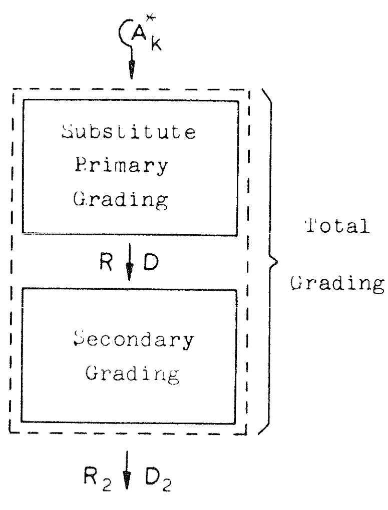


Fig.4: Fictitious Arrangement for the Design of Secondary Gradings.

$$B_{tot} = \frac{R_2}{A_k^*} = \frac{E_{n_1^*+n_2}(A_0)}{E_{n_1^*+n_2-k_1^*-k_2}(A_0)} \qquad (3)$$

Be given the number of lines n_2. Then the call congestion of the secondary grading becomes:

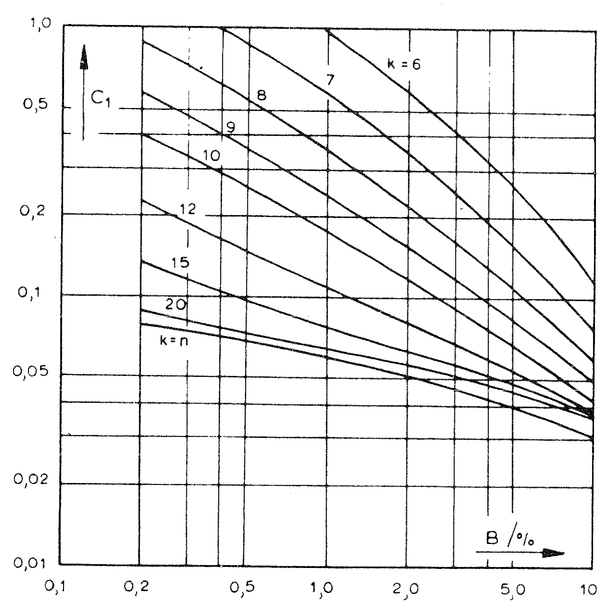
$$B_2 = \frac{R_2}{R} = \frac{A_k^*}{R} \cdot B_{tot} \qquad (4)$$

Conversely, if there is given the loss B_2 the necessary number of lines n_2 can be determined by equations (3) and (4).

Evaluating the necessary amount of lines for a secondary grading by means of a digital computer, R.SCHEHRER has found, that the additionally needed amount Δn_2 of lines -compared with offered overflow traffic- can be well approximated by the following equation:

$$\Delta n_2 = \frac{D}{R} \cdot \left[C_1(R-20) + C_2 \right] \qquad (5)$$

Diagrams 3 and 4 contain the coefficients C_1 and C_2. Therefore, the number of lines

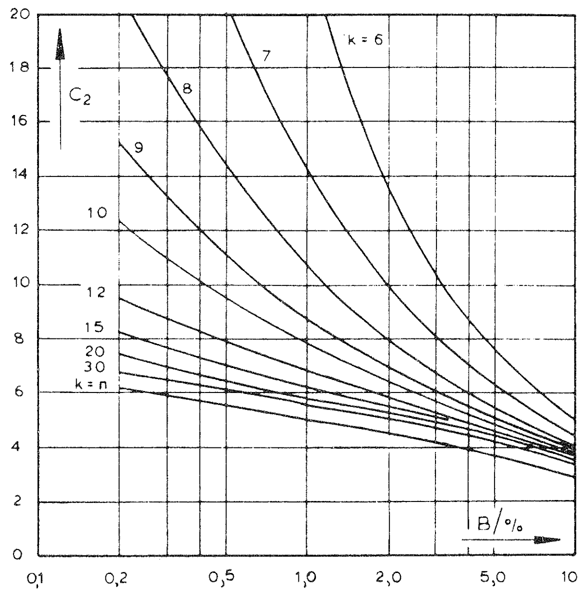


Diagr.3: Coefficient C_1 for the Additionally Needed Amount Δn_{ov}.

in the secondary grading becomes

$$n_2 = n_{20} + \Delta n_2 \quad (6)$$

The number of lines n_{20} can be looked up in loss tables for pure chance traffic (for example /7,11/).



