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TOKEN-RING PERFORMANCE: MEAN-DELAY APPROXIMATION

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This paper investigates mean delays in local-area ring systems using a permission token access technique. An approximate analysis is performed for an arbitrary number of stations, non-zero token-passing overhead, exhaustive service discipline, Poisson arrival processes, and general message-length distributions. Neither interarrival time, nor message-length distributions have to be identical for the different stations. A major value of our approximation lies in the simplicity of the numerical evaluation for an arbitrary number of stations and any traffic pattern. Extensive comparisons with simulation results show the accuracy of the approximation over a wide range of parameters.

1. INTRODUCTION

Local-area networks with either ring or bus topology can use a token mechanism for medium access [1-8]. For a bus topology, the token consists of a special control frame. A station terminates its own transmission period by sending a token frame addressed to its successor in the access sequence. In contrast to this, tokens on a ring system do not carry an address, but are realized by one bit position (or more) within a well-defined bit pattern. After a station has finished its transmission, it generates this pattern which then circulates around the ring until a station ready to transmit changes the token from "free" to "busy" and appends its data. In the present paper, we study the performance of token-controlled ring systems. However, the queueing model presented and the analysis also apply to token-controlled buses.

Queueing models applicable to token rings have been extensively studied, primarily in the context of polling systems [8-26]. In the present paper, we provide an approximate analysis for an important subset of the different schemes possible for token rings. The corresponding queueing model has been treated exactly in the literature [15-19]; however, the numerical evaluation of the exact solution is restricted to a rather small number of stations attached to a ring. The reason is that — except for very special cases — a system of equations of order n^2 to n^3 has to be solved (n is the number of stations) to calculate the mean queueing delays.

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Experience with the numerical evaluation of the exact solution for a larger number of stations has not been reported in the literature; in particular, no simple algorithm is known to determine the coefficients of these equations in case of many stations.

In designing and configuring ring networks with token access, there is a practical need for an accurate delay formula which can be easily computed for an arbitrary number of stations. The objective of the present paper is to provide such a result.

2. THE MODEL

Fig. 1 shows a queueing model of a token ring with four active stations represented by their transmit queues. For convenience, we assume that both transmit and receive buffers of the stations are not limited in size. The stations are serviced in a cyclic manner by a single server standing for the ring. On token rings, different token-generation strategies can be employed; the general impact on the delay-throughput characteristic of these strategies has been shown in [6,26]. In the present paper, we assume what is called "multiple-token" operation in [26], namely, that stations issue a free token immediately after the end of their transmission.

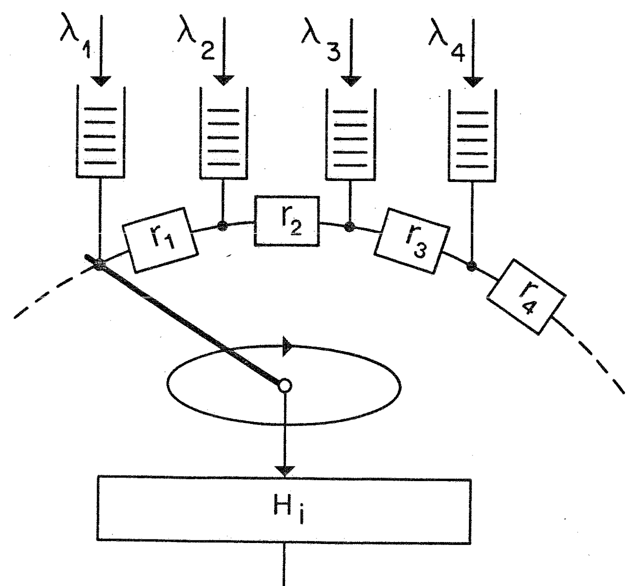

Fig. 1. Token-ring queueing model.

In the model of Fig. 1, the time needed to pass the token from station i to station $(i + 1)$ is modeled by a constant delay r_i. On an actual ring, the delay r_i corresponds to the propagation delay of the signals between stations i and $(i + 1)$ plus the latency caused within station i by the repeater and by actions such as alteration of the token bit. The station latency is usually in the order of one bit time. Subsequently, we denote by R the total ring latency, i.e., the sum of all token-passing times $r_1 + r_2 + \ldots + r_n$.

