

Diagram illustrating the calculation of the output delay time (t_{out}) for a priority encoder. It shows two input buses, A to B and B to A, with their respective priority levels. The output delay time t_{out} is the time interval between the first valid input on bus A and the first valid input on bus B.

Input Bus	Priority Levels (from left to right)
A to B	11.4, 12.5, 13.6, 14.7, 15.0, RR1.P, 11.4, ...
B to A	15.1, 16.1, 17.1, 10.1, 11.1, 12.1, 13.1, RR1.F, 14.1, ...

The output delay time t_{out} is indicated as the time interval between the first valid input on bus A (11.4) and the first valid input on bus B (15.1).

Fig. 4: Example of Balanced operation - Time-out recovery in case of a disturbed retransmission.
(t_{out} : duration of time-out)

If a retransmitted I-frame is again disturbed then the REJ-recovery must not be repeated according to the rules of HDLC but the error situation is usually resolved by time-out recovery. This situation is illustrated in Fig.4, which can be conceived as continuing the sequence of Fig.3, if the retransmitted I-frame with $N(S)=1$ is again disturbed. As shown in Fig.4 station A continues to transmit I-frames until a system-specified time-out expires. After station A has timed out it inquires status by transmitting an RR (Receive Ready)-frame with the P-bit set to 1, RR1,P. In response to this command station B transmits an RR response with the F-bit set to 1, RR1,F. Upon receipt of this RR-frame, station A retransmits the I-frame with $N(S)=1$ once more, according to the rules of P/F-bit recovery (check-pointing).

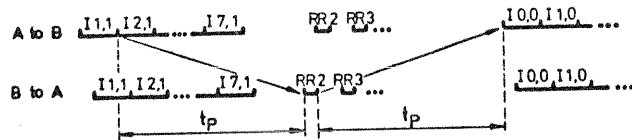


Fig. 5: Example of Balanced operation - Influence of the modulus M of the sequence numbers.

2.2 Data Link Model

Fig.6 shows the structure of the data link model. It consists of two data stations connected by a full-duplex circuit. The link is controlled by HDLC, Balanced Class of Procedures including the optional function REF. Messages to be transmitted from station A to station B, or vice versa, are stored in the send buffer of the sending station where they have to wait for transmission. Messages are transmitted according to first-come, first-served, one message per I-frame. Throughout this paper we assume that the size of the message buffers is unlimited and that the messages are of constant length l . The transmission channels are characterized by their transmission rate v , their bit-error probability p_{bit} (independent bit errors), and their (one-way) propagation delay t_{prop} . Furthermore, we assume that for the processing of a received frame a constant time t_{proc} is required. For the analysis, we combine propagation and processing delay in a constant but arbitrarily prescribable delay

$$t_p = t_{\text{prog}} + t_{\text{prop}} \quad (1)$$

With respect to the traffic conditions on the link we distinguish between two situations. First, the saturated case where a station always has information to send (c.f. Section 3), and, second, the non-saturated case where the amount of information to be transmitted varies statistically (c.f. Section 4). In the first case, the maximum throughput is the essential measure of performance, whereas in the second case, average delays or delay distribution functions characterize the performance of an HDLC-controlled data link.

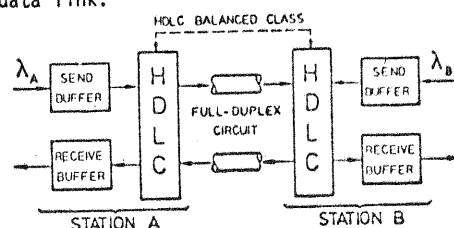


Fig. 6: Structure of the data link model.
(λ_A, λ_B : message arrival rates, c.f. Section 4.1)

3. THROUGHPUT ANALYSIS

In this section we derive analytic expressions for the maximum information throughput of a link, which is achieved if both stations have information to send at any time (saturated message queues). The value of the maximum throughput and its dependence on the relevant system parameters is interesting for batch applications in data communication.

The various time components considered subsequently are defined in such a way that the definitions are valid for the non-saturated case, too (c.f. Section 4).

3.1 The Concept of the Virtual Transmission Time

We address the problem by using the concept of the virtual transmission time which we have already successfully applied to the performance analysis of terrestrial HDLC links /4/. In order to cope with propagation and processing delays and their interaction with the finite modulus value we extend the notion of the virtual transmission time, as defined in Ref./4/, in the following way:

The virtual transmission time of an I-frame with $N(S)=i$ begins with the start of its transmission provided the I-frame with $N(S)=(i-1)$ is (or will be) received in sequence and without transmission error. It terminates at the end of its transmission at the sending station provided this transmission of the frame is successful. If the I-frame with $N(S)=i$ cannot be transmitted due to M-1 simultaneously outstanding I-frames its virtual transmission time is prolonged by the time it has to wait until the "modulus window opens again" (c.f. Fig.7 and Section 3.2).

This definition enables us to replace the complicated sequence of I- and S-frames in case of errors by an equivalent but much simpler sequence of virtual transmission times. The expectation t_v of the virtual transmission time corresponds to the mean time required to successfully transmit one I-frame from station A to station B, or vice versa. Therefore, the maximum information throughput T is given by:

$$T = l / t_v \quad (2)$$

In what follows, we derive an expression for the mean virtual transmission time in case of saturated message queues by taking into account all mechanisms having the major impact on the performance.

Throughout the analysis it is assumed that the probability that an S-frame is disturbed can be neglected. (The S-frame length is 48 bit for the unextended and 56 bit for the extended Control Field Format.)

The considerations pertain to the direction of transmission from A to B. Due to the Balanced Procedure the results for the other direction can be obtained in the same way. The principle of analysis allows to handle symmetrical and asymmetrical situations of the link ; in favour of a simpler notation we describe the analysis for symmetrical conditions of both directions of the link.