Diagr.4: Coefficient C2 for the Additionally Needed Amount Δn_{ov}.

III. The RDA-Method for Multistage Link Systems

The availability of link systems is not constant, but depends on the instantaneous state of occupation. Calculating the effective or average availability (definitions c.f. III.1 and III.2) one can describe very accurately the characteristics of multistage link systems by means of a grading with the same availability.

The stucture of these link systems is allowed to be arbitrary, that is to say with preselection or group selection and furthermore with graded link- or group-lines.

III.1 Calculation of Variance Coefficient

The effective availability /1/ for the considered route is equal to the availability of a grading, which carries the same traffic Y on the same number of lines n with the same call congestion B.

Be known the carried traffic Y and - by traffic measurement or calculation - the loss B (which the traffic Y suffers in the considered route). Then, the variance V or variance coefficient D = V - R of the overflow traffic can be calculated directly by means of the effective availability k_{eff} = = f(n,B,Y). According to the theory for gradings, published in /12,5-8,14-17/ and outlined shortly in section II, the variance coefficient of the overflow traffic is given by:

$$D = p \cdot R^2 \cdot \frac{k_{eff}}{n} \quad (7)$$

and the variance

$$V = D + R \quad (8)$$

with $R = B \cdot Y/(1-B)$

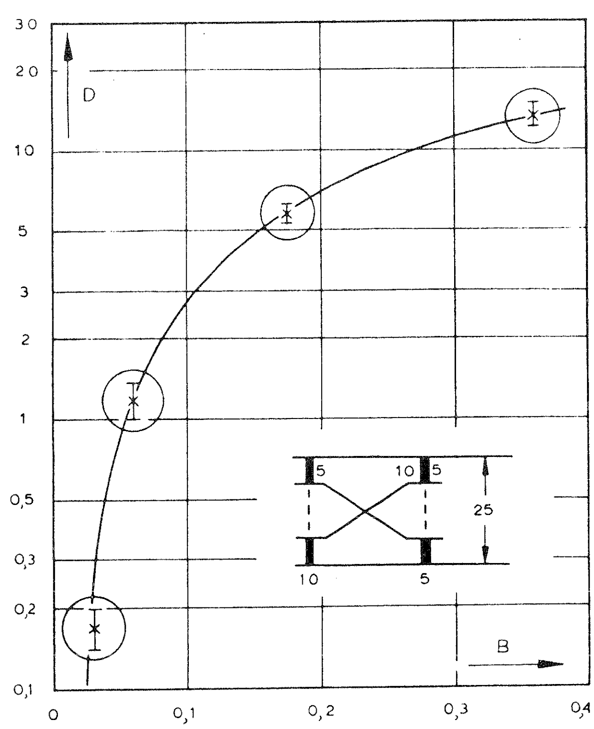
The peakednesscoefficient p, characterizing the peakedness of the overflow traffic can be looked up in diagram 2 as a function of the availability k_{eff} and the loss B.

For very good gradings calculation of the variance coefficient D according to equation (7) leads to an inferior limit. For two- and morestage link systems, however, this equation allows a very accurate calculation of the variance and variance coefficient (c.f. the following section).

