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#### Beitrag der Arbeit / Achievement

Definition and comparison of QoS metrics for TCP-based IP traffic (elastic traffic) with short TCP connections

#### Kurzfassung / Abstract

In this report, we introduce a variety of possible performance measures for elastic traffic and discuss how these measures simplify when they are applied to a processor sharing model and a packet-level model using TCP, respectively.

Furthermore, case studies are presented in which we compare the impact of various parameters on some of the introduced measures for the processor sharing model and the TCP model.

# **1** Introduction

For many years, research community focused on determining quality of service (QoS) by using packet level performance measures like loss probability, delay and delay variation. With increasing importance of data transfer and WWW services producing elastic traffic based on the transmission control protocol (TCP), the lack of models and measures considering retransmission and rate adaptation became obvious.

A first step towards an adequate model for elastic traffic is to assume that network resources are shared among a fixed number of long-lasting TCP connections (greedy sources) and to measure long-term throughput. While this way of modelling may be appropriate in case of traditional file transfer services it is not adequate for describing WWW traffic using HTTP. The latter kind of traffic is characterised by small to medium size objects transmitted over TCP connections. The dynamic character of WWW traffic has, however, not only some influence on source modelling. Finding appropriate measures to describe the quality received by such a service has also become a major issue, which is addressed in this report.

An important question in this context is the grade of detail that is needed to describe elastic traffic. A rather abstract model which is able to cover TCP's fair rate adaptation effect is the processor sharing (PS) model. This burst level model is well-known from computer system modelling and can be evaluated analytically under quite general assumptions. However, it does neither consider the details of TCP nor the special constraints occurring on the packet level. Therefore the question comes up whether neglecting those details has a significant impact on the performance results. Besides the processor sharing model, we consider a simulation model based on a TCP implementation including a huge amount of configuration parameters.

A more detailed description of both the processor sharing and the TCP model can be found in Section 2. Section 3 contains a comprehensive collection of possible performance measures. Special emphasis is put on relative measures following the notion of delay and fun factors introduced in [4] and [1], respectively. In Section 4, we show how the previously defined measures can be applied to the processor sharing model and to the TCP model. The results of a comparative performance evaluation using analysis and simulation are presented in Section 5.

# 2 Models

# **2.1 Processor sharing**

The processor sharing model as considered in the context of this report is closely related to the theoretical processor sharing model known from literature [3]. We assume an infinite number of users and thus characterise the arriving traffic by request interarrival times and request size distributions. The bandwidth on the link is shared equally among all active requests. Furthermore, the link bandwidth of each request can be bounded before the request is passed to the link.

# **2.2 TCP simulation**

Fig. 2.1 shows the principle structure of the simulation model. An application block is situated on top of the TCP stack and controls transmission of data. For the present simulation study, we assume an infinite number of users and thus model the traffic generated by the applications as



Fig. 2.1: Global TCP simulation model considering applications

independent requests characterised by the request size distribution and request interarrival time distribution, respectively. Thus, the correlation between successive/parallel requests is not considered in this model.

Data packets and acknowledgements are transmitted in a network that is modelled as depicted in Fig. 2.2. Data packets are transmitted via two single server queues that represent the network access and the bottleneck link, respectively. Propagation delay is only considered on the acknowledgement path and is modelled as an infinite server with constant service time. After each successful transmission of a request, statistics considering the actual request size and the measured transfer time are updated.

Our TCP model does not consider connection setup and release. For computing a fun factor that depends on both connection setup and data transfer delay, the setup times are drawn from an independent distribution. We will further elaborate on this in Section 4.3.

# **3** General performance measures for elastic traffic

After an introduction of the model assumptions in Section 3.1 and an introduction of the applied random variables in Section 3.2, performance measures that are based on the introduced random variables are presented. In Sections 3.3 to 3.5 a variety of possible performance measures are introduced. They differ in the way the previously defined random variables are



Fig. 2.2: TCP simulation model considering possible delays

combined. First, mean values are presented in Section 3.3. Section 3.4 presents mean values that are conditioned on the actual request size. Finally, Section 3.5 introduces distributions.