3.2 Principle of Analysis

To cope with the interaction among the finite modulus M and transmission errors we determine the mean virtual transmission time t_v by considering first the conditional virtual transmission time given that the considered I-frame "sees" a certain width w of the modulus window. The window width is defined in this context as the maximum number of I-frames which at the beginning of the virtual transmission time of the considered I-frame can be transmitted until the maximum number of $M-1$ simultaneously outstanding I-frames is reached (c.f. Fig. 7).

We denote by $\phi(w)$ the probability that I-frames see a window width w at the beginning of their virtual transmission time and by $t_v(w)$ the mean virtual transmission time of these I-frames. Then the mean virtual transmission time can be written as

$$t_v = \sum_{w=0}^{M-1} \phi(w) t_v(w) \quad (3)$$

In Sections 3.3 and 3.4 we derive expressions for the probabilities $\phi(w)$ and the conditional expectation values $t_v(w)$ for those cases where the modulus M affects the throughput. The latter depends on the actual values of the modulus M , the propagation plus processing delay t_p and the I-frame transmission time t_I . More precisely, the modulus influences the throughput if the acknowledgment time T_{ack} between the end of a virtual transmission time of an I-frame and the receipt of its acknowledgment is greater than the time to transmit $M-2$ I-frames (c.f. Fig. 7).

The derivation of the virtual transmission time in case of $T_{ack} \leq (M-2)t_I$ is briefly discussed in Section 3.5 as a simple special case of the general considerations in Sections 3.3 and 3.4.

3.3 The Process of the Window Width

Fig. 7 illustrates how transmission errors and the modulus interact and affect the virtual transmission time of the I-frames. It shows in particular the values of the window width W defined at the beginning of the virtual transmission times. For the sake of a simplified representation, it is assumed that the modulus M is equal to 4 and that station B has no I-frames to transmit. Therefore, station B acknowledges in the example of Fig. 7 correctly received I-frames by using RR-frames.

We can see that due to the specific definition of the virtual transmission time the channel A to B is "virtually" occupied without any gap in time in the saturated case. Therefore, the window width W as defined above can have only the values $0, 1, 2, \dots, M-2$ because at the beginning of a virtual transmission time the preceding I-frame cannot yet have been acknowledged.

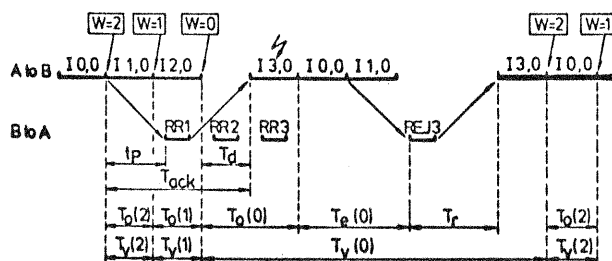


Fig. 7: Impact of modulus M and transmission errors on virtual transmission time T_v . ($M=4$; abbreviations c.f. text)

Under our assumptions the process of the window width W forms a Markov-chain which is illustrated in Fig. 8 and described as follows:

The window width W is reduced by 1 every time an I-frame is successfully transmitted (probability $1-p_B$, where p_B represents the block-error probability of the I-frames) until the minimum achievable value of $W=0$ is reached (note that we consider here the case $T_{ack} > (M-2)t_I$). I-frames, which see a window width $w=0$ are delayed for a certain time T_d until they can be transmitted due to a received acknowledgment. The value of T_d is approximately determined in Section 3.4.3.

Following a disturbed I-frame (see e.g. I-frame with $N(S)=3$ in Fig. 7), the window width W returns to $M-2$ because by definition the next observation epoch of the window process is located just after the error recovery action is finished.

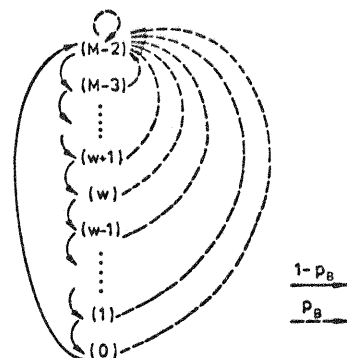


Fig. 8: State-transition diagram of the window width process.

From the state-transition diagram of the window process shown in Fig. 8 the probabilities $\phi(w)$ can be easily determined:

$$\phi(w) = \frac{p_B(1-p_B)^{M-2-w}}{1-(1-p_B)^{M-1}} \quad \text{for } w \in \{0, 1, \dots, M-2\} \quad (4)$$

3.4 Conditional Expectation Values of the Virtual Transmission Time

With probability $1-p_B$ the considered I-frame is not disturbed. Since an I-frame which sees a window width $w=0$ is delayed for a time T_d until the window opens again, the virtual transmission time $T_v(w)$ is given by (see Fig. 7):

$$T_v(w) = T_0(w) = \begin{cases} t_I + T_d & w=0 \\ t_I & w \in \{1, 2, \dots, M-2\} \end{cases} \quad (5)$$

With probability $p_B^n(1-p_B)$ exactly n transmissions of the considered I-frame are disturbed before it is correctly received. As described in Section 2.1.3, if a retransmission of an I-frame is disturbed, then the error situation is resolved by time-out recovery. Therefore, in this case $T_v(w)$ is equal to (c.f. Fig. 7):

$$T_v(w) = T_0(w) + T_e(w) + T_r + (n-1)(t_I + t_{out} + t_s + t_p + t_r) + t_I \quad (6)$$

The precise definitions of the time components $T_e(w)$ and T_r are given in Sections 3.4.1 and 3.4.2.

