

INTERNAL TRAFFIC
AND ITS EFFECT ON THE CONGESTION
IN SWITCHING SYSTEMS

by

D. BOTSCH

INSTITUT FÜR NACHRICHTENVERMITTLUNG
UND DATENVERARBEITUNG
TECHNISCHE HOCHSCHULE STUTT GART

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by D. Botsch

Institute for Switching Systems and Data Processing
Technical University Stuttgart

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Summary

Two methods are described to calculate the loss of a switching system carrying external and internal traffic and hunted with full or limited availability.

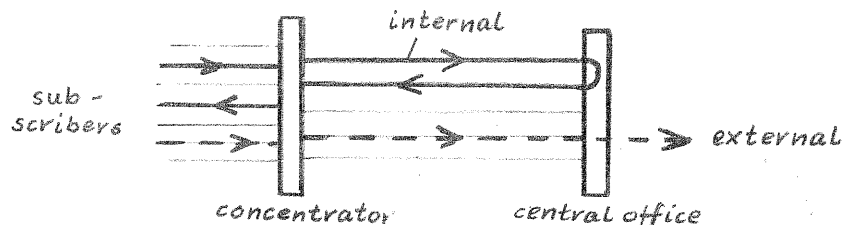
The first method (chapters 3,4,5) takes only the mean values of traffic into account, but the results are in most cases sufficiently accurate. Familiar loss tables may be used.

The second method (chapters 6,7) is based on the conditions for statistical equilibrium leading to the exact solution. The formulae can only be evaluated with electronic computers.

1. Introduction

This paper deals with switching arrangements, that can be loaded by the outgoing as well as incoming traffic of a subscriber group. A certain part of the traffic - i.e. the traffic between subscribers of the same group - runs twice through the switching arrangement, outgoing as well as incoming. This kind of traffic will be called *internal traffic*. Each call of this internal traffic needs two devices. Furthermore, there also exists a traffic to and from subscribers of other groups. This traffic runs only once through the switching arrangement - either in outgoing or incoming direction - and will be called *external traffic*.

For example, we find both external and internal traffic on the trunks between a line concentrator and the central office:



The familiar methods to determine the number of devices N for given values A (offered traffic), k (availability), and B (probability of loss) can be used only for external traffic, i.e. traffic, that occupies only one device for each call. Internal traffic has different statistical properties, because every successful call occupies two devices simultaneously.

By writing down equations of state, the loss $B = f(A_e, A_i, N)$ in a selector stage with full availability, loaded with mixed external plus internal traffic, can be calculated exactly (see chapter 6).

Equations of state and hence the loss $B = f(A_e, A_i, N, k)$ can also be determined exactly for selector stages with limited availability, if certain assumptions concerning the nature of the grading are made. Erlangs Interconnection Formula and the modified Palm-Jacobaeus Formula [3] appeared to be a useful approach to this problem. See chapter 7.

These solutions yield rather complicated formulae, which- at least for more than 3 or 4 trunks - can only be calculated by means of electronic computers.

The exact formulae in chapter 6 and 7 have been checked on an electronic computer by artificial traffic tests, the result being, that the exact formulae are not always necessary for practical use. Approximate formulae yield values, that are sufficiently close to the real values, obtained by artificial traffic tests. The familiar loss tables for full available groups (e.g. [2]) or for gradings ([3]) may be used for this purpose. These simple approximate formulae will be derived in the following three chapters.

2. General

Let us suppose that Poisson traffic may be offered, which means an infinite number of sources. This also applies to the exact formulae in chapter 6 and 7. Exact solutions for a finite number of sources have also been found and will be published later.

- 2.1 Offered external traffic means pure chance traffic, each successful call occupies one trunk.
- 2.2 Offered internal traffic means pure chance traffic too, but since every successful call occupies two trunks, the traffic carried as a whole is no longer Erlang traffic. But if only one trunk out of each pair is considered, for example the trunk occupied by the outgoing internal call, the traffic on these trunks may well be supposed to be pure Erlang traffic. So, the equations of state for the outgoing internal calls in a group of N trunks carrying internal traffic only are exactly the same as those, obtained by external calls in a group of N/2 trunks.