Messages arrive from all stations according to Poisson processes with rates $\lambda_1, \lambda_2, \ldots, \lambda_n$. The entity of transmission on the ring is called a "frame". It consists of the user message and a constant overhead for frame delimitation, control information, addresses, and a frame check sequence. The message lengths are determined through an arbitrarily selectable distribution. We denote by h_i and $h_i^{(2)}$, respectively, the mean and second moments of the service/transmission times H_i of frames transmitted by station i. The traffic $\lambda_i h_i$ offered by station i is denoted by ρ_i, and we assume that the sum ρ of all ρ_i is less than one.

Several service disciplines can be distinguished. In this paper, we concentrate on the exhaustive type of service discipline where a station is allowed to completely empty its transmit queue when it holds the token. (Alternative disciplines would be to allow a station to transmit either one frame per access opportunity, or up to a specified number of frames, or for a given maximum period of time.)

In token rings, the sender is responsible for removing the frames it transmitted from the ring. Therefore, the location of frame destinations on the ring relative to the location of the sender does not affect performance. Consequently, we have only to specify the input traffic of the stations. Hence, qualifiers, such as "symmetrical/asymmetrical" or "balanced/unbalanced" traffic, are meant with respect to traffic sources, not sinks.

3. MEAN WAITING TIME ANALYSIS

3.1 Approximation

To determine the waiting time of messages in station i, it is appropriate to view the model of Fig. 1 as an M/G/1 queue with server vacation times. The server vacation time A_i corresponds to what in [15] is called "intervisit time", i.e., the time interval from the server's departure from queue i until its return to the same queue. This consideration leads direct to the following relation for the mean waiting time of messages in queue i, see also [15]:

$$w_i = \frac{E[A_i]}{2} + \frac{\mathrm{Var}[A_i]}{2E[A_i]} + \frac{\lambda_i h_i^{(2)}}{2(1-\rho_i)} . \quad (1)$$

This means that the average delays can be determined if the first two moments of the intervisit times are known. Whereas it is straightforward to determine the first moment of A_i, calculation of the second moment is fairly involved. Our approach is to employ an approximation for the second moment based on a heuristic extrapolation from the exact result for $n = 2$ stations to the case of an arbitrary value of n.

Let C_i be the cycle time defined as the time between subsequent visits of the server to queue i, and T_i be the time which the server spends to service queue i. Furthermore, let $\phi_{A_i}(s)$ and $\phi_{T_i}(s)$ be the Laplace transforms of the probability density functions of A_i and T_i. The time T_i for which the server is serving queue i can be conceived as the sum of M_i independent busy periods B_i, where M_i is the number of arrivals at queue i during A_i. Therefore, if we denote by $\phi_{B_i}(s)$ the Laplace transform of the p.d.f. of the busy periods, and by $G_{M_i}(z)$ the generating function of the distribution of M_i, we obtain

$$\phi_{T_i}(s) = G_{M_i}(\phi_{B_i}(s)) \quad (2)$$

and

$$G_{M_i}(z) = \phi_{A_i}(\lambda_i - \lambda_i z). \quad (3)$$

Insertion of (3) into (2) yields

$$\phi_{T_i}(s) = \phi_{A_i}(\lambda_i - \lambda_i \phi_{B_i}(s)). \quad (4)$$

By differentiating Eq. (4) twice, we obtain

$$E[T_i] = \lambda_i \, E[A_i] \, E[B_i], \quad (5)$$

$$E[T_i^2] = \lambda_i \, E[A_i] \, E[B_i^2] + \lambda_i^2 \, E[A_i^2] \, E[B_i]^2. \quad (6)$$