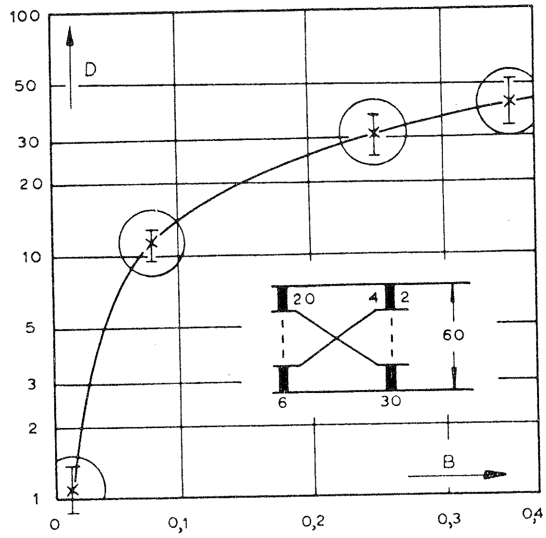
III.1.1 Comparison with Simulation Results

A large number of traffic trials on a digital computer verify the accuracy of the RDA-method extended to link systems with preselection or groupselection.

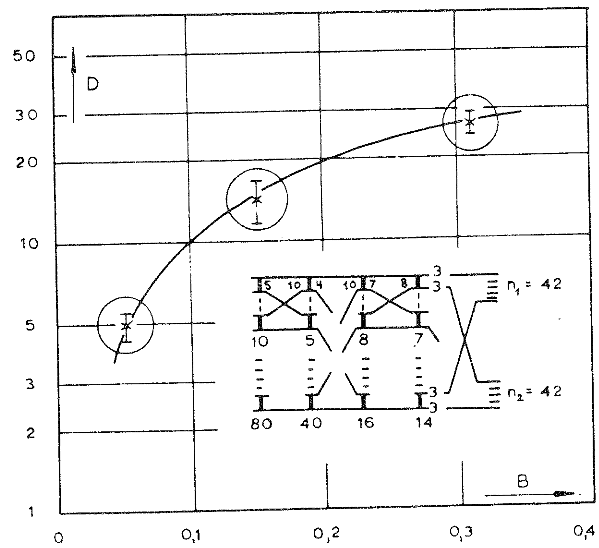
In the following diagrams 5,6,7 and 8, some of these traffic trials are compared with calculated results.



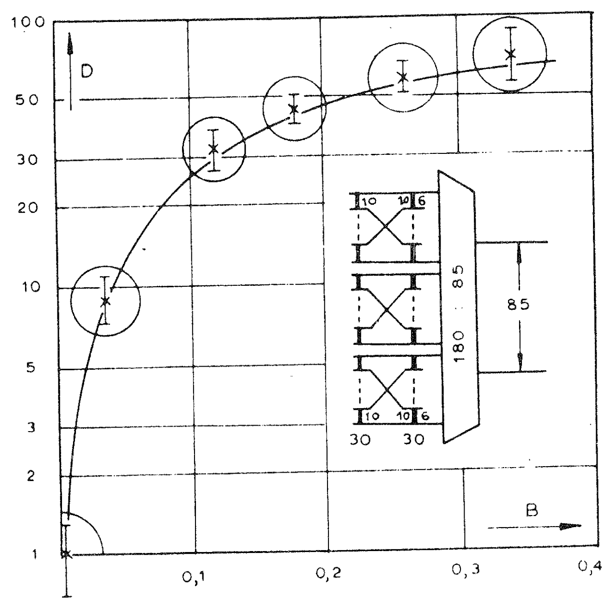
Diagr.5: Variance Coefficient D for a Two-Stage Link System with Preselection (⁑ Test with Confidence Interval; S = 95% in all Simulation Results)



Diagr.6: Variance Coefficient D for a Two-Stage Link System with Preselection



Diagr.8: Variance Coefficient D for Trunk Group No.1 of a Four-Stage Link System with Group Selection.



Diagr.7: Variance Coefficient D for a Two-Stage Link System with Grading.