3. Full availability

- 3.1 According to 2.2 one can obtain the loss of a circuit group with N trunks and an offered pure internal traffic A_1 (= outgoing traffic) from the familiar loss tables to be

$$B_i = E_{i, \frac{N}{2}} = \frac{A_i^{\frac{N}{2}}}{\sum_{r=0}^{\frac{N}{2}} \frac{A_i^r}{r!}} \quad (1)$$

Evidently, the whole internal traffic carried will occupy an even number of trunks. So, N has to be rounded if necessary:

$$N = N - N(\text{mod}2)$$

The loaded traffic y_i on the N trunks will be

$$y_i = 2 \cdot A_i \cdot (1 - B_i)$$

where outgoing and incoming traffic are of the same value

$$y_{ig} = y_{ic} = y_i/2$$

3.2 The overall loss of a group, to which a mixed external plus internal traffic is offered, can be computed in a similar way.

To begin with, the loaded traffic y^* consisting of independent calls will be determined, i.e. the external plus outgoing or incoming internal traffic :

$$y^* = y_e + \frac{y_i}{2} = (1 - \frac{b}{2}) y \quad (2)$$

b means the ratio of the internal traffic and the entire traffic carried :

$$b = \frac{y_i}{y_e + y_i} \quad (3)$$

Since the second half of the internal traffic needs trunks simultaneously, only $N^* < N$ trunks are available for the traffic y^* and the following holds good on an average :

$$\frac{N}{y} = \frac{N^*}{y^*} \Rightarrow \frac{N}{y_e + y_i} = \frac{N^*}{y_e + \frac{y_i}{2}}$$

Using relation (3)

$$\boxed{N^* = (1 - \frac{b}{2}) N} \quad (4)$$

The loss of a full available group with given carried traffic $y = y_e + y_i$ and a given number of trunks N can be determined with good approximation

$$B = f(y^*, N^*) \quad (5)$$

where y^* , N^* are the values given by equations (2) and (4)
B can be drawn immediately from the tables of the mPJ [3]

or from tables or graphs of the Post Offices.

If N^* is a fraction, a linear interpolation may be sufficient. For $b=1$ (internal traffic only), N^* must be rounded to obtain an integer:

$$N^* = \frac{N - N(\text{mod } 2)}{2}$$

From traffic y^* and loss B the offered traffic A is found to be

$$A = \frac{y^*}{1-B}$$

Hence, the total loss is defined by $B = B_e + B_i = \frac{A - y^*}{A}$

Only this total loss B can be determined with the approximate formulae. To get the external loss

$$B_e = \frac{A_e - y_e}{A} = \frac{A_e - y_e}{A_e + A_i}$$

and the internal loss

$$B_i = \frac{A_i - \frac{y_i}{2}}{A} = \frac{A_i - \frac{y_i}{2}}{A_e + A_i}$$

separately, refer to the exact solutions in chapter 6.

3.3 Some curves obtained from (5) are given in Fig.1. For each value of N a carried traffic y has been chosen having about $B = 1\%$ loss for the ratio $b = 0$ (external traffic only). With constant value of carried traffic but increasing part of internal traffic, the loss increases more or less, dependent on the number of trunks.

3.4 In the preceding, we started the computation by assuming a certain carried traffic $y = y_e + y_i$. With another approximation, the number of trunks N^* can also be determined, if the offered traffic $A = A_e + A_i$ is taken as a starting point.

Exactly :

$$\frac{y_i}{y_e + y_i} = \frac{2(A_i - B_i A)}{A_e - B_e A + 2(A_i - B_i A)} = \frac{2A_i}{A_e + 2A_i} \cdot \frac{1 - \frac{B_i A}{A_i}}{1 - \frac{(B_e + 2B_i) A}{A_e + 2A_i}}$$

For $B_e, B_i \ll 1$ or $\frac{A_e}{A_i} \approx \frac{B_e}{B_i}$ the following approximation holds :

$$\frac{y_i}{y_e + y_i} \approx \frac{2A_i}{A_e + 2A_i}$$

By introducing

$$c = \frac{A_i}{A_e + A_i} \tag{6}$$

we get

$$b = \frac{y_i}{y_e + y_i} \approx \frac{2c}{c+1} \tag{7}$$

and

$$N^* = \frac{N}{c+1} \quad (8)$$

Hence

$$B = f(A, N^*) = \frac{\frac{A^{N^*}}{N^*!}}{\sum_{s=0}^{N^*} \frac{A^s}{s!}} \quad (9)$$

The formulae (4) and (8) for N^* apply also for the extreme cases $b=c=0$ (external traffic only) and $b=c=1$ (internal traffic only). For $b=c=1$, N^* has to be an integer - see 3.2. If N^* is a fraction, an interpolation is necessary between the two closest integers.