## 3.1 Model assumptions

For the performance values presented in this section the following assumption are made:

- requests for object transmission arrive in a Poisson stream
- variable object size with arbitrary distribution
- TCP-based transmission over a shared backbone link
- limited access rate for each transfer

## 3.2 Random variables

The traffic parameters presented in the following are characteristics of the object itself and thus are independent of the network. In contrast to that, the performance results represent measures that also consider experienced/minimum transfer time.

#### **Traffic parameters**

- S object size
- h(s) object size probability density function
- H(s) object size probability distribution function

#### **Performance results**

- *T* transfer time
- *G* per-transfer goodput

 $T_{min}$  minimum transfer time

*G<sub>max</sub>* maximum goodput

 $\Delta$  delay factor

 $\Phi$  fun factor

The performance measures are defined by the following equations:

$$G = \frac{S}{T}$$
(3.1)

$$T_{min} = t_{min}(S) \tag{3.2}$$

$$G_{max} = \frac{S}{T_{min}}$$
(3.3)

$$\Delta = \frac{T}{T_{min}} \tag{3.4}$$

$$\Phi = \frac{G}{G_{max}} = \frac{T_{min}}{T} = \frac{1}{\Delta}$$
(3.5)

## 3.3 Mean values

### 3.3.1 Absolute measures

• mean transfer time:

$$\dot{t} = \mathbf{E}[T] \tag{3.6}$$

• mean goodput:

$$\overline{g} = \mathrm{E}[G] = E\left[\frac{S}{T}\right]$$
 (3.7)

• weighted mean goodput:

$$\bar{g}_w = \frac{\mathrm{E}[S \cdot G]}{\mathrm{E}[S]} = \frac{\mathrm{E}\left[\frac{S^2}{T}\right]}{\mathrm{E}[S]}$$
(3.8)

#### 3.3.2 Relative measures

Mean fun factors are always abbreviated with  $\phi_i$  and mean delay factors with  $\delta_i$ . The general relationship between fun and delay factor is

$$\varphi_i = \frac{1}{\delta_i}.$$
(3.9)

Fun factors always relate an optimum measure to an experienced measure and thus yield a value smaller or equal to one. On the contrary, delay factors relate an actual measure to the optimum measure and thus are larger of equal to one. The considered measure here is either the transfer time or the goodput.

Non-weighted mean fun/delay factors:

$$\varphi_{\Delta} = \frac{1}{\mathrm{E}[\Delta]} = \frac{1}{\mathrm{E}\left[\frac{T}{T_{min}}\right]}, \, \delta = \delta_{\Delta} = \mathrm{E}[\Delta] = \mathrm{E}\left[\frac{T}{T_{min}}\right]$$
(3.10)

$$\varphi_T = \frac{1}{\delta_T} = \frac{\mathbf{E}[T_{min}]}{\mathbf{E}[T]}, \, \delta_T = \frac{\mathbf{E}[T]}{\mathbf{E}[T_{min}]}$$
(3.11)

$$\varphi_{\Phi} = \mathbf{E}[\Phi] = \mathbf{E}\left[\frac{G}{G_{max}}\right] = \mathbf{E}\left[\frac{T_{min}}{T}\right], \ \delta_{\Phi} = \frac{1}{\varphi_{\Phi}} = \frac{1}{\mathbf{E}\left[\frac{T_{min}}{T}\right]}$$
(3.12)

$$\varphi_G = \frac{\mathrm{E}[G]}{\mathrm{E}[G_{max}]} = \frac{\mathrm{E}\left[\frac{S}{T}\right]}{\mathrm{E}\left[\frac{S}{T_{min}}\right]}, \ \delta_G = \frac{1}{\varphi_G} = \frac{\mathrm{E}\left[\frac{S}{T_{min}}\right]}{\mathrm{E}\left[\frac{S}{T}\right]}$$
(3.13)