Hence the conditional expectation values of the virtual transmission time are equal to

$$\begin{aligned} t_v(w) &= E[T_v(w)] \\ &= E[T_0(w)] + \frac{p_B}{1-p_B} t_I + p_B E[T_e(w)] \\ &\quad + \frac{p_B}{1-p_B} E[T_r] + \frac{p_B^2}{1-p_B} (t_{out} + t_s + t_p) \end{aligned} \quad (7)$$

The values $E[T_e(w)]$, $E[T_r]$, and $E[T_d]$ which are necessary to evaluate $E[t_v]$ according to Eqs. (3) and (7) are subsequently derived.

3.4.1 Time Component $T_e(w)$

The time component $T_e(w)$ in Eq. (7) is in general defined as follows (c.f. Fig. 7): Let the I-frame with $N(S)=i$ be the considered frame for which the window width is equal to w at the beginning of its virtual transmission time.

Then $T_e(w)$ is equal to the interval between the end of transmission of this I-frame at station A and the receipt of the first error-free I-frame at station B following the I-frame with $N(S)=i$, or the receipt of an RR-frame with P-bit set to 1 following the expiration of the time-out (duration t_{out}).

In order to determine the expectation of the component $T_e(w)$ we must differentiate among the cases where REJ-recovery is performed and the cases where time-out recovery is performed.

REJ-recovery is possible only if, following the disturbed I-frame with $N(S)=i$, one of the I-frames with $N(S)=i+j \pmod{M}$, $j \geq 1$, is received without transmission error.

For the number j there exist two upper bounds: One is clearly given by $M-2$, the other one is given by the maximum number h of I-frames which can be transmitted before the time-out expires. Since the time-out duration certainly includes the time interval T_d , it holds

$$h = [(t_{out} - T_d)/t_I] + 1 \quad (8)$$

where $[y]$ is defined as the greatest integer not exceeding y . The maximum value c of the number j is therefore equal to:

$$j \leq c = \inf\{h, M-2\} \quad (9)$$

Considering first the case $w=0$ we can distinguish between the following two cases:

- (1) With probability $p_B^x(1-p_B)$, $x = 0, 1, \dots, c-1$, the first error-free I-frame following the considered one has a send sequence number $N(S)=i+x+1 \pmod{M}$. In this case the error situation is resolved by REJ recovery and it holds:

$$T_e(0) = (x+1)t_I + t_p \quad (10)$$

- (2) With probability p_B^c all c I-frames following the considered one are disturbed. In this case the error situation is resolved by time-out recovery. Therefore, the component $T_e(0)$ is equal to

$$T_e(0) = t_{out} + t_S + t_p \quad (11)$$

From this it follows:

$$E[T_e(0)] = t_I(1-p_B) \sum_{x=0}^{c-1} (x+1)p_B^x + p_B^c(t_{out} + t_S) + t_p \quad (12)$$

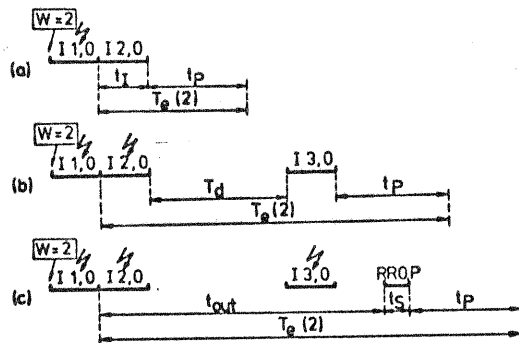


Fig. 9: Derivation of $E[T_e(w)]$ for $1 \leq w \leq c$ ($M=2$, $C=M-2=2$, $w=2$, $i=1$).

For $w > 0$ we must differentiate among two cases, $1 \leq w \leq c$ and $w \geq c+1$.

In case of $1 \leq w \leq c$ we consider three situations (a), (b) and (c), see Fig.9:

- (a) With probability $p_B^x(1-p_B)$, $x = 0, 1, \dots, w-2$, the first error-free I-frame following the considered one has a send sequence number $N(S)=i+x+1 \pmod{M}$. Then it holds:

$$T_e(w) = (x+1)t_I + t_p \quad (13)$$

- (b) With probability $p_B^x(1-p_B)$, $x = w-1, w, \dots, c-1$, exactly x consecutive I-frames following the considered one are disturbed. Then it holds:

$$T_e(w) = (x+1)t_I + T_d + t_p \quad (14)$$

because the I-frame with $N(S)=i+w \pmod{M}$ is delayed for the time T_d until the window opens again.

- (c) With probability p_B^c all c I-frames following the considered one are disturbed. The error situation is resolved by time-out recovery and therefore it holds:

$$T_e(w) = t_{out} + t_S + t_p \quad (15)$$

In case of $w \geq c+1$ it holds:

$$T_e(w) = (x+1)t_I + t_p \quad (16)$$

if $x=0, 1, \dots, c-1$ consecutive I-frames following the considered one are disturbed (probability $p_B^x(1-p_B)$).

If c I-frames following the considered one are disturbed (probability p_B^c), then

$$T_e(w) = t_{out} + t_S + t_p \quad (17)$$

Summarizing these considerations we obtain

$$E[T_e(w)] = \begin{cases} t_I(1-p_B) \sum_{x=0}^{c-1} (x+1)p_B^x + p_B^c(t_{out} + t_S) + t_p \\ \quad + E[T_d](p_B^{w-1} - p_B^c) & \text{for } 1 \leq w \leq c \\ t_I(1-p_B) \sum_{x=0}^{c-1} (x+1)p_B^x + p_B^c(t_{out} + t_S) + t_p & \text{for } w \geq c+1 \end{cases} \quad (18)$$

The value of c is given by Eq. (9), the value $E[T_d]$ is determined in Section 3.4.3.