With the offered traffic A and the loss B , the traffic y^* becomes

$$y^* = A(1-B)$$

So, the whole traffic carried on the N trunks will be

$$y = \frac{y^*}{1 - \frac{b}{2}} \approx \frac{y^*}{1 - \frac{c}{c+1}} = (c+1)y^*$$

4. Limited availability

4.1 Just as in the full available group, only independent calls will be considered. On the N trunks carrying a total traffic of $y = y_e + y_i$ Erlang, the traffic that consists of independent calls is (exactly as in the full available group) :

$$(2) \quad y^* = y_e + \frac{y_i}{2} = (1 - \frac{b}{2}) y$$

and the equivalent number of trunks

$$(4) \quad N^* = (1 - \frac{b}{2}) N$$

From one out of all multiples, which offer traffic to the grading, only certain k out of the N trunks can be reached. Double occupations by the internal traffic may exist on these k trunks. That means, that the availability k has to be reduced too.

Two trunks out of k trunks will be occupied simultaneously, if the incoming internal call occupies one out of the same k trunks, which have been hunted by the corresponding outgoing internal call.

The part of the internal traffic returning to the same

k trunks is given by $y_{ic} = \frac{1}{2} y_i \cdot \frac{k}{N} \cdot \frac{k}{N}$

An equal distribution of the traffic on all multiples is assumed.

Subtracting y_i from the total traffic carried on the N trunks, the traffic y_k^* consisting of independent calls only remains

$$y_k^* = \frac{k}{N} y - y_c = \frac{k}{N} y \left(1 - \frac{b}{2} \frac{k}{N}\right)$$

Therefore, the availability has to be reduced according to y_k^*

$$\frac{k}{N} y = \frac{k^*}{y_k^*} = \frac{k^*}{\frac{k}{N} y \left(1 - \frac{b}{2} \frac{k}{N}\right)}$$

Hence

$$k^* = \left(1 - \frac{b}{2} \frac{k}{N}\right) k \quad (10)$$

The entire loss B_k of a route with N trunks, the availability k and a given carried traffic of y Erlang can now be determined from familiar tables (e.g. mFJ [3])

$$B_k = f(N^*, k^*, y^*) \quad (11)$$

where N^* , k^* , y^* have been computed with formulae (4), (10), (2) respectively.

The real offered traffic A_k can now be determined:

$$A_k = \frac{y^*}{1 - B_k}$$

4.2 Fig. 2 shows some curves obtained from (11) with $k = 6$ and $y = \text{const.}$ for each curve. With increasing ratio of internal traffic, the loss increases.

4.3 Again we can start the approximation by assuming the offered traffic A_k instead of the carried traffic y_k . By analogy with 3.3, the total loss will be

$$B_k = f(N^*, k^*, A) \quad (12)$$

where

$$(8) \quad N^* = \frac{N}{c+1}$$

and

$$k^* = \left(1 - \frac{c}{c+1} \frac{k}{N}\right) \cdot k \quad (3)$$

From offered traffic A_k and total loss B_k we obtain the traffic y^*

$$y^* = A_k (1 - B_k)$$

and the total traffic carried on the N trunks

$$y = \frac{y^*}{1 - \frac{b}{2}} \approx (c+1) y^*$$

5. Link systems

Approximate formulae for the congestion in link systems can also be derived, if the above mentioned methods are used together with the method of combined input and route blocking CIRB [5]. Since these formulae have not yet been checked by artificial traffic tests, they will be published later.

6. Exact solutions for full availability

Abbreviations :

- A offered traffic (Erlang)
- y carried traffic (Erlang)
- Index e : external
- " i : internal
- N total number of trunks
- k availability
- x number of busy trunks
- p(x) probability, x trunks busy
- B probability of loss, time congestion blocking probability

$$c = \frac{A_i}{A_e + A_i} \quad \text{ratio of internal and total traffic offered}$$

$$b = \frac{y_i}{y_e + y_i} \quad \text{ratio of internal and total traffic carried}$$