III.2 Design of Link Systems to which Overflow Traffic (R,D) is Offered

Be prescribed the data of the offered overflow traffic (R,D) and the structure of the link system (c.f. figure 5). Then, the number of lines n_{ov} has to be calculated for a given loss

$$B_{ov} = R_{ov}/R \tag{9}$$

As the carried traffic by the link system is known according to the formula

$$Y_{ov} = R\cdot(1-B_{ov}) \tag{10}$$

the average availability can be calculated. One obtains /15/ for a two-stage link system with preselection:

$$k_{ov} = (k_A - Y_{ov}/g_A)\cdot k_B + \frac{Y_{ov}}{g_A} \tag{11}$$

with k_A, k_B = availability in the A- and B-stage respectively

g_A = number of selector groups in the A-stage

Therefore, all conditions are met to make use of the RDA-method. The manual calculation or the evaluation by computer has to be carried out in the following manner:

a) According to equation (10) one obtains the carried traffic Y_{ov}. The average availability k_{ov} is determined by equation (11).

b) For a prescribed call congestion B_{ov} and the average availability k_{ov} one may draw the coefficients C_1 and C_2 from diagram 3 and 4. Hence, the additionally needed amount Δn_{ov} of lines - compared with pure chance traffic - is

$$\Delta n_{ov} = \frac{D}{R}\cdot\left[C_1(R-20) + C_2\right] \tag{12}$$

c) The number of lines n_p in case of pure chance traffic has to be calculated with /7/ or /11/.

d) Finally, the actual needed number of lines is

$$\boxed{n_{ov} = n_p + \Delta n_{ov}} \tag{13}$$

Remark: The method as presented here is not only suitable for two-stage link systems with preselection, but applies generally to multistage link systems with more than two stages and also to group selection. Then, the average availability has to be calculated according to /15/.

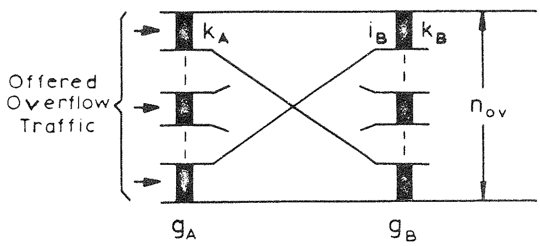


Fig.5: Two-Stage Link Arrangement with Preselection.

III.2.1 Example of Calculation. Comparison with Traffic Test.

An overflow traffic (R,D) is offered to a two-stage link system with preselection. For a prescribed call congestion B_{ov} the number of lines n_{ov} and selector groups g_B for the B-stage has to be calculated (c.f. fig.5).

Be given:

$R = 29{,}8$ Erl $\quad D = 29{,}4 \quad B_{ov} = 0.0435$

$k_A = 20 \quad k_B = 2 \quad g_A = 6$

Calculation:

a) Carried traffic $Y_{ov} = R\cdot(1-B_{ov}) = 28{,}5$ Erl

b) Average availability

$$k_{ov} = (k_A - Y_{ov}/g_A)\cdot k_B + Y_{ov}/g_A = 35{,}25$$

c) Number of lines
For the values ($k = 35{,}25; B = 4{,}35\%$) the diagrams 3 and 4 yield

$$C_1 = 0{,}043 \qquad C_2 = 4{,}15$$

Hence

$$\Delta n_{ov} = \frac{D}{R}\cdot\left[C_1(R-20) + C_2\right] \approx 4{,}5$$

In case of pure chance traffic the necessary number n_p of lines would be /11/:

$$n_p = f(A=R, k_{ov}, B_{ov}) \approx 35{,}7$$

Finally the actually needed number of lines is

$$n_{ov} = n_p + \Delta n_{ov} = 40{,}2 \sim 40 \text{ lines}$$

d) Number of selector groups for the B-stage

$$g_B = n_{ov}/k_B = 20$$

Remark: In general, the calculated proportion n_{ov}/k_{ov} is not an integer. Approximating the number g_B of selector groups to an integer, k_B will not change substantially. Therefore, verification of the average availability k_{ov} will not be necessary.