3.4.2 Time Component T_r

The time component T_r in Eq. (7) begins at the receipt of the first error-free I-frame following the considered one or the receipt of the RR-command with the P-bit set to 1 following the expiration of the time-out. It ends when the considered I-frame is retransmitted (c.f. Fig.7). In order to get a conservative estimation of $E[T_r]$ we exclude for this consideration the case where station B has $M-1$ unacknowledged I-frames outstanding. Then T_r is composed of the processing and propagation delay t_p , the transmission time t_S of an S-frame, and the delays $T_{res,A}$ and $T_{res,B}$ until the channels become available:

$$T_r = t_p + t_S + T_{res,A} + T_{res,B} \quad (19)$$

$$\text{Let } a = [(2t_p + t_S)/t_I] + 1 \quad (20)$$

We define the integer random variable Z as the number of I-frames transmitted from A to B during the time $t_p + T_r$ and $T_{res,B}$ as the residual transmission time of an I-frame transmitted from B to A. Then it holds with probability 1:

$$Z = \begin{cases} a & \text{if } T_{res,B} \leq at_I - (2t_p + t_S) \\ a + 1 & \text{otherwise} \end{cases} \quad (21)$$

We assume that due to the random nature of the transmission errors the arrival times at station B of the first error-free I-frame following the considered one and the RR,P-command following a time-out are purely random with respect to the data flow on the channel B to A. Then the residual transmission time $T_{res,B}$ is uniformly distributed between 0 and t_I . Hence we get for the expectation of Z :

$$E[Z] = a \frac{at_I - (2t_p + t_S)}{t_I} + (a+1)(1 - \frac{at_I - (2t_p + t_S)}{t_I}) = (t_I + 2t_p + t_S)/t_I \quad (22)$$

And finally the expectation of T_r becomes:

$$E[T_r] = t_I + t_p + t_S \quad (23)$$

3.4.3 Time Component T_d

As discussed in Section 3.3, an I-frame which sees a window width $w=0$ at the beginning of its virtual transmission time is delayed for the time T_d until the window opens again. From Fig.7 it can be seen that

$$T_d = T_{ack} - (M-2)t_I \quad (24)$$

where T_{ack} is the time interval between the end of a virtual transmission time of an I-frame and the receipt of its acknowledgment.

We assume for simplicity that the channel B to A is fully loaded by I-frames. Then T_{ack} is given by two times the processing and propagation delay t_p , plus the residual transmission time of the I-frame which occupies channel B to A at the arrival of the considered I-frame at station

B, plus the transmission time of the I-frame carrying the acknowledgment. We can therefore estimate the expectation of T_{ack} by

$$E[T_{ack}] = 2t_p + 1.5t_I \quad (25)$$

3.5 Special Case: No Impact of Modulus

Up to now we have considered the case where the modulus M has an impact on the throughput, namely if $T_{ack} > (M-2)t_I$, i.e. $T_d > 0$.

If $T_{ack} \leq (M-2)t_I$ the virtual transmission time is no longer affected by the modulus M . In this case the time component T_d is equal to zero and it can be seen from Eqs. (5), (12) and (18) that the time components $T_s(w)$ and $T_p(w)$ are independent of the window width w . The mean virtual transmission time can then be written as

$$t_v = t_I \frac{1}{1-p_B} + p_B E[T_e] + \frac{p_B}{1-p_B} E[T_r] + \frac{p_B^2}{1-p_B} (t_{out} + t_s + t_p) \quad (26)$$

with

$$E[T_e] = t_I (1-p_B) \sum_{x=0}^{c-1} (x+1) p_B^x + p_B^c (t_{out} + t_s) + t_p$$

The mean value of T_r is still given by Eq. (23).

3.5 Throughput Results

Figures 10, 11, and 12 show typical calculation and simulation results for the throughput characteristic of the HDLC link as the function of the essential parameters: message length L , processing plus propagation delay t_p , modulus M , transmission rate v , and bit-error probability p_{bit} . Two different link types are considered: a terrestrial link with processing plus propagation delay of 50 msec and transmission rates $v=4.8$ kbit/sec and $v=48$ kbit/sec, and a satellite link with processing plus propagation delay of 350 msec and a transmission rate of 48 kbit/sec. These values for t_p have been borrowed from Ref. /6/.

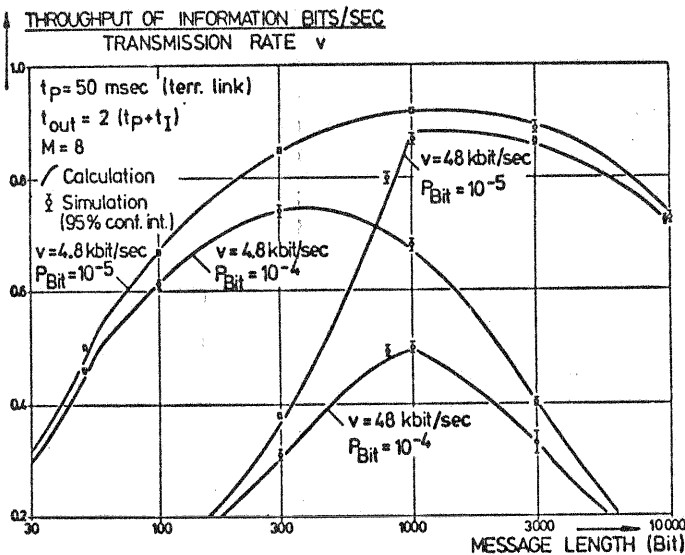


Fig. 10: Throughput efficiency vs message length (terrestrial links).

Fig. 10 shows for the terrestrial link the maximum throughput of information bits per second relative to the transmission rate v as a function of the message length L , i.e. the information field length of the I-frames for two different values of the bit-error probability: 10^{-4} and 10^{-5} . The curves for $v=4.8$ kbit/sec show the typical throughput behavior of link control procedures employing an error detection and retransmission strategy: For short message lengths the throughput of information bits/sec is low due to the relatively large overhead of flag, address, control, and frame checking sequence bits (48 bits in case of $M=8$). For longer message lengths the relative overhead decreases but the block-error probability p_B increases according to

$$p_B = 1 - (1 - p_{bit})^{L+48} \quad (27)$$

Therefore, the throughput curves show a maximum.