Weighted mean fun factors:

$$\delta_{\Delta, w} = \frac{\mathbf{E}[S \cdot \Delta]}{\mathbf{E}[S]} = \frac{\mathbf{E}\left[S \cdot \frac{T}{T_{min}}\right]}{\mathbf{E}[S]}$$
(3.14)

$$\delta_{T,w} = \frac{\mathrm{E}[S \cdot T]}{\mathrm{E}[S] \cdot \mathrm{E}[T_{min}]} \tag{3.15}$$

$$\varphi_{\Phi,w} = \frac{\mathrm{E}[S \cdot \Phi]}{\mathrm{E}[S]} = \frac{\mathrm{E}\left[S \cdot \frac{G}{G_{max}}\right]}{\mathrm{E}[S]} = \frac{\mathrm{E}\left[S \cdot \frac{T_{min}}{T}\right]}{\mathrm{E}[S]}$$
(3.16)

$$\varphi_{G,w} = \frac{\mathrm{E}[S \cdot G]}{\mathrm{E}[S] \cdot \mathrm{E}[G_{max}]} = \frac{\mathrm{E}\left[\frac{S^2}{T}\right]}{\mathrm{E}[S] \cdot \mathrm{E}\left[\frac{S}{T_{min}}\right]}$$
(3.17)

## **3.4 Conditional mean values**

In this section, the measures introduced in Section 3.3 are conditioned on the actual length of the request. This allows to discuss whether the performance experienced by a request depends on the length of the considered request or not.

#### 3.4.1 Absolute measures

• conditional mean transfer time:

$$t(s) = E[T|S=s]$$
 (3.18)

• conditional mean goodput:

$$g(s) = \mathbb{E}[G|S=s] = E\left[\frac{S}{T}\middle|S=s\right] = s \cdot \mathbb{E}\left[\frac{1}{T}\middle|S=s\right]$$
(3.19)

## 3.4.2 Relative measures

- $\varphi_i(s)$  conditional mean fun factors
- $\delta_i(s)$  conditional mean delay factors

General relationship:  $\varphi_i(s) = \frac{1}{\delta_i(s)}$ 

$$\varphi_{\Delta}(s) = \frac{t_{min}(s)}{\mathbb{E}[T|S=s]},$$
  

$$\delta(s) = \delta_{\Delta}(s) = \mathbb{E}[\Delta|S=s] = \mathbb{E}\left[\frac{T}{T_{min}}\middle|S=s\right] = \frac{\mathbb{E}[T|S=s]}{t_{min}(s)}$$
(3.20)

$$\begin{aligned}
\phi_T(s) &= \frac{t_{min}(s)}{E[T|S=s]} = \phi_{\Delta}(s), \\
\delta_T(s) &= \frac{E[T|S=s]}{E[T_{min}|S=s]} = \frac{E[T|S=s]}{t_{min}(s)} = \delta_{\Delta}(s)
\end{aligned}$$
(3.21)

$$\varphi(s) = \varphi_{\Phi}(s) = \mathbf{E}[\Phi|S=s] = \mathbf{E}\left[\frac{G}{G_{max}}\middle|S=s\right] = t_{min}(s) \cdot \mathbf{E}\left[\frac{1}{T}\middle|S=s\right],$$

$$\delta_{\Phi}(s) = \frac{1}{t_{min}(s) \cdot \mathbf{E}\left[\frac{1}{T}\middle| S = s\right]}$$
(3.22)

$$\varphi_G(s) = \frac{\mathbf{E}[G|S=s]}{\mathbf{E}[G_{max}|S=s]} = \frac{\mathbf{E}\left[\frac{S}{T}\middle|S=s\right]}{\mathbf{E}\left[\frac{S}{T_{min}}\middle|S=s\right]} = t_{min}(s) \cdot \mathbf{E}\left[\frac{1}{T}\middle|S=s\right] = \varphi_{\Phi}(s),$$

$$\delta_G(s) = \frac{1}{t_{min}(s) \cdot \mathbf{E}\left[\frac{1}{T} \middle| S = s\right]} = \delta_{\Phi}(s)$$
(3.23)