Simulation Result:

The above calculated link system was tested with artificial traffic on a digital computer. When the offered overflow traffic was ($R = 29{,}8$ Erl, $D = 29{,}4$) the call congestion became $B_{ov} = 0{,}0435 + 0{,}0055$ in the traffic test.

III.3 Calculation of Link Systems with Group Selection and Alternate Routing

The principle of the connection array is shown in figure 6: At first a call will try to find a free way through the link system to the direct route in the wanted direction. If there is blocking, the traffic will be reversed to the final route.

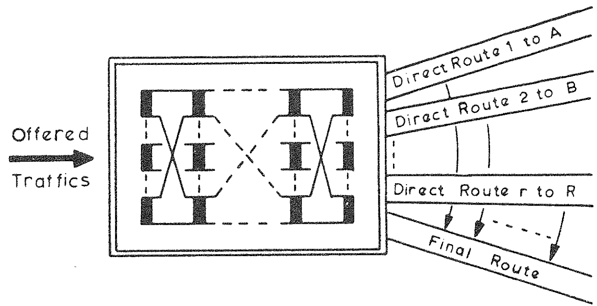


Fig.6: Multi-Stage Link Arrangement with Group Selection and Alternate Routing.

Be prescribed for each direct route not only the carried traffic but also the permissible loss B and for the final route the loss B_{fin}. Then, the number of lines for all direct routes and for the final route can be determined.

III.3.1 Calculation of Direct Routes and Parameters of the Overflow Traffic

The design of the direct routes is possible according to the method of "Combined Inlet- and Route-Blocking" by A.LOTZE, published in /11/ and /15/.

The variances or variance coefficients of the overflow traffics, rejected by this direct routes, has to be computed according to section III.1.

III.3.2 Calculation of Total Overflow Traffic

The mean value of the total overflow is determined by the relation:

$$\boxed{R_{tot} = \sum_j R_j} \tag{14}$$

i.e., mean values have to be added. Obviously, calculating the total variance coefficient (or variance) the correlation caused by the commom link lines for the different traffics must be taken into account.

One gets an approximate formula for this correlation by splitting the traffic overflowing one direct route into two proportions: $D_j(in)$ being generated by the inlet blocking of the A-stage and $D_j(r)$ being originated by route blocking (if there is no inlet blocking at the same time).

The variance coefficients $D_j(in)$ of all routes ($j=1\ldots r$) are fully correlated, which means

$$D_{tot}(in) = \left[\frac{R_{tot}(in)}{R_j(in)}\right]^2 \cdot D_j(in) \tag{15}$$

As a suitable approximation one can suppose that the variance coefficients $D_j(r)$ are independent. Hence

$$D_{tot}(r) = \sum_j D_j(r) \qquad (16)$$

Therefore, the variance coefficient of the total overflow traffic becomes:

$$\boxed{D_{tot} = D_{tot}(in) + D_{tot}(r)} \qquad (17)$$

and the variance is:

$$\boxed{V_{tot} = R_{tot} + D_{tot}} \qquad (18)$$

For r equivalent direct routes with the same offered traffics one obtains

$$\boxed{D_{tot} = r^2 \cdot D_j(in) + r \cdot D_j(r)} \qquad (19)$$

The variance coefficients may be calculated according to section III.1. Thus,

$$D_j(in) = p\{[k_A], k_A\} \cdot R_j^2(in) \cdot \frac{k_A}{g_B \cdot i_B} \qquad (20)$$

The proportion $D_j(r)$, generated by route blocking (if there is no inlet blocking at the same time) is given by the difference between $D_j = f(Y_j, k_{eff}, n_j)$ -c.f. sec.III.3.1- and $D_j(in)$. Hence

$$D_j(r) = D_j - D_j(in) \qquad (21)$$

III.3.3 Calculation of the Final Route.