An additional characteristic - distinct in particular for $v=48$ kbit/sec - is the impact of the modulus causing a drastic throughput degradation for short message lengths. The reason for this behavior is the aforementioned HDLC rule that a station must stop sending further I-frames if it has 7 unacknowledged I-frames simultaneously outstanding.

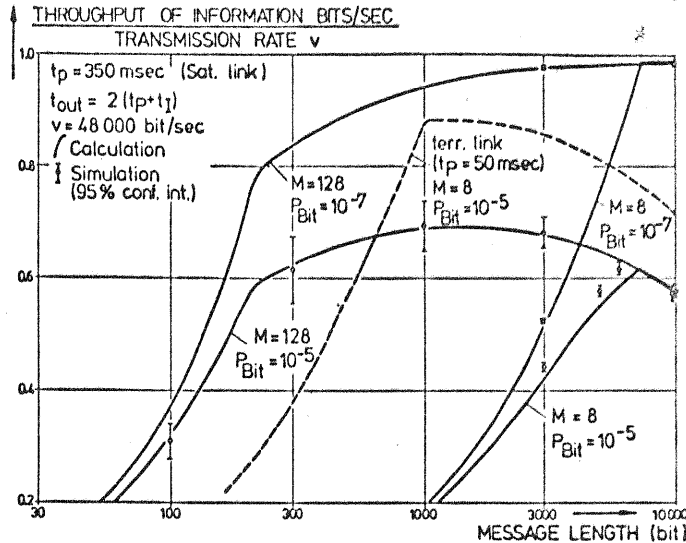


Fig. 11: Throughput efficiency vs message length (satellite links).

The explanation of the throughput behavior of the satellite link in Fig. 11 is virtually the same. Two bit-error probabilities are considered, 10^{-7} and 10^{-5} , where the higher of these values may be caused by a poor quality terrestrial extension of the satellite channels. Of course, the impact of the modulus is more distinct due to the long propagation delay of the satellite channels. Increasing the modulus from 8 to 128 yields a substantial improvement of the throughput. However, as can be seen from the reference curve for the terrestrial link with $M=8$ (dashed line) we cannot again obtain by this means the maximum throughput of the terrestrial link.

The latter effect can be most obviously explained with the aid of Fig. 12. It shows the throughput of information bits per second relative to the transmission rate versus the processing plus propagation delay relative to the I-frame transmission time. For reasons of clarity the length of the S-frames and accordingly the number of overhead bits of the I-frames is held constant at 56 bits in Fig. 12, irrespective of the modulus value employed.

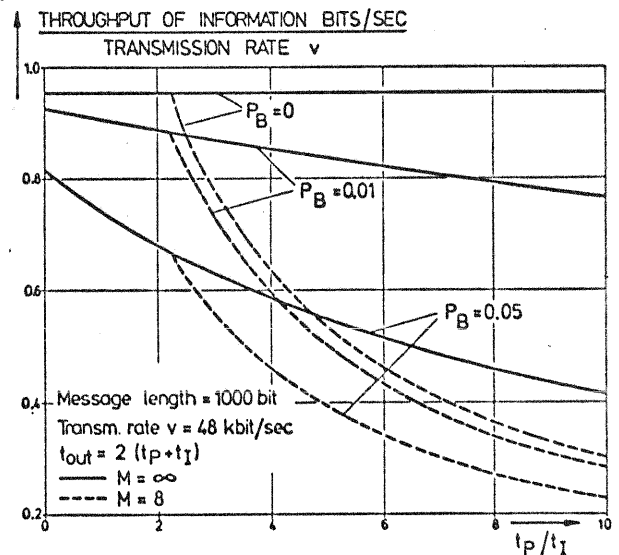


Fig. 12: Throughput efficiency vs processing plus propagation delay relative to I-frame transmission time.

Obviously, the normalized throughput of information bits per second for $M=\infty$ and $p_B=0$ is equal to $1000/1056$, independent of the value of t_p . Considering first the case $M=\infty$ (bold lines in Fig. 12) we can see that for $p_B > 0$ the throughput reduction increases with growing processing and propagation delay t_p . The reason for this effect is that

the error recovery takes longer for longer processing and propagation delays. Formally expressed, this dependency is mainly due to the terms $p_B E[T_e(w)]$ and $p_B/(1-p_B) E[T_r]$ in Eq.(7) where both the expectation of $T_e(w)$ and of T_r include the processing and propagation delay t_p (c.f. Eqs.(12), (18) and (23)).

Furthermore, the dashed lines in Fig.12 reflect the additional throughput reduction caused by the finite modulus $M=8$, which we have discussed above.

4. TRANSFER TIME ANALYSIS

In contrast to the considerations in Section 3 we assume now that the channels are only loaded corresponding to a fraction of their full capacity and that the traffic over the link varies statistically. This situation is typical of interactive traffic, where the transfer time of the transmitted messages represents the essential measure of performance. The transfer time is defined here as the time from the arrival of a message at one station until its successful receipt at the other station. That means, the transfer time encompasses the waiting time of a message and all its transmissions, that are necessary for a successful transfer.

Besides the traffic conditions on the link the following main effects may influence the transfer time of the messages: the channel characteristics (transmission rate, propagation delay, bit-error probabilities), processing delays, and the modulus of the sequence numbers. Generally speaking, the modulus has an impact on performance only if the round-trip delay augments the time to transmit $M-1$ frames. Apart from terrestrial applications with short message lengths this can only be the case with satellite links. However, for real interactive applications where a short transfer time is required, the one-way propagation time of 270 msec of a satellite link is hardly acceptable /10/, especially if an additional delay were caused by a too small modulus M . Therefore, the most important applications, for which the transfer time is of interest, are those cases considered subsequently, where the modulus value has no significant impact. Nevertheless, propagation and processing delays may affect the transfer time; therefore, they are included in our analysis.