The first two moments of the busy periods are given by (see, e.g., [11,27])

$$E[B_i] = \frac{h_i}{1-\rho_i} , \quad (7)$$

$$E[B_i^2] = \frac{h_i^{(2)}}{(1-\rho_i)^3} . \quad (8)$$

Insertion of (7) into (5) yields

$$E[T_i] = \frac{\rho_i}{1-\rho_i} E[A_i] = E[C_i] - E[A_i] \quad (9)$$

and hence

$$E[A_i] = (1-\rho_i) \, E[C_i]. \quad (10)$$

It can be shown that the average cycle time for all queues is given by (see e.g. [15])

$$E[C_i] = \frac{R}{1-\rho} . \tag{11}$$

This leads to

$$E[A_i] = \frac{1-\rho_i}{1-\rho} R . \tag{12}$$

If we insert (7), (8), and (12) into (6), we obtain

$$\mathrm{Var}[T_i] = \lambda_i \frac{1-\rho_i}{1-\rho} R \frac{h_i^{(2)}}{(1-\rho_i)^3} + \lambda_i^2 \mathrm{Var}[A_i] \frac{h_i^2}{(1-\rho_i)^2} . \tag{13}$$

For an arbitrary number of stations n, the above considerations are apparently insufficient to determine the variance of the intervisit times A_i needed in (1) to calculate the mean waiting times. In the special case of $n = 2$ stations, however, the intervisit times of one queue and the server's sojourn times at the other queue differ only by the constant time R; hence, we have

$$\mathrm{Var}[T_i] = \mathrm{Var}[A_k] \; ; \; i = 1,2, \quad k = 3-i. \tag{14}$$

Inserting (14) into (13) and solving for Var (A_i) leads to

$$\mathrm{Var}[A_i] = \frac{R}{(1-\rho)^2} \frac{\lambda_k h_k^{(2)} (1-\rho_i)^2 + \lambda_i h_i^{(2)} \rho_k^2}{1-\rho_i-\rho_k + 2\rho_i\rho_k} . \tag{15}$$

$$i = 1,2, \quad k = 3-i$$

Returning to our general model with $n > 2$ stations, we now establish our major heuristic assumption. We assume (a) that the impact of the messages from any queue $k \neq i$ on the variance of the intervisit time of queue i can be approximately described by an expression corresponding to Eq. (15), and (b) that the total variance of A_i is obtained by superposition of the individual components of all queues, i.e.,

$$\mathrm{Var}[A_i] = \frac{R}{(1-\rho)^2} \sum_{\substack{k=1 \\ k\neq i}}^{n} \frac{\lambda_k h_k^{(2)} (1-\rho_i)^2 + \lambda_i h_i^{(2)} \rho_k^2}{1-\rho_i-\rho_k + 2\rho_i\rho_k} . \tag{16}$$

By inserting (12) and (16) into (1), we finally obtain the following approximate formula for the mean waiting time at station i:

$$w_i = \frac{1-\rho_i}{2(1-\rho)} R + \frac{\lambda_i h_i^{(2)}}{2(1-\rho_i)} + \frac{1}{2(1-\rho)(1-\rho_i)} \sum_{\substack{k=1 \\ k\neq i}}^{n} \frac{\lambda_k h_k^{(2)} (1-\rho_i)^2 + \lambda_i h_i^{(2)} \rho_k^2}{1-\rho_i-\rho_k + 2\rho_i\rho_k} . \tag{17}$$

3.2 Discussion

In Section 4, by comparison with simulation results we shall show that the accuracy of our approximation is adequate for all practical purposes. Furthermore, as subsequently discussed, Eq. (17) has the attractive property of yielding the exact solution for a number of special cases.

1) In case of two stations with arbitrary service-time distributions and arrival rates, our delay formula yields the exact result known from [11] and [12].

2) For an arbitrary number of stations but symmetrical traffic conditions, i.e., identical service-time distributions with mean h and second moment $h^{(2)}$, and equal arrival rates $\lambda_1 = \ldots = \lambda_n = \lambda/n$, from (17) we obtain the following mean delay for all stations:

$$w_{sym} = \frac{1-\rho/n}{2(1-\rho)} R + \frac{\lambda h^{(2)}}{2(1-\rho)} . \tag{18}$$

Comparison with the results derived in [16] and [17] shows that (18) represents the exact result for this particular case.