The values of mean R_{tot} and variance coefficient D_{tot} of the total overflow traffic being known, the necessary number of lines for the final route can be calculated according to section III.2. Of course more than one final route in the link arrangement can also be treated.

III.3.4 Example of Calculation. Comparison with Traffic Test.

Two-Stage Link System with one Direct Route and a Final Route.

Be given the structure of the link system, the carried traffic Y(d) and the loss B(d) of the direct route and the loss B(fin) of the final route:

$k_A = 15 \quad g_A = 15 \quad k_B(d) = 4 \quad k_B(fin) = 2$

$Y(d) = 56$ erlang $\quad B(d) = 0.19 \quad B(fin) = 0.02$

Then, the number of lines for the direct and for the final route has to be determined.

1. Mean value of the overflow traffic

   $R(d) = Y(d) \cdot B(d)/1-B(d) \approx 13,1$ erlang

2. Carried traffic of the final route

   $Y(fin) = R(d) \cdot (1-B(fin)) \approx 12,9$ erlang

3. Total carried traffic.

   $Y = Y(d) + Y(fin) = 68,9$ erlang

4. Average availability of the direct route

   $k(d) = (k_A - Y/g_A) \cdot k_B(d) + Y(d)/g_A \approx 45,3$

5. Average availability of the final route

   $k(fin) = (k_A - Y/g_A) \cdot k_B(fin) + Y(fin)/g_A$

   $k(fin) \approx 21,7$

6. Design of the direct route.

   Calculating the number of lines, inlet-blocking can be neglected ($[k_A] \approx 0,7 \cdot 10^{-4}$). Therefore, one gets

   $n(d) = f(B(d), k(d), Y(d)) \approx 60,3 \rightarrow 60$ lines

7. Variance coefficient D(d).

   $D(d) = p \cdot R(d)^2 \cdot k(d)/n(d) \approx 29,9$

   where $p = 0,23$

8. Final route.

   Obviously, in case of only one direct route, the total overflow traffic is identical with (R(d),D(d)).

   For the values k(fin) and B(fin) the diagrams 3 and 4 yield

   $C_1 = 0,055 \qquad C_2 = 5,15$

   Therefore, the additionally needed amount $\Delta n(fin)$ -compared with pure chance traffic- is

   $\Delta n(fin) = D(d)/R(d) \cdot (C_1 \cdot (R(r)-20) + C_2)$

   $\Delta n(fin) \approx 10,9$ lines

   In case of pure chance traffic, the necessary number of lines would be

   $n_p = f(B(fin), k(fin), R(d)) \approx 19,5$ lines

   Finally, the actual needed amount of lines for the final route is

   $n(fin) = \Delta n(fin) + n_p = 30,4 \sim 30$ lines

   Remark: For pure chance traffic, offered to the final route, this route would be full available. However for the actual offered overflow traffic, the final route becomes limited accessible.

Simulation Results.

The above calculated two-stage link system was tested on a digital computer with artificial traffic.

When the carried traffic of the direct route was

$Y(d) = 56,180 \pm 0,009$ erlang,

the call congestion of this route became

$B(d) = 0,1929 \pm 0,0062$,

the carried traffic of the final route was

$Y(fin) = 13,164 \pm 0,403$ erlang

and the call congestion of this route

$B(fin) = 0,0208 \pm 0,0058$

From this results one can see, that simulation results and calculation are in good agreement.

Four Stage Link System with four Direct Routes and one Final Route.

Be given the structure of the link system the carried traffic Y(i) and the loss B(i) for each direct route (i = 1..4) and the prescribed loss B(fin) for the final route.

$$k_A = 5 \quad k_B = 4 \quad k_C = 7 \quad k_D(i) = 1$$
$$g_A = 80 \quad g_B = 40 \quad g_C = 16 \quad k_d(\mathrm{fin}) = 2$$

$$Y(i) = 12 \text{ erlang}$$
$$B(i) = 0.255 \mathrel{\hat{=}} 25.5\ \%$$
$$B(\mathrm{fin}) = 0.03 \mathrel{\hat{=}} 3\ \%$$

Then, the number of lines for all direct routes and for the final route has to be determined.