4.1 Principle of Analysis

Our objective is to determine the mean transfer time of the messages transmitted over a data link with non-zero error probability and non-zero propagation and processing delays which operates under HDLC, Balanced Class of Procedures including the function REJ. The analysis presented subsequently is a straightforward extension of the approach in Ref. /4/ in the sense that now also propagation and processing delays are taken into consideration.

We assume that the messages to be transmitted to station B (A) arrive at station A(B) according to a Poisson process with rate λ_A (λ_B) (c.f. Fig.6) such that the total channel utilizations are less than 1.

Again, the considerations pertain to one direction of transmission, namely, from station A to station B. Due to the Balanced Class of Procedures the results for the other direction can be obtained in the same way.

The transfer time analysis is also based on the notion of the virtual transmission time, as defined in Section 3. This approach allows to replace the complicated sequence of I- and S-frames in case of transmission and sequence errors by the much simpler sequence of the virtual transmission times, because we can conceive that the transmitted I-frames occupy the channel for the duration of their virtual transmission time.

Therefore, the mean transfer time of the messages can be determined by using well-known results from the theory of M/G/1 queues. The mean transfer time t_f of the messages can be evaluated with the aid of the Pollaczek-Khinchine formula /9/:

$$t_f = \frac{\lambda_A E[T_v^2]}{2(1-\lambda_A E[T_v])} + E[T_v] + t_p \quad (28)$$

Our main problem is to determine the first two moments of the virtual transmission time T_v . This is described in the following subsection.

4.2 Virtual Transmission Time

4.2.1 Decomposition of the Virtual Transmission Time

Without loss of generality we assume that the following relation holds among the time-out duration t_{out} and the I-frame transmission time t_I :

$$kt_I \leq t_{out} < (k+1)t_I \quad (29)$$

As shown in Fig.13 we decompose the virtual transmission time of a disturbed I-frame into appropriate components.

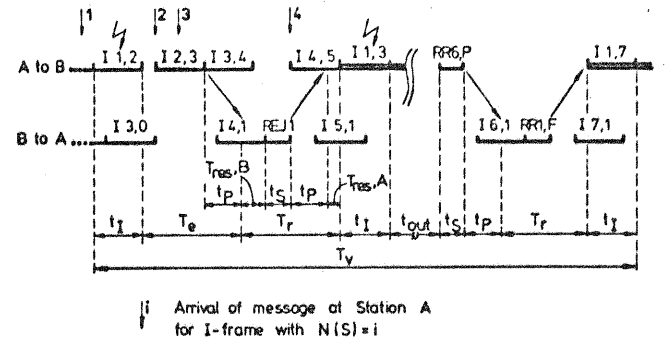


Fig. 13: Decomposition of the virtual transmission time.

If the considered I-frame is received without transmission and sequence error, its virtual transmission time is equal to t_I . If N transmissions of the considered I-frame are disturbed ($N \geq 1$), its virtual transmission time is composed as follows (c.f. Fig.13):

$$T_v = 2t_I + T_e + T_r + (N-1)(t_I + t_{out} + t_s + t_p + T_r) \quad (30)$$

Since the number of transmissions N is geometrically distributed with parameter p_B , the first two moments of T_v are given by

$$E[T_v] = \frac{1}{1-p_B} t_I + p_B E[T_e] + \frac{p_B}{1-p_B} E[T_r] + \frac{p_B^2}{1-p_B} (t_{out} + t_s + t_p) \quad (31)$$

$$E[T_v^2] = (1+3p_B)t_I^2 + p_B E[T_e^2] + \frac{p_B^2(1+p_B)}{(1-p_B)^2} \theta^2 + \frac{p_B}{1-p_B} E[T_r^2] + 4t_I(p_B E[T_e] + \frac{p_B}{1-p_B} \theta + \frac{p_B}{1-p_B} E[T_r]) + 2E[T_e](\frac{p_B}{1-p_B} \theta + \frac{p_B}{1-p_B} E[T_r]) + 4\frac{p_B^2}{(1-p_B)^2} \theta E[T_r] + \frac{2p_B^2}{(1-p_B)^2} E[T_r]^2 \quad (32)$$

$$\text{with } \theta = t_{out} + t_I + t_s + t_p$$

For Eq.(32) it is assumed that T_e and T_r are stochastically independent. This assumption is not valid in a strict sense, but it represents an acceptable working basis, as can be seen from our simulation results.

In the two subsections that follow, it is shown how the first two moments of T_e and T_r , which are required to evaluate $E[T_v]$ and $E[T_v^2]$ in Eqs.(31),(32) can be determined.

4.2.2 Time Component T_e

In Section 3.4.1 the general definition of the time components $T_e(w)$ was given. As discussed above we do not take into account the finite modulus for our derivation of the mean transfer time. Therefore, only a unique time component T_e must be considered here, irrespective of the actual window width w . Apart from this simplification the definition of the time component T_e is identical to that of the times $T_e(w)$. To determine its first two moments we proceed as follows.

Let ξ_x be the time between end of transmission of the considered I-frame with $N(S)=i$ and the end of transmission of the I-frame with $N(S)=i+x \pmod{M}$. We define a new random variable $T_{e,x}$ by

$$T_{e,x} = \begin{cases} \xi_x + t_p & \xi_x \leq t_{out} + t_I \\ t_{out} + t_S + t_p & \text{otherwise} \end{cases} \quad (33)$$

As can be seen from Fig.14 $T_{e,x}$ is equal to T_e , provided the I-frame with $N(S)=i+x \pmod{M}$ is the first I-frame without transmission error following the considered I-frame with $N(S)=i$. Furthermore, Fig.14 illustrates the two cases which must be distinguished: If $\xi_x \leq t_{out} + t_I$ then the first error-free I-frame is transmitted before the time-out expires (REJ-recovery), otherwise the time-out expires (REJ-recovery), otherwise the time-out expires (REJ-recovery).