6.1 External traffic

The well known statistical equilibrium for external traffic (pure chance traffic offered), as presented by A.K.Erlang, is given by

$$\begin{aligned} (x+1) \cdot p(x+1) &= A \cdot p(x) \\ \text{or} \quad (x+2) \cdot p(x+2) &= A \cdot p(x+1) \end{aligned} \quad (14)$$

6.2 Internal traffic

Internal traffic occupying (x+2) trunks consists of only (x+2)/2 conversations. Consequently, the density

of the probability, that one out of $(x+2)/2$ conversations terminates, is

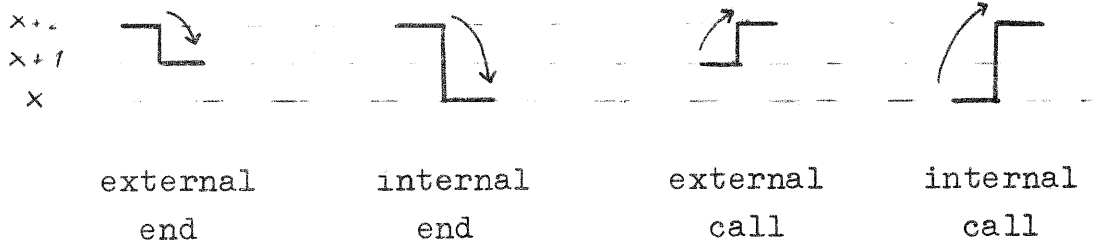
$$\frac{x+2}{2} p(x+2)$$

(average holding time equal to unity)

Hence, the statistical equilibrium is found to be :

$$\frac{x+2}{2} p(x+2) = A \cdot p(x) \quad (15)$$

The left hand sides of eq. (4) and (15) represent the transition from state $(x+2)$ either to $(x+1)$ - termination of an external conversation - or to state x - termination of an internal conversation. The right hand sides represent the transition to state $(x+2)$ either from state $(x+1)$ - new successful external call - or from state x - new successful internal call.



6.3 External plus internal traffic

If a group carries external plus internal traffic and if $(x+2)$ trunks are busy, the average number of trunks occupied by internal traffic is $b \cdot (x+2)$ and occupied by external traffic $(1-b) \cdot (x+2)$

With reference to the total number of incoming calls the part of the internal calls is given by the factor b the part of the external calls is given by the factor $(1-b)$

Hence, the statistical equilibrium becomes :

$$(1-b) \cdot (x+2) \cdot p(x+2) + b \cdot \frac{x+2}{2} \cdot p(x+2) = (1-b) \cdot A \cdot p(x+1) + b \cdot A \cdot p(x) \quad (16)$$

For $b=c=0$ or $b=c=1$ the recurrence relation for pure external traffic (14) or pure internal traffic (5) are obtained.

b and c are not independent.

Therefore, the equations of state and hence the loss can only be evaluated by an iterative method. For example the offered traffic A and the part c of the offered internal traffic may be given. Then, a first approximate value for b has to be chosen; formula (7)

$$b = \frac{2c}{c+1}$$

may be a good approach.

With the additional condition $\sum_{x=0}^N p(x) = 1$ and the abbreviations :

$$a_e = \frac{1-c}{1-\frac{b}{2}} \cdot A \quad ; \quad 2a_i = \frac{c}{1-\frac{b}{2}} \cdot A$$

the above mentioned statistical equilibrium (16) yields the probabilities of state :

$$p(x) = \frac{\sum_{r=0}^{\lfloor \frac{x}{2} \rfloor} \frac{a_i^r a_e^{x-2r}}{r! (x-2r)!}}{\sum_{x=0}^N \sum_{r=0}^{\lfloor \frac{x}{2} \rfloor} \frac{a_i^r a_e^{x-2r}}{r! (x-2r)!}} \quad \left[\frac{x}{2} \right] = \frac{x - X(\text{mod } 2)}{2} \quad (17)$$

It must be pointed out, that the values a_e and a_i are different from $A_e = (1-c) \cdot A$ and $A_i = c \cdot A$

Formula (17) is published in [1] and [4], containing, however, the offered traffic A_e, A_i instead of a_e, a_i .

