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Titel / Title **Non-Stationarity of Walker Movability in Random Walk Mobility Models**

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Kurzfassung / Abstract

Simulation scenarios using either the Random Waypoint mobility model or some flavors of Random Direction mobility models often exhibit speed distributions of their mobile entities that actually differ from the chosen speed distribution functions. Furthermore, this speed distribution may change over simulation time thus breaking the stationarity criteria. This report surveys the problem and contributes an adjusted model generating speed distribution function to obtain an observable uniform speed distribution.

1 Problem Statement

This report considers mobility models, which are based on individual mobile entities walking on walk segments that are defined by

- a walk segment length or a destination point
- a walk speed v for a whole walk segment.

These models show a different probability density function (PDF) $f_V(v)$ for actual walk speeds at a random time t than the PDF $f_V^*(v)$ used for the model generating embedded Markov process. For typical simulation scenarios used in many publications, this PDF even shows non-stationary behaviour.

Certain mobility models such as the *Random Waypoint Model (RWP)* (see Fig. 1) and some flavors of the *Random Direction Model (RD)* (see Fig. 2) define walk segments i with a given segment length l_i and draw a random speed v_i for this walk segment. The segment length is either defined via the choice of the destination point or directly chosen according to a distribution function. A frequently used distribution function for walker speeds, but not the only one affected by this phenomenon, is the uniform distribution with $V_{min} = 0$ and $V_{max} > 0$.

Now, since slower walk speeds result in longer segment times according to

$$t_i = l_i/v_i, \tag{1}$$

the share of slow walkers is higher than the one of fast walkers. This leads to an often unexpected shift of the actual speed distribution towards lower values and, in extreme cases with speed values near 0, to approximately infinite segment times and starving mobile entities that cannot finish their segments during the remaining simulation time.

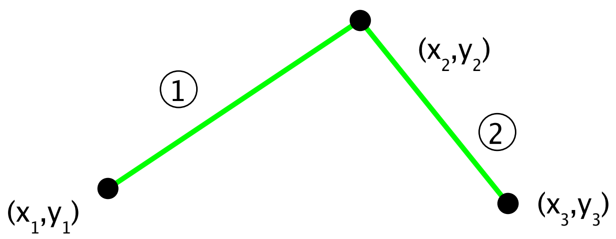

Figure 1: RWP Walk Segment

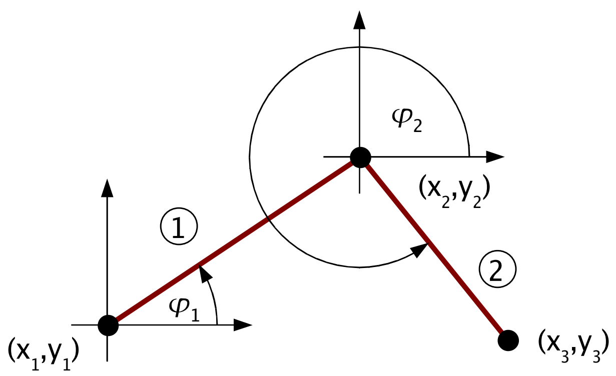

Figure 2: RD Walk Segment

For cases with non-negligible minimal walk speed, the speed mean value approximates a certain value above that minimum. Therefore, a long enough transient phase is required to sufficiently meet the stationarity criteria. For a minimal walk speed $V_{min} = 0$, the speed mean value of a population of mobile entities approximates 0 in ∞ time. Thus the stationarity criteria will never be reached.

2 Example Study

An example simulation scenario is presented in Appendix A. The mobile entities of that study follow a Random Direction mobility model with a parameterization as listed in Tab. 1. The most important parameter to demonstrate this effect is the model generating speed distribution, which is a uniform distribution covering a range from 0.01m/s to 20.0m/s. The diagram in Fig. 3 depicts the evolution of the average speed of a population of 10000 walkers. At $t = 0$, all walkers start a new walk segment according to the given speed distribution. Thus the mean walker speed at simulation start is ~ 10m/s. According to the argumentation before, the speed mean value decays rapidly. Fig. 4 shows the speed distribution of the same scenario and visualizes very well the increase of the share of slow mobile entities.