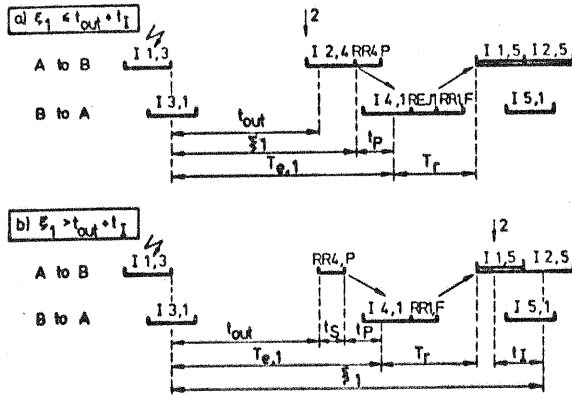


Fig. 14: Definition of $T_{e,x}$ (here $x=1$).

Using the random variables $T_{e,x}$ defined in such a manner, the distribution and the first two moments of T_e can be written as

$$P(T_e \leq t) = \sum_{x=1}^{\infty} (1-p_B) p_B^{x-1} P(T_{e,x} \leq t) \quad (34)$$

$$E[T_e] = \sum_{x=1}^{k+1} (1-p_B) p_B^{x-1} E[T_{e,x}] + p_B^{k+1} (t_{out} + t_S + t_p) \quad (35)$$

$$E[T_e^2] = \sum_{x=1}^{k+1} (1-p_B) p_B^{x-1} E[T_{e,x}^2] + p_B^{k+1} (t_{out} + t_S + t_p)^2 \quad (36)$$

The random variable ξ_x defined above can be conceived as the time until the x -th departure of a customer from an M/D/1 queue measured from a departure epoch. The distribution of ξ_x has been derived by PACK [11]:

$$F_x(t) = P(\xi_x \leq t) = \begin{cases} 1 - \sum_{m=0}^{x-1} (1-Q_{x-m}(t)) \pi_m & t \geq x t_I, x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

where $Q_n(t)$ is defined as

$$Q_n(t) = \begin{cases} 1 - \sum_{k=1}^n \lambda_A^{n-k} e^{-\lambda_A(t-kt_I)} \frac{(t-kt_I)^{n-k-1}}{(n-k)!} & t \geq n t_I, n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (38)$$

The quantities π_m in Eq. (37) are the steady-state probabilities of the queue length at the departure instants:

$$\pi_m = \begin{cases} \bar{\pi}_m - \bar{\pi}_{m-1} & m \geq 1 \\ \bar{\pi}_0 & m = 0 \end{cases} \quad (39)$$

$$\bar{\pi}_m = (1-\lambda_A t_I) \sum_{i=0}^m e^{i \lambda_A t_I} \frac{(-i \lambda_A t_I)^{m-1}}{(m-1)!} \quad (40)$$

Herewith, the following expressions for the first two moments of $T_{e,x}$ can be derived:

$$E[T_{e,x}] = t_{out} + t_S + (t_I - t_S) F_x(t_{out} + t_I) + F_x^*(x t_I) - F_x^*(t_{out} + t_I) + t_p \quad (41)$$

$$E[T_{e,x}^2] = (t_{out} + t_S + t_p)^2 + 2 t_p (t_I - t_S) F_x(t_{out} + t_I) + ((t_{out} + t_I)^2 - (t_{out} + t_S)^2) F_x(t_{out} + t_I) - 2(t_{out} + t_I + t_p) F_x^*(t_{out} + t_I) + 2 F_x^{**}(t_{out} + t_I) + 2(x t_I + t_p) F_x^*(x t_I) - 2 F_x^{**}(x t_I) \quad (42)$$

$$\text{with } F_x^*(t) = \int_0^t F_x(t) dt, \quad F_x^{**}(t) = \int_0^t F_x^*(t) dt$$

In Ref./4/ it is shown that a simple recursive relation holds for the integral expressions F_x^* and F_x^{**} in Eqs. (41) and (42). Therefore, $E[T_e]$ and $E[T_e^2]$ can be straightforwardly computed by substituting Eqs. (41) and (42) into Eqs. (35) and (36).

4.2.3 Time Component T_r

The general definition of the time T_r is given in Section 3.4.2. According to Fig.13 we decompose T_r into four components:

$$T_r = T_{res,B} + t_S + T_{res,A} + t_p \quad (43)$$

Assuming independence of $T_{res,A}$ and $T_{res,B}$ the first two moments of T_r are given by

$$E[T_r] = E[T_{res,B}] + t_S + E[T_{res,A}] + t_p \quad (44)$$

$$E[T_r^2] = E[T_{res,B}^2] + t_S^2 + E[T_{res,A}^2] + t_p^2 + 2E[T_{res,B}](t_S + E[T_{res,A}] + t_p) + 2E[T_{res,A}](t_S + t_p) + 2t_S t_p \quad (45)$$

In order to estimate the first two moments of $T_{res,A}$ and $T_{res,B}$, we assume that the arrival times at station B of the first correct I-frame following the considered one and the arrival times of the RR-frames with the P-bit set to 1 following a time-out as well as the arrival times at station A of the REJ-frames or the RR-frames with the F-bit set to 1 are purely random. Then $T_{res,A}$ and $T_{res,B}$ can be considered as residual life times of a renewal process with constant interevent time t_I , if the channel is occupied. This consideration leads to

$$E[T_{res,A}] = \lambda_A t_I^2 / 2, \quad E[T_{res,A}^2] = \lambda_A t_I^3 / 3 \quad (46)$$

$$E[T_{res,B}] = \lambda_B t_I^2 / 2, \quad E[T_{res,B}^2] = \lambda_B t_I^3 / 3 \quad (47)$$

4.3 Transfer Time Results

Figure 15 shows the mean transfer time of the messages as the function of the useful channel load

$$Y_u = \lambda_A \ell / v \quad (48)$$

It should be noted that the increase of the mean transfer time due to transmission errors is significant even for the rather small block-error probability of 0.01 corresponding to a bit-error probability of 2×10^{-6} . This is again caused by the multiplicative interaction of block-error probability and processing plus propagation delay.