3) Summation over the station-specific delays weighted with the corresponding utilization yields

$$\tilde{w} = \sum_{i=1}^{n} \frac{\rho_i}{\rho} w_i = \frac{1 - \frac{1}{\rho} \sum_{i=1}^{n} \rho_i^2}{2(1-\rho)} R + \frac{\sum_{i=1}^{n} \lambda_i h_i^{(2)}}{2(1-\rho)} . \tag{19}$$

This result is noteworthy in two respects. First, for zero token-passing overhead, i.e., $R = 0$, it yields the correct M/G/1 delay, as required by the conservation law [27]. Second, it suggests that the more unbalanced the traffic, the smaller the impact of the token-passing overhead on the delay averaged over all stations. This is an intuitively appealing result since the more unequal the utilizations of the different stations, the higher the probability that a newly arriving message will join a non-empty queue or one which is currently being serviced. Such a message, however, will be serviced without any token-passing overhead.

4. NUMERICAL RESULTS

In this section, we present numerical examples in order to discuss some general characteristics of token-ring models and, in particular, to show the accuracy of our approximate solution.

In Fig. 2, we show a comparison between analytic and simulation results for a token-ring model with sixteen stations. The parameters chosen are 1 Mbps transmission rate, 1 km cable length, and 1 bit latency per station, which results in a total ring latency of $R = 21$ μs (see [26]). Message lengths are exponentially distributed with mean 2 kbit; the framing overhead is 48 bit per message.

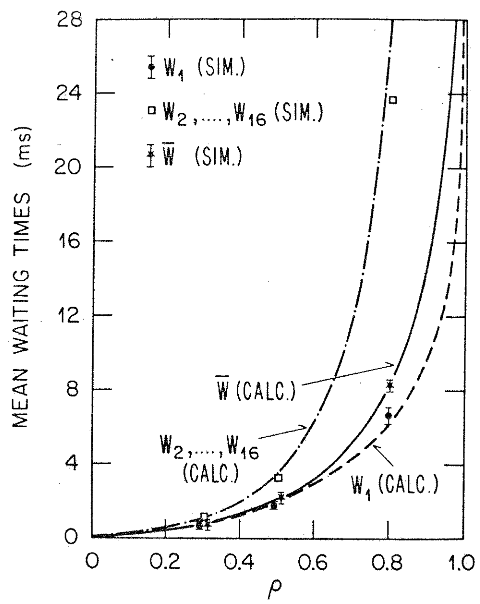

Fig. 2. Comparison of analytic and simulation results for a 16-station token-ring model with one heavy-traffic station: $\rho_2 = \rho_3 = \ldots = \rho_{16} = \rho_1/150$. Parameters: 11 Mbps transmission rate, 1 km cable length, 1 bit latency per station (queue), i.e., R = 21 µs; exponentially distributed message lengths (mean 2 kbit), 48 bit framing overhead per message.

For this example, a very unbalanced traffic flow was assumed, viz., that 10/11 of the total ring traffic is generated by one station; the other 15 stations together generating only 1/11 of the traffic in equal amounts.

We observe that messages from the heavy-traffic station 1 suffer a smaller delay than those from the light-traffic stations 2 to 16. This — at first glance — surprising effect can be explained by the fact that messages being generated by a heavily loaded station have a better chance that their station is currently transmitting than those ones from lightly loaded stations. It should be noted that the simulation values shown for stations 2-16 represent the average of the individual simulation results for these stations. The simulated specific delays for stations 2 - 16 vary slightly around this average. The simulation does not reveal any significant interdependence of location relative to the heavy-traffic source and delay, although such differences do exist in principle [28].

Furthermore, Fig. 2 shows that our approximate solution yields results of satisfactory accuracy. We can generally observe that it has the tendency to slightly underestimate the delay of heavy-traffic stations, and slightly overestimate the delay of light-traffic stations. Except for very high utilizations, the relative error stays well below 10%. (The relative error is defined as the difference between calculated and simulated delay divided by the simulated delay.) The accuracy of the waiting time averaged over all stations $\bar{w}$ is very high.

For a token-ring model of the same kind, but with 20 stations, Table 1 compares analytic and simulation results in another unbalanced traffic flow situation. The underlying assumption of this example is that stations 1 and 8 generate 40% of the total traffic each; the rest is generated by the other 18 stations. Analytic and simulation results again agree very well; the accuracy of the approximation is slightly better than for the case of Fig. 2. This suggests that a highly unbalanced situation such as the one underlying Fig. 2 represents an unfavorable case with respect to accuracy of our approximation.