3 Problem Solutions

To avoid this kind of instationary behaviour and to obtain statistically valid results, one of the following guidelines has to be followed:

- Choose a mobility model drawing segment times instead of segment lengths and walker speeds.
- Choose a constant walker speed $V_{min} = V_{max} = V$.
- Choose a significant lower bound V_{min} for walker speeds and configure a sufficiently long transient phase.
- Apply steady state model initializations to obtain *The Perfect Simulation* as described in [1, 2].
- Choose an adjusted speed distribution for the embedded Markov process to obtain the intended uniform distribution function (see Appendix B).

Furthermore, for analytical approaches, the following guideline is important:

- Consider the biased actual speed distribution of walkers for analytical approaches according to Xie and Goodman [3].

4 Literature

There are several publications covering the speed decay effect discussed in this report. Xie and Goodman [3] describe parts of the problem analytically and provide solutions to calculate the biased distribution function of walker speeds, both for randomly sampled walkers and for walkers crossing cell boundaries. The latter is also discussed by Schweigel in [4]. The problem of applying non-stationary model configurations is denounced by Yoon, Liu and Noble in [5], in which they describe the problem and list publications with that mistake. In [1] Yoon, Liu and Noble, and in [2] Le Boudec and Vojnović propose solutions to start mobility models in steady state to obtain *"The Perfect Simulation"*.

References

[1] J. Yoon, M. Liu, and B. Noble, "Sound Mobility Models," in *MobiCom '03: Proceedings of the 9th annual international conference on Mobile computing and networking*. San Diego, CA, USA: ACM Press, New York, NY, USA, 2003, pp. 205–216.

[2] J.-Y. Le Boudec and M. Vojnović, "Perfect Simulation and Stationarity of a Class of Mobility Models," EPFL, Tech. Rep. EPFL/IC/2004/59, July 2004. [Online]. Available: http://icwww.epfl.ch/publications/list.php

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[4] M. Schweigel, "Beiträge zur Berechnung bewegungsabhängiger Kenngrößen von Mobilfunknetzen," Ph.D. dissertation, TU Dresden, Feb. 7 2005, in German.

[5] J. Yoon, M. Liu, and B. Noble, "Random Waypoint Considered Harmful," in *Proceedings of Infocom 2003*, San Francisco, California, USA, April 2003, pp. 1312–1321. [Online]. Available: http://www.ieee-infocom.org/2003/technical_programs.htm

A Example Study

With this example study, the evolution of walker movability of a population of 10000 walkers is demonstrated. The mobility pattern applied by the walkers is a Random Direction model as depicted in Fig. 2. Walk segments are determined via the random variables (RV) Φ for the absolute heading, L for the walk segment length and V for the walker speed. Walk segments start at the stop point of the previous walk segment. An optional pause time drawn from the RV T is introduced at the beginning of walk segments. All RVs follow a uniform distribution function with ranges as listed in Tab. 1. Fig. 3 shows the evolution of the population's mean speed and Fig. 4 shows the evolution of the corresponding speed probability density function.

Parameter	Value
Model	Random Direction
N_{Walker}	10000
Walk area	1000m $\times$ 800m
Border behaviour	Opposite Enter
$f_V^*(v)$	$\frac{v}{V_{max} - V_{min}}$ for $V_{min} \leq v \leq V_{max}$
V_{min}	0.01m/s
V_{max}	20.0m/s
L_{min}	100m
L_{max}	500m
Φ_{min}	0
Φ_{max}	2π
$T_{pause,min}$	0s
$T_{pause,max}$	0s