External loss arises, if an external call finds all trunks busy :

$$B_e = (1-c) \cdot p(N) \quad (18)$$

Internal loss arises, if an internal call finds (i) all N trunks busy - internal loss outgoing B_{ig} - or (ii) N-1 trunks busy - internal loss incoming B_{ic} :

$$\begin{aligned} B_i &= B_{ig} + B_{ic} \\ &= c \cdot p(N) + c \cdot p(N-1) \end{aligned} \quad (19)$$

From these loss values the loaded traffic is obtained :

$$\begin{aligned} \text{external : } y_e &= A_e - B_e A = A_e [1 - p(N)] \\ \text{internal : } y_i &= 2[A_i - B_i A] = 2A_i [1 - p(N) - p(N-1)] \end{aligned}$$

Consequently, the ratio b becomes

$$b = \frac{y_i}{y_e + y_i}$$

This new value for b will surely be a little different from the initial one obtained by $b = 2c/c + 1$. With the new improved value the above mentioned procedure can be repeated. Finally, after sufficient iterations, a value b is obtained, which fits the given values A and c . From the corresponding probabilities of state the desired loss can be determined.

7. Limited availability

7.1 Extension of Erlangs Interconnection Formula

According to Erlangs Interconnection Formula, the following is defined :

$\mu(x)$ = probability, that a call finds a free trunk, if x trunks are busy

$\sigma(x)$ = probability, that a call finds no free trunk, if x trunks are busy = $1 - \mu(x)$

The equation $\sigma(x) = \frac{\binom{x}{k}}{\binom{N}{k}}$ holds exactly for ideal Erlang gradings or for gradings of arbitrary type, if an equal distribution of the load over all trunks is assumed.

From the conditions for the statistical equilibrium the following recurrence relation is obtained :

$$p(x+2) = \frac{1-c}{1-\frac{b}{2}} \cdot \frac{A}{x+2} \cdot \mu(x+1) \cdot p(x+1) + \frac{c}{1-\frac{b}{2}} \cdot \frac{A}{x+2} \cdot \mu(x) \cdot \mu(x+1) \cdot p(x)$$

(20)

The state $(x+2)$ will arise

- a.) out of the state $(x+1)$ by an external call with the probability $\mu(x+1)$, that this call will find a free outlet
- b.) out of the state x by an internal call with the probability $\mu(x) \cdot \mu(x+1)$, that not only in state x a free internal outgoing trunk can be found, but also in state $(x+1)$ a free internal incoming trunk.

With the additional condition $\sum_{x=0}^N p(x) = 1$ and the abbreviations

$$a_e = \frac{1-c}{1-\frac{b}{2}} A \quad ; \quad 2a_i = \frac{c}{1-\frac{b}{2}} A$$

the probabilities of state for limited availability become :

$$p(x) = \frac{\prod_{\xi=0}^{x-1} \mu(\xi) \sum_{r=0}^{\lfloor \frac{x}{2} \rfloor} \frac{a_i^r a_e^{x-2r}}{r!(x-2r)!}}{\sum_{x=0}^N \prod_{\xi=0}^{x-1} \mu(\xi) \sum_{r=0}^{\lfloor \frac{x}{2} \rfloor} \frac{a_i^r a_e^{x-2r}}{r!(x-2r)!}} \quad (21)$$

Computation of loss from the pr. of state :

External loss arises, if in the state x an external call finds no free trunk

$$B_e = (1-c) \sum_{x=k}^N p(x) \cdot \sigma(x) \quad (22)$$

Internal loss outgoing arises, if in the state x an internal outgoing call finds no free trunk

$$B_{ig} = c \sum_{x=k}^N p(x) \cdot \sigma(x) \quad (23)$$

Internal loss incoming arises, if in the state x an internal outgoing trunk can be found, but no internal incoming trunk in the new state $(x+1)$

$$B_{ic} = c \cdot \sum_{x=k-1}^{N-1} p(x) \cdot \mu(x) \cdot \sigma(x+1) \quad (24)$$

In all 3 loss values B_e , B_{ig} , B_{ic} the summation may be extended from $x=0$ to $x=N$. Because of $\mu(x)$ or $\sigma(x)$, the corresponding terms will be zero.

From offered traffic A and loss B , the carried traffic becomes

$$\text{External } y_e = A_e - B_e \cdot A = (1-c-B_e) \cdot A$$

$$\text{Internal } y_i = 2[A_i - (B_{ig}+B_{ic}) \cdot A] = 2A \cdot (c-B_i)$$

Just as in the full available group, the pair b, c can now be found by iteration.

For $b=c=0$, formula (21) is reduced to the well known inter-connection formula for gradings, for $k=N$ to the formula (17) for full available groups.