1. Mean value of the overflow traffic for one direct route.

$$R(i) = Y(i)\cdot B(i)/[1-B(i)] \approx 4{,}1 \text{ erlang}$$

2. Mean value of the total overflow traffic

$$R_{tot} = \sum R(i) = 4\cdot 4{,}1 = 16{,}4 \text{ erlang}$$

3. Carried traffic of the final route

$$Y(\mathrm{fin}) = R_{tot}\cdot(1-B(\mathrm{fin})) = 15{,}9 \text{ erlang}$$

4. Total carried traffic of all routes

$$Y = \sum Y(i) + Y(\mathrm{fin}) = 63{,}9 \text{ erlang}$$

5. Average availability for the direct route

$$k(i) = \left(k_A-\frac{Y}{g_A}\right)\cdot\left(k_B-\frac{Y}{g_B}\right)\cdot\left(k_C-\frac{Y}{g_C}\right)\cdot k_D(i) + \frac{Y(i)}{g_A}$$

$$k(i) \approx 30{,}4$$

6. Average availability for the final route

$$k(\mathrm{fin}) = \left(k_A-\frac{Y}{g_A}\right)\cdot\left(k_B-\frac{Y}{g_B}\right)\cdot\left(k_C-\frac{Y}{g_C}\right)\cdot k_D(\mathrm{fin}) + \frac{Y(\mathrm{fin})}{g_A} \approx 60{,}7$$

7. Design of the direct routes

The inlet-blocking, caused by the carried traffic Y/g_A of each selector group in the A-stage is very small ($[k_A] = 0.0012$,/15/). Comparing route- and inlet-blocking, therefore inlet-blocking may be neglected.

Looking for the number of lines n(i) for one direct route for the values B(i) = 0.255 and k(i) = 30,4 we will find, that the route is full available with n(i) = 14 lines.

Taking into account that the average availability -evaluated at point 5- is 30,4, the link system will have a meshed structure

8. Variance coefficient D(i) of one overflow traffic

$$D(i) = p\cdot R(i)^2\cdot\frac{n(i)}{n(i)} \approx 4{,}95 \qquad (p=0{,}295)$$

9. Total variance coefficient

According to equation (19) we will find

$$D_{tot} = 4^2\cdot 0 + 4\cdot 4{,}95 = 19{,}8$$

10. Final route

In case of pure chance traffic the necessary number of lines would be (c.f. remark to point 7 of this example)

$$n_p \approx 22{,}8 \text{ lines}$$

and the additionally needed amount $\Delta n(\mathrm{fin})$ of lines is

$$\Delta n(\mathrm{fin}) = \frac{19{,}8}{16{,}4}\cdot(0{,}047\cdot(16{,}4.20)+4{,}1) \approx 4{,}7$$

Finally the actual needed number of lines n(fin) for the final route is

$$n(\mathrm{fin}) = n_p + \Delta n(\mathrm{fin}) = 27{,}5 \rightarrow 28$$

Simulation Results

The above calculated four stage link system was tested with artificial traffic.

Carried traffic and loss for the direct routes:

Y(1) = 12,02 erl.	B(1) = 0.261 ± 0.010
Y(2) = 11,98 "	B(2) = 0.258 ± 0.009
Y(3) = 11,91 "	B(3) = 0.251 ± 0.009
Y(4) = 11,92 "	B(4) = 0.257 ± 0.011

Carried traffic and loss for the final route

Y(fin) = 15,93 erl. B(fin) = 0.0293 ± 0.005

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Bibliography

/1/ Bininda,N., Wendt,A. Die effektive Erreichbarkeit für Abnehmerbündel hinter Zwischenleitungsanordnungen. NTZ 12 (1959),p. 579-585

/2/ Brockmeyer,E. The simple Overflow Problem in the Theory of Telephone Traffic. Teleteknik 5 (1954), p. 361 - 374.