Table 1: Model Settings

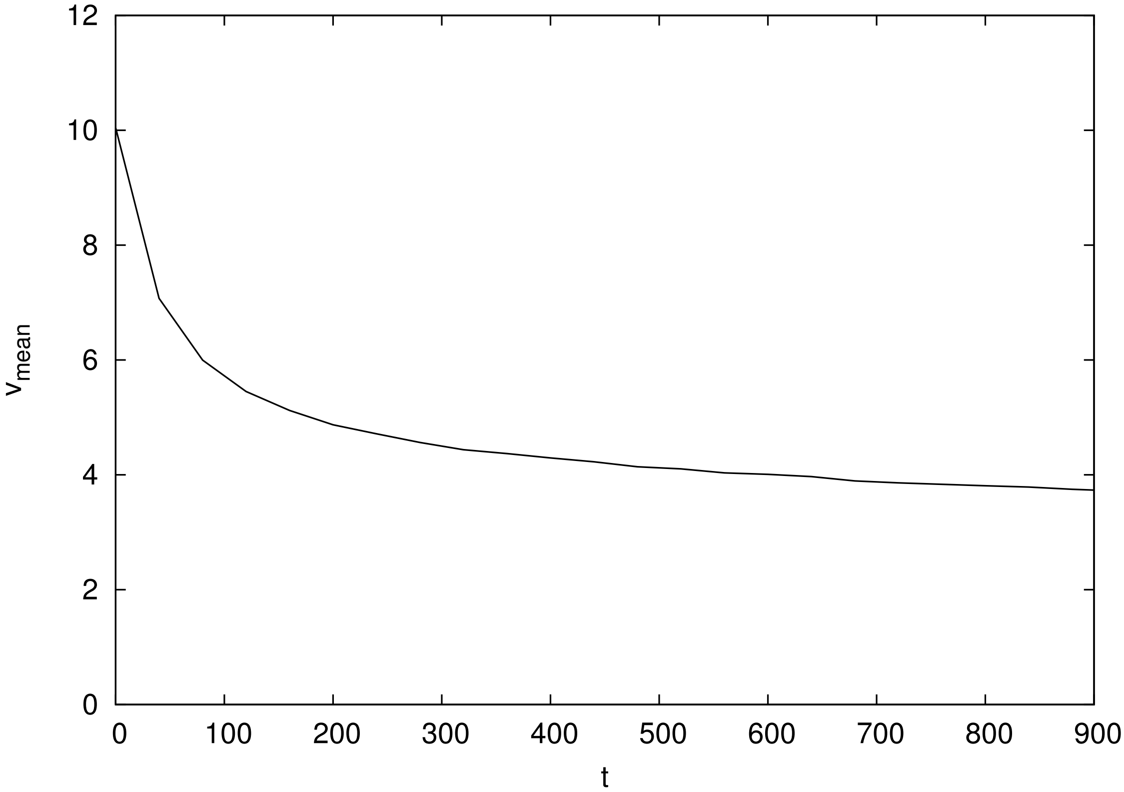

Figure 3: Speed Mean Trace

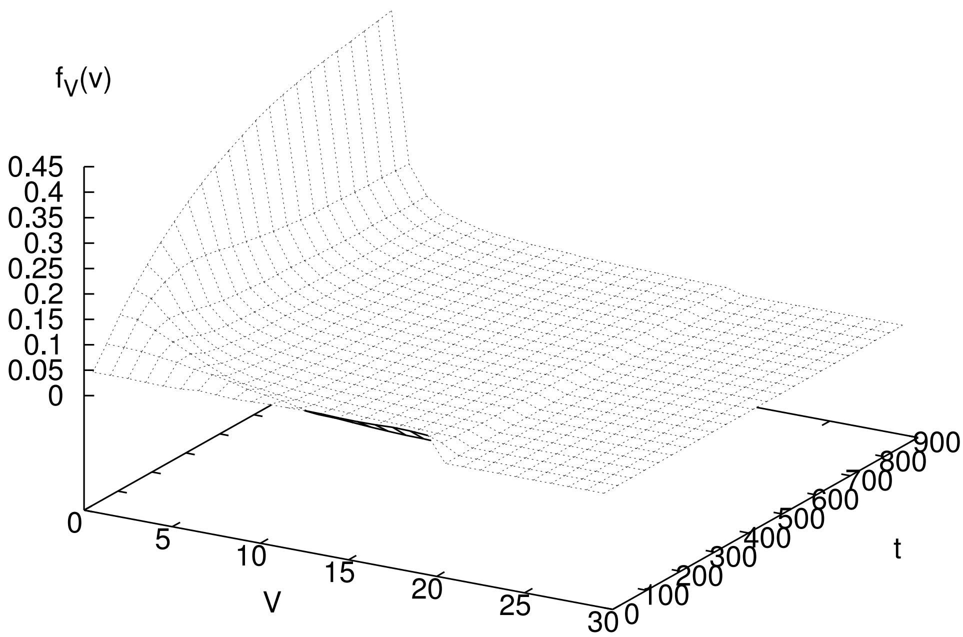

Figure 4: Speed PDF Trace

B Adjustment of the Model Generating Embedded Markov Process

One possibility is to adjust the model generating speed distribution function used in the embedded Markov process. For this, Xie and Goodman [3] derived an adjusted probability density function according to

$$f_V^*(v) = \frac{v}{\mathrm{E}[V]} f_V(v) \tag{2}$$

with $f_V(v)$ being the resulting probability density function and $f_V^*(v)$ the one used for the model generating embedded Markov process.

Assuming the intended speed distribution to be uniform between V_{min} and V_{max}, i.e.,

$$f_V(v) = \begin{cases} \dfrac{1}{V_{max} - V_{min}} & \text{for } V_{min} \leq v \leq V_{max} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\mathrm{E}[V] = \frac{V_{max} + V_{min}}{2} \tag{4}$$

This speed PDF will be obtained, if