	ρ	0.3	0.5	0.8
w_1 [ms]	Calc.	0.444	0.994	3.760
	Sim.	0.445 (±0.018)	1.009 (±0.057)	3.947 (±0.425)
	Error	-0.2%	-1.6%	-4.7%
w_8 [ms]	Calc.	0.444	0.994	3.760
	Sim.	0.448 (±0.021)	0.997 (±0.046)	3.896 (±0.316)
	Error	-0.9%	-0.3%	-3.5%
$w_2,\ldots,w_7$, $w_9,\ldots,w_{20}$, [ms]	Calc.	0.505	1.248	5.660
	Sim.	0.499	1.236	5.693
	Error	1.1%	1.0%	-0.6%
$\bar{w}$ [ms]	Calc.	0.456	1.044	4.140
	Sim.	0.457 (±0.024)	1.049 (±0.062)	4.276 (±0.448)
	Error	-0.2%	-0.4%	-3.2%

Table 1. Comparison of analytic and simulation results for a 20-station token-ring model with two heavy-traffic stations: $\rho_1 = \rho_8 = 0.4\rho$; $\rho_2 = \ldots = \rho_7 = \rho_9 = \ldots = \rho_{20} = \rho/90$. Parameters: 1 Mbps transmission rate, 1 km cable length, 1 bit latency per station (queue), i.e., R = 25 µs; exponentially distributed message lengths (mean 1024 bit), 48 bit framing overhead per message; (95% confidence intervals in parenthesis). (Note that the calculation results are based on the following values for ρ: 0.3003, 0.4996, 0.79973.)

In Table 2, we show the delays occurring on 32 Mbps rings with 5 and 10 km cable lengths which represent extremely large values for local-area token rings. Traffic pattern and message-length distribution are the same as for Table 1. Furthermore, it is assumed that each station causes a latency of 8 bits. This results in total ring latencies of R = 30 µs and R = 55 µs, respectively; hence, the ratios of ring latency to mean frame transmission time are rather high, namely, 0.90 and 1.64. Comparison of Tables 1 and 2 shows that the

a) $R = 30\ \mu s$

	ρ	0.3	0.5	0.8
w_1 [μs]	Calc.	32.221	54.470	167.354
	Sim.	32.205	54.595	168.45
		(±0.880)	(±0.392)	(±3.584)
	Error	0.05%	-0.2%	-0.7%
w_8 [μs]	Calc.	32.221	54.470	167.354
	Sim.	32.360	54.498	167.37
		(±0.796)	(±0.538)	(±3.932)
	Error	-0.4%	-0.1%	-0.1%
$w_2,\ldots,w_7,$ $w_9,\ldots,w_{20},$ [μs]	Calc.	36.546	68.122	249.588
	Sim.	36.884	68.187	246.77
	Error	-0.9%	-0.1%	1.1%
$\bar{w}$ [μs]	Calc.	33.086	57.200	183.801
	Sim.	33.210	57.284	183.66
		(±0.746)	(±0.464)	(±4.132)
	Error	-0.4%	-0.1%	0.1%

b) $R = 55\ \mu s$

	ρ	0.3	0.5	0.8
w_1 [μs]	Calc.	47.935	74.470	209.854
	Sim.	47.887	74.367	209.98
		(±0.666)	(±0.624)	(±4.148)
	Error	0.1%	0.1%	-0.1%
w_8 [μs]	Calc.	47.935	74.470	209.854
	Sim.	48.328	74.449	209.84
		(±0.836)	(±0.563)	(±3.819)
	Error	-0.8%	0.03%	0.01%
$w_2,\ldots,w_7,$ $w_9,\ldots,w_{20},$ [μs]	Calc.	54.344	92.983	311.533
	Sim.	54.789	92.831	309.079
	Error	-0.8%	0.2%	0.8%
$\bar{w}$ [μs]	Calc.	49.217	78.173	230.190
	Sim.	49.449	78.100	229.72
		(±0.771)	(±0.305)	(±4.456)
	Error	-0.5%	0.1%	0.2%

Table 2. Comparison of analytic and simulation results for a 20-station token ring model with two heavy-traffic stations and greater relative token-passing overhead: $\rho_1 = \rho_8 = 0.4\rho$; $\rho_2 = \ldots = \rho_7 = \rho_9 = \ldots = \rho_{20} = \rho/90$. Parameters: 32 Mbps transmission rate, 5 and 10 km cable length, 8 bit latency per station (queue), i.e., $R = 30\ \mu s$ and $R = 55\ \mu s$; exponentially distributed message lengths (mean 1024 bit), 48 bit framing overhead per message; (95% confidence intervals in parenthesis).

accuracy of the approximation improves when the ratio of latency to frame transmission time is increased. This indicates that our approximation reflects very accurately the impact of the token-passing overhead on delay.