/3/ Bretschneider,G. Die Berechnung von Leitungsgruppen für überfliessenden Verkehr in Fernsprechwählanlagen. NTZ 9 (1956), p. 533-540.

/4/ Bridgford,J. The Geometric Group Concept and its Application to the Dimensioning of Link Acess Systems. ITC London (1964), Doc.No. 13

/5/ Herzog,U. Näherungsverfahren zur Berechnung des Streuwerts von Überlaufverkehr hinter Mischungen. Monograph of the Institute for Shwitching and Data Technics, Technical University Stuttgart Germany, (1964).

/6/ Herzog,U., Lotze,A. Das RDA-Verfahren, ein Streuwertverfahren für unvollkommene Bündel. NTZ 19 (1966), p. 640-646

/7/ Institute for Switching and Data Technics, Technical University Stuttgart, Tables for Overflow Variance Coefficient and Loss of Gradings and Full Available Groups. Second edition 1966.

/8/ Institute for Switching and Data Technics, Technical University Stuttgart, Tables for Variance Coefficient D and Overflow Traffic R of one stage gradings with limited access. Calculation of Secondary Route in case of offered Overflow Traffic (R,D). (1965).

/9/ Lotze,A. Loss Formula, Artificial Traffic Checks and Quality Standards for Characterizing One Stage Gradings. ITC Paris (1961), Doc.No.28.

/10/ Lotze,A. Verluste und Gütemerkmale einstufiger Mischungen. NTZ 14 (1961), p. 449-453.

/11/ Lotze,A., Wagner,W. Table of the Modified Palm-Jacobaeus-Loss-Formula. Institute for Switching and Data Technics, Technical University Stuttgart, (1962) and Proceedings No.3 (1963).

/12/ Lotze,A. A Traffic Variance Method for Gradings of Arbitrary Type. ITC London (1964), Doc.No.80.

/13/ Lotze,A. Verkehrstheoretische Fragen bei der Gestaltung internationaler Fernwählnetze. NTZ 19 (1966), p. 633-639

/14/ Lotze,A., Schehrer,R. Die streuwertgerechte Bemessung von Leitungsbündeln in Wählnetzen mit Leitweglenkung. NTZ 19 (1966), p. 719-724

/15/ Lotze,A.. Computation of Time- and Call-Congestion in link systems with two and more selector-stages and with preselection or group-selection according to an approximation method, which is named "Combined Inlet- and Route-Blocking". Institute for Switching and Data Technics, Technical University Stuttgart, (1962) and Proceedings No.3 (1963).

/16/ Schehrer,R. Die Berücksichtigung des Streuwerts bei der Bemessung von Querleitungsbündeln und Kennzahlwegbündeln in der Landesfernwahl. Monograph of the Institute for Switching and Data Technics, Technical University Stuttgart 1964.

/17/ Schehrer,R. Ein Verfahren zur wirtschaftlich optimalen Bemessung von Wählnetzen mit Leitweglenkung. Monograph of the Institute for Switching and Data Technics, Technical University Stuttgart (1965).

/18/ Smith,N.M.H. More accurate calculation of overflow traffic from link trunked crossbar group selectors. Austral. Telecommun. Monographs No.1.

/19/ Wallstrøm,B. Artifical Traffic Trials on a Two Stage Link System. Congestion Theory and Simulated Traffic Studies. Ericsson Technics (1961), No.2

/20/ Wilkinson,R.I.,(Riordan,J.) Theories for Toll Traffic Engineering in the U.S.A. ITC Paris (1955) and B.S.T.J. 35 (1956), p. 421-514

/21/ Wilkinson,R.I. Simplified Engineering of single stage Alternate Routing Systems. ITC London (1964), Doc.No. 75