$$f_V^*(v) = \begin{cases} \dfrac{2\,v}{V_{max}^2 - V_{min}^2} & \text{for } V_{min} \leq v \leq V_{max} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

is chosen as adjusted speed PDF (see Fig. 5 and Fig. 6).

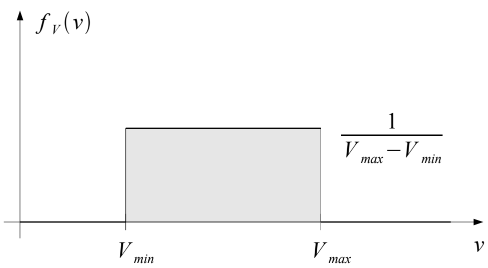

Figure 5: Intended walker speed distribution

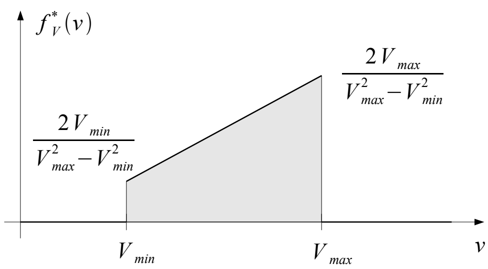

Figure 6: Adjusted speed distribution used for the model generating embedded Markov process

Results of simulation studies with such an adjusted model generating speed distribution function are shown in Fig. 7 and 8 for a speed range from 0.01m/s to 20.0m/s (see Tab. 2) and in Fig. 9 and 10 for a speed range from 10.0m/s to 40.0m/s (see Tab. 3). Both studies show a short transient phase since no *Perfect Simulation*-techniques are applied, but adopt the intended uniform speed distribution function quickly.

B.1 Uniform(0.01m/s, 20.0m/s)

This study shows traces of the speed mean value and speed distribution for an adjusted model generating speed distribution function. The intended speed distribution is Uniform(0.01m/s, 20.0m/s).