7.2 Extension of the modified Palm-Jacobaeus Formula

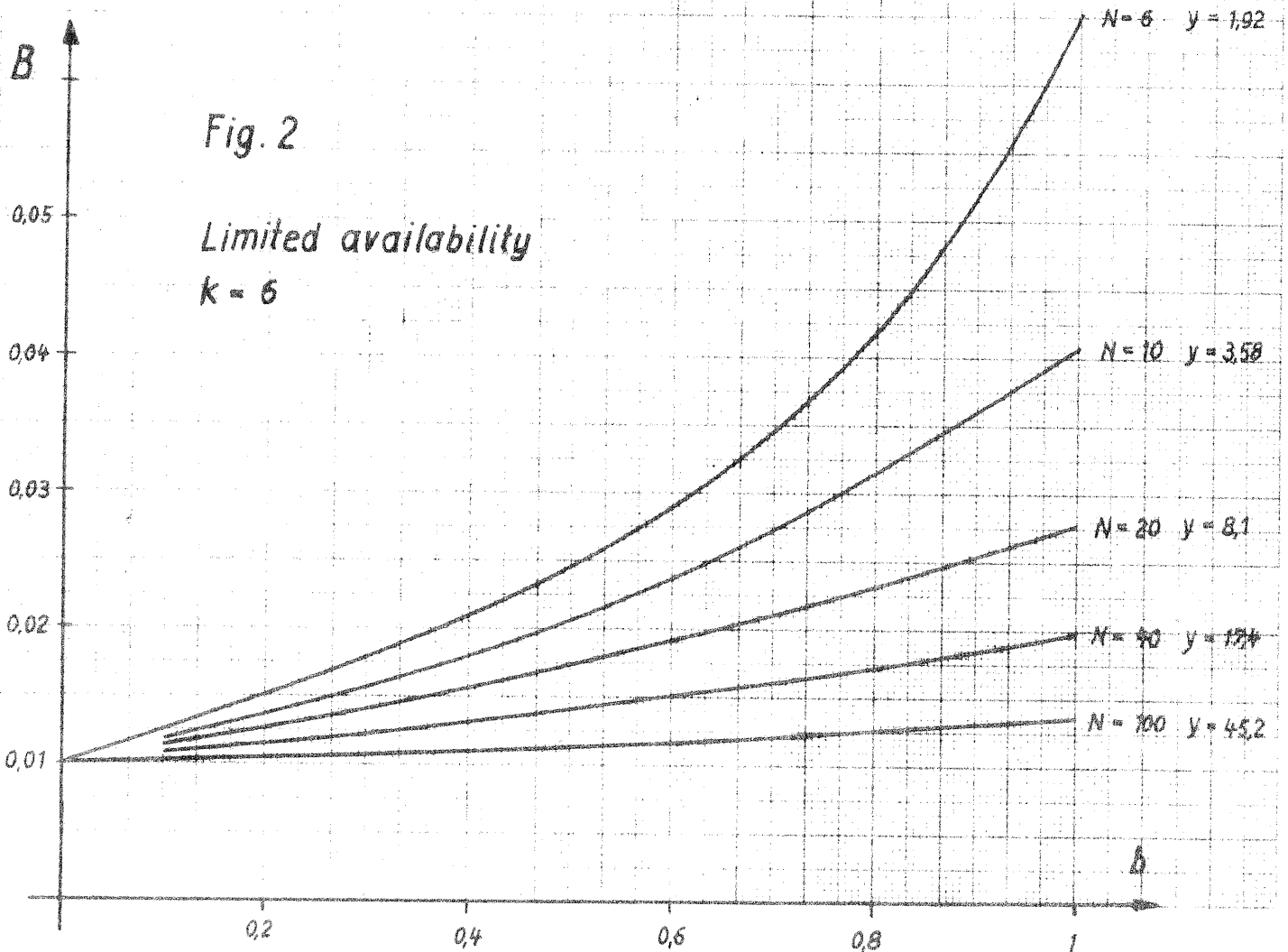
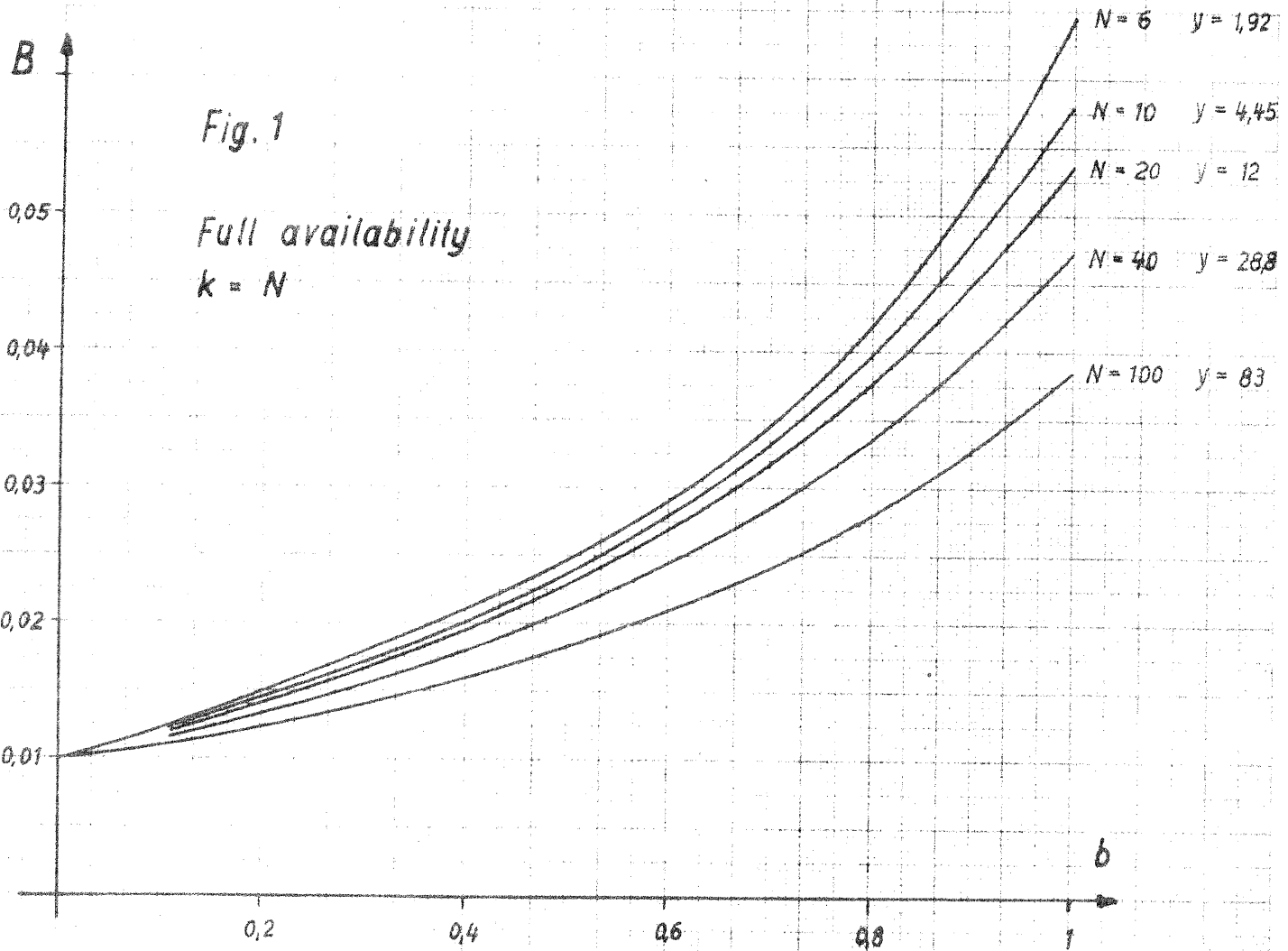
Erlangs interconnection formula makes demands on the type of grading, that often cannot be fulfilled by familiar gradings. Therefore, the computed loss values are sometimes too small. The modified Palm-Jacobaeus Formula [3] yields results closer to reality.

Suppose the N trunks to be hunted with full availability. The probabilities of state are then given by eq. (17). The fictitious traffic A_0 offered to the full available group is defined such, that the carried traffic y is as great as in the real system, where the offered traffic A_k hunts the N trunks with the availability k .

Hence $y = f (A_0, N) = f (A_k, N, k)$

The loss is computed with formulae (22), (23), (24), but with the pr. of state $p(X)$ given by (17). The real offered traffic A_k can be determined only after the computation of the loss B_k by

$$A_k = y / 1 - B_k$$



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