The latter effect is illustrated in Fig.16. It shows the mean transfer time t_f relative to the I-frame transmission time t_I as a function of the processing plus propagation delay t_p relative to t_I . Since the queueing delay in case of $p_B=0$ is constant irrespective of the value of t_p the increase in this case is only caused by the growing value of t_p . For non-zero block-error probability p_B the curves show an increasing slope which is caused by the fact that the recovery takes longer for higher values of t_p . Here the performance degradation is even more distinct than in the saturated case because - in terms of our analytic approach - a higher value of t_p increases not only the mean of the virtual transmission time but also its variation. This in turn leads to longer waiting and transfer times, see Eq. (28).

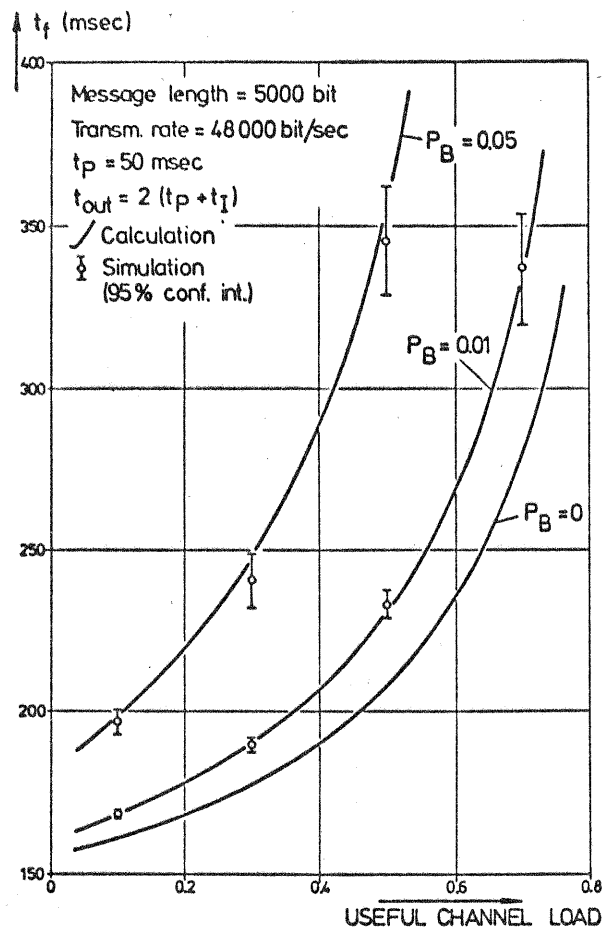


Fig. 15: Mean transfer time vs useful channel load (terrestrial link).

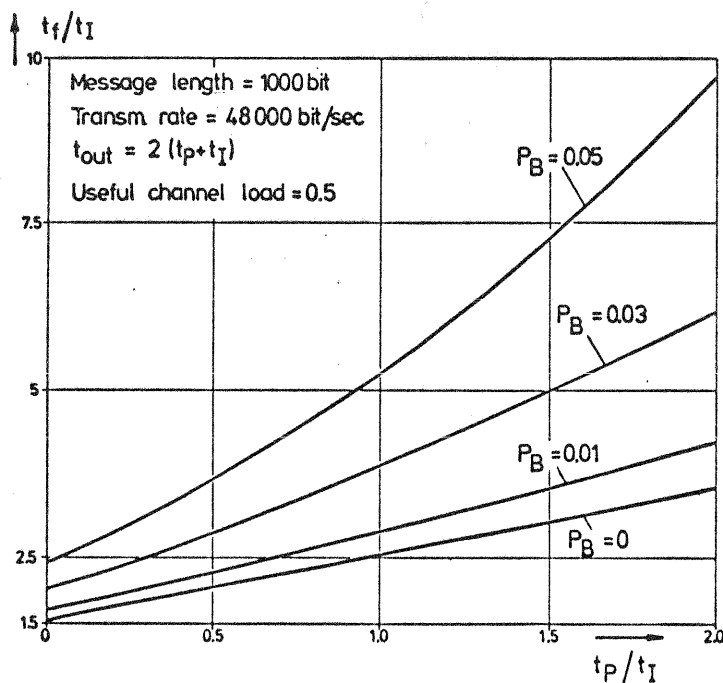


Fig. 16: Mean transfer time relative to I-frame transmission time vs processing plus propagation delay relative to I-frame transmission time.

5. CONCLUSION

The major contribution of this paper is to extend the performance analysis of HDLC-controlled data links to those cases where the processing and propagation delays cannot be neglected. It could be demonstrated that the notion of the virtual transmission time of the information frames can be generalized to handle also links with non-zero processing and propagation delays.

The developed approach possesses three main features:

- (1) It provides a method for a fast and accurate performance evaluation of HDLC-controlled data links.
- (2) The concept of the virtual transmission time allows to replace the complex model of an HDLC link by a much simpler queueing model with approximately the same performance characteristics. This property can be used to advantage in the performance analysis of protocols above the link control level.
- (3) The notion of the virtual transmission time provides a sound explanation of the performance properties of HDLC links, like e.g. the multiplicative effect with respect to performance degradation of processing and propagation delays, and the block-error probability of the Information frames.

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