Parameter	Value
Model	Random Direction
N_{Walker}	50000
Walk area	1000m $\times$ 800m
Border behaviour	Opposite Enter
$f_V^*(v)$	$\frac{2\,v}{V_{max}^2 - V_{min}^2}$ for $V_{min} \leq v \leq V_{max}$
V_{min}	0.01m/s
V_{max}	20.0m/s
L_{min}	100m
L_{max}	500m
Φ_{min}	0
Φ_{max}	2π
$T_{pause,min}$	0s
$T_{pause,max}$	0s

Table 2: Model Settings for $f_V(v) = \text{Uniform}(0.01\text{m/s}, 20.0\text{m/s})$

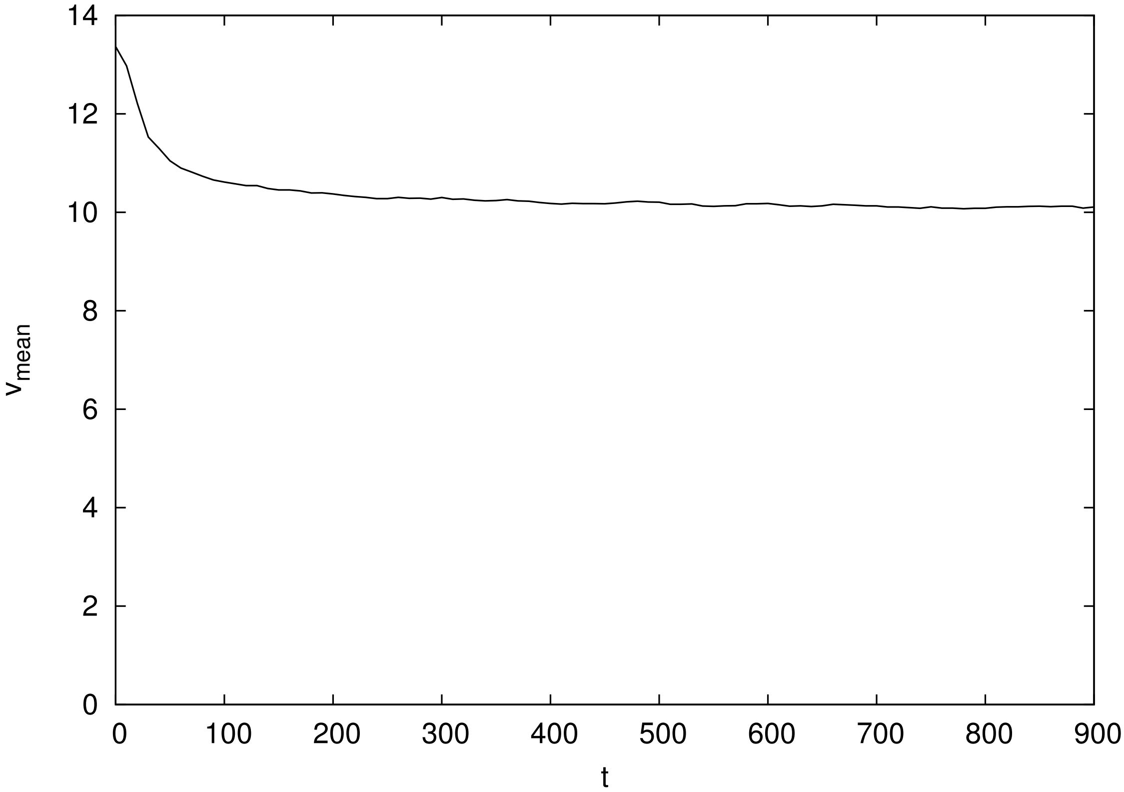

Figure 7: Speed Mean Trace

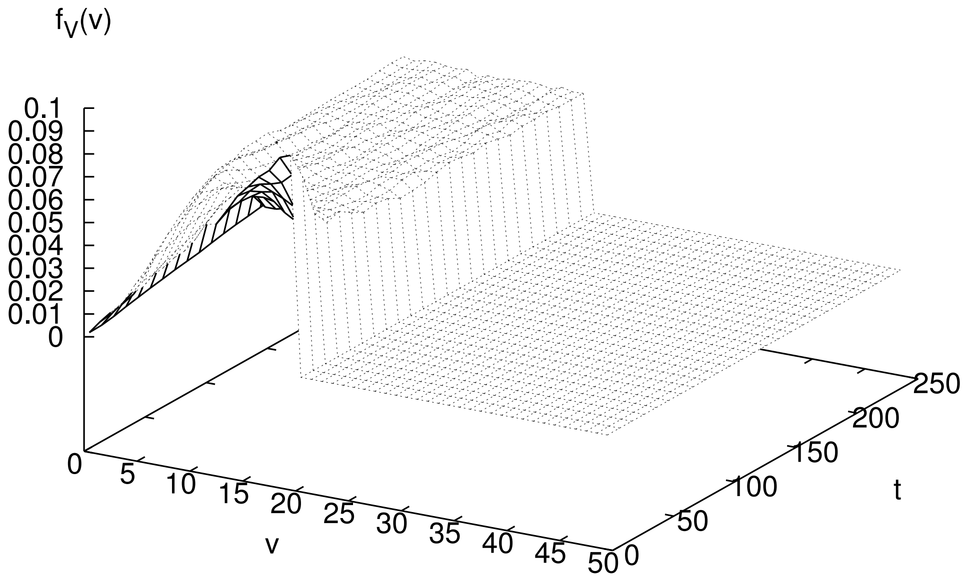

Figure 8: Speed PDF Trace

B.2 Uniform(10.0m/s, 40.0m/s)

This study shows traces of the sped mean value and speed distribution for an adjusted model generating speed distribution function. The intended speed distribution is Uniform(10.0m/s, 40.0m/s).

Parameter	Value
Model	Random Direction
N_{Walker}	50000
Walk area	1000m $\times$ 800m
Border behaviour	Opposite Enter
$f_V^*(v)$	$\frac{2\,v}{V_{max}^2 - V_{min}^2}$ for $V_{min} \leq v \leq V_{max}$
V_{min}	10.0m/s
V_{max}	40.0m/s
L_{min}	100m
L_{max}	500m
Φ_{min}	0
Φ_{max}	2π
$T_{pause,min}$	0s
$T_{pause,max}$	0s