The examples considered so far were based on identical message-length distributions for all stations. With the final example, we demonstrate that our approximation yields reasonably accurate results also for cases with different message-length distributions for different stations. Table 3 shows the mean waiting times in a queueing model of a token ring with 20 stations. Utilization values are the same as for Tables 1 and 2. Messages from the heavy-traffic stations are assumed to be eight times longer on the average than those from light-traffic stations. Moreover, the coefficients of variation of the message lengths are also different: 1.0 and 2.0 for the heavy-traffic stations 1 and 8, respectively, and 0.5 for the light-traffic stations. Comparing results for stations 1 and 8 reveals an interesting effect: although both stations generate the same amount of traffic and

	ρ	0.3	0.5	0.8
w_1 [ms]	Calc.	1.372	3.118	12.001
	Sim.	1.347	2.896	13.126
		(±0.194)	(±0.276)	(±2.199)
	Error	1.8%	7.7%	-8.6%
w_8 [ms]	Calc.	1.218	2.545	8.927
	Sim.	1.198	2.513	10.922
		(±0.146)	(±0.205)	(±1.261)
	Error	1.7%	1.3%	-18.3%
$w_2,\ldots,w_7,$ $w_9,\ldots,w_{20},$ [ms]	Calc.	1.515	3.744	17.182
	Sim.	1.489	3.512	17.863
	Error	1.7%	6.6%	-3.8%
$\bar{w}$ [ms]	Calc.	1.434	3.409	14.716
	Sim.	1.408	3.215	15.710
		(±0.229)	(±0.277)	(±2.219)
	Error	1.8%	6.0%	-6.3%

Table 3. Comparison of analytic and simulation results for a 20-station token-ring model with two heavy-traffic stations and different service-time distributions: $\rho_1 = \rho_8 = 0.4\rho$; $\rho_2 = \ldots = \rho_7 = \rho_9 = \ldots = \rho_{20} = \rho/90$. Parameters: 1 Mbps transmission rate, 1km cable length, 8 bit latency per station (queue), i.e., $R = 165\ \mu s$; mean message lengths: 2048 bit for queues 1 and 8, 256 bit for all other queues; coefficient of variation of message-length distributions: 1.0 for queue 1, 2.0 for queue 8, 0.5 for all other queues; 48 bit framing overhead per message; (95% confidence intervals in parenthesis).

the coefficient of the message-length distribution is greater for station 8, the delay in station 1 is always slightly longer than in station 8. An intuitive explanation of this phenomenon is that, due to the greater variance of the service times of station 8, the chance that a message finds its own station being served upon its arrival is greater for station 8. Our analysis reflects this effect correctly, although its accuracy decreases at higher utilizations.

5. CONCLUSIONS

Modeling of token rings leads to single-server models with cyclic service among an arbitrary number of queues. For the exhaustive service discipline, rigorous analyses under the assumption of Poisson input, generally distributed message lengths, and non-zero token-passing overhead have appeared in the literature. From a practical traffic-engineering point of view, a drawback in the application of these results is that their numerical evaluation is difficult, except for certain special cases.

To overcome these limitations, the present paper suggests a rather simple approximate solution for the mean delays of individual stations. The important features of our solution are:

1) Its overall accuracy is good. Comparison with simulation results shows that, except for very high utilizations and extremely unbalanced traffic flow, the relative error is less than 10%.

2) For the special cases of (a) two stations and arbitrary traffic, and (b) an arbitrary number of stations but symmetrical traffic conditions, the solution is exact. Furthermore, in case of zero token-passing overhead, it yields the correct result for the delay averaged over all stations, as required by the M/G/1 conservation law.

3) The numerical evaluation of the approximate delay formula is very simple for an arbitrary number of stations and any kind of traffic pattern.

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