Table 3: Model Settings for $f_V(v) =$ Uniform$(10.0$m/s$, 40.0$m/s$)$

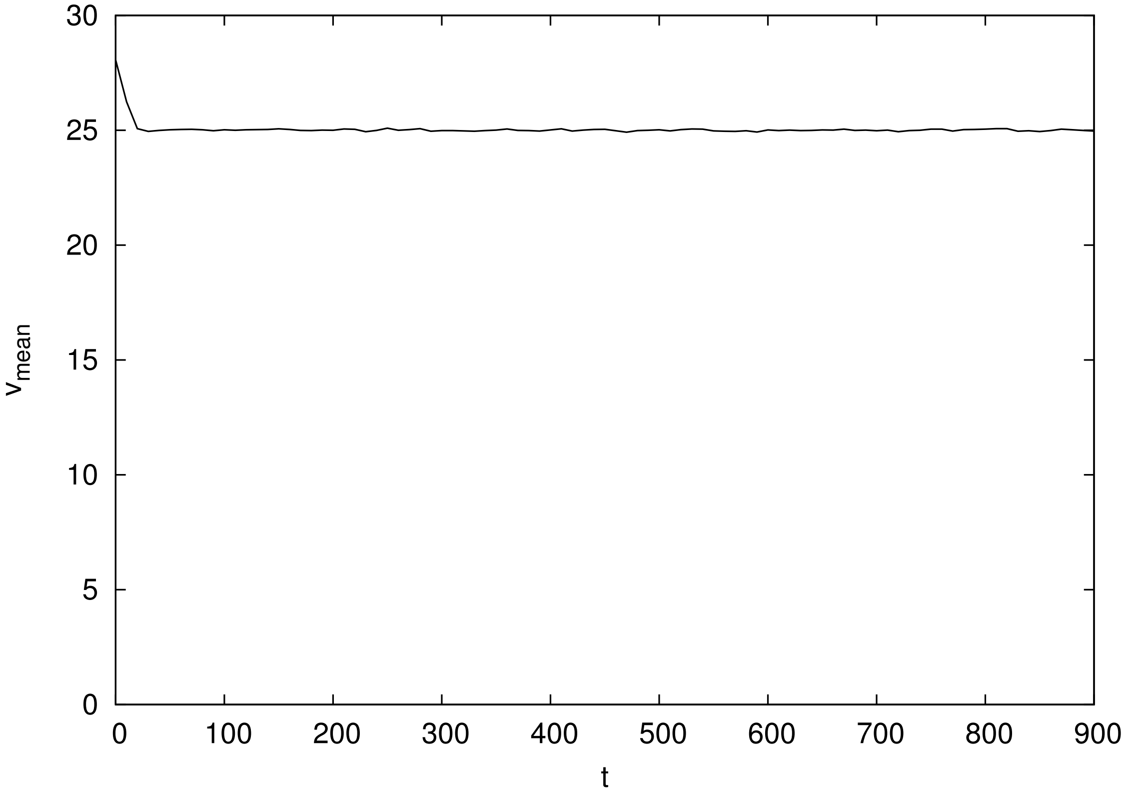

Figure 9: Speed Mean Trace

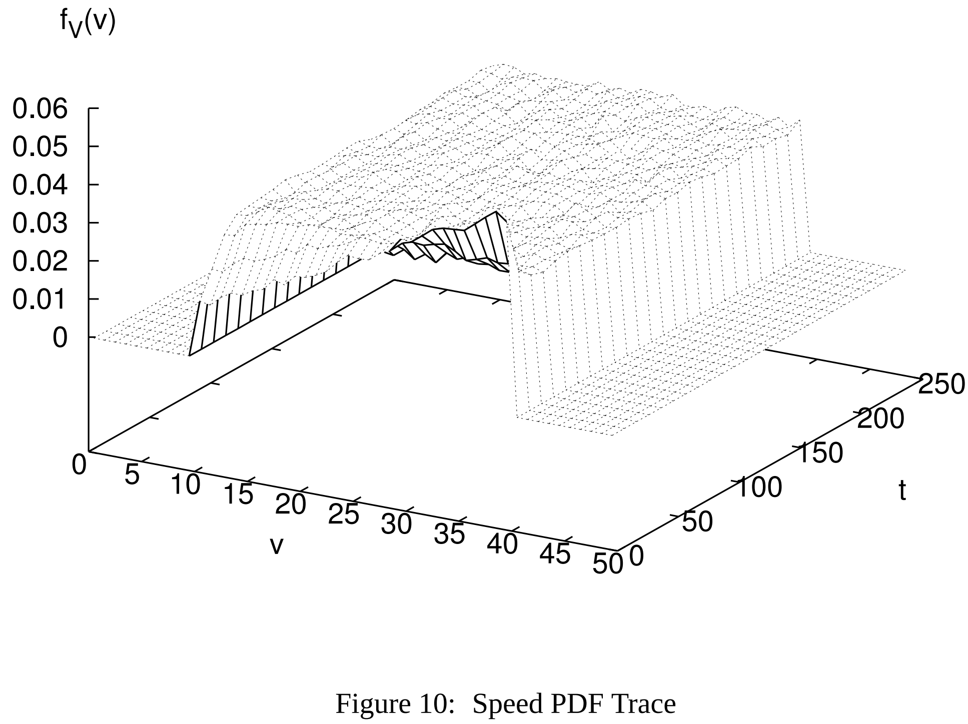

Figure 10: Speed PDF Trace