## **3.5 Distributions**

#### 3.5.1 Absolute measures

• transfer time distribution

$$Q_T(t) = \mathbf{P}(T \le t) \tag{3.24}$$

• goodput distribution:

$$Q_G(y) = \mathbf{P}(G \le y) = \mathbf{P}\left(\frac{S}{T} \le y\right)$$
(3.25)

• weighted goodput distribution:

$$Q_{G,w}(y) = \frac{1}{\mathrm{E}[S]} \cdot \int_{0}^{\infty} (s \cdot \mathrm{P}(G \le y | S = s) \cdot h(s)) ds$$
(3.26)

## 3.5.2 Relative measures

## Non-weighted fun/delay factor distributions

$$F_{\Delta}(x) = P(\Delta \le x) = P\left(\frac{T}{T_{min}} \le x\right), \quad x \in [1, \infty)$$
(3.27)

$$F_T(x) = \mathbb{P}\left(\frac{T}{\mathbb{E}[T_{min}]} \le x\right) = Q_T(x \cdot \mathbb{E}[T_{min}]), x \in [0, \infty)$$
(3.28)

$$F_{\Phi}(x) = P(\Phi \le x) = P\left(\frac{T_{min}}{T} \le x\right) = P\left(\frac{T}{T_{min}} \ge \frac{1}{x}\right) = 1 - F_{\Delta}\left(\frac{1}{x}\right), x \in (0, 1] \quad (3.29)$$

$$F_{G}(x) = P\left(\frac{G}{E[G_{max}]} \le x\right) = P\left(\frac{S}{T \cdot E\left[\frac{S}{T_{min}}\right]} \ge x\right) = Q_{G}(x \cdot E[G_{max}]),$$
$$x \in (0, \infty) \qquad (3.30)$$

## Weighted fun factor distribution:

$$F_{\Delta,w}(x) = \frac{1}{E[S]} \cdot \int_{0}^{\infty} (s \cdot P(\Delta \le x | S = s) \cdot h(s)) ds = 1 - F_{\Phi,w}\left(\frac{1}{x}\right), x \in [1,\infty) \quad (3.31)$$

$$F_{\Phi,w}(x) = \frac{1}{\mathrm{E}[S]} \cdot \int_{0}^{\infty} (s \cdot \mathrm{P}(\Phi \le x | S = s) \cdot h(s)) ds = 1 - F_{\Delta,w}\left(\frac{1}{x}\right), x \in (0,1] \quad (3.32)$$

# 4 Application of performance measures to special models

In this chapter, a specialisation of the performance measures introduced in the previous section is presented. Whereas Section 4.1 focuses on the processor sharing model, Section 4.2 discusses aspects of the TCP model.

## 4.1 M/G/*n* processor sharing model

### **Model parameters**

*C* link rate

 $r_{max}$  maximal access rate

### ρ utilization

### **Random variables**

$$T_{min, PS} = \frac{S}{r_{max}} \tag{4.1}$$

$$G_{max, PS} = r_{max} \tag{4.2}$$

$$\Phi_{PS} = \frac{S}{T \cdot r_{max}} \tag{4.3}$$

$$\Delta_{PS} = \frac{T \cdot r_{max}}{S} \tag{4.4}$$

#### **Known results**

$$\mathbf{E}[T_{min}] = \frac{\mathbf{E}[S]}{r_{max}} \tag{4.5}$$

$$E[T|S=s] = \frac{s}{r_{max}} \cdot f\left(\frac{C}{r_{max}}, \rho\right)$$
(4.6)

$$E[T] = \frac{E[S]}{r_{max}} \cdot f\left(\frac{C}{r_{max}}, \rho\right)$$
(4.7)

$$E\left[\frac{T}{S}\right] = \int_{0}^{\infty} E\left[\frac{T}{s}\middle|S=s\right] \cdot h(s) \cdot ds = \int_{0}^{\infty} \frac{E[T|S=s]}{s} \cdot h(s) \cdot ds$$
$$= \frac{f\left(\frac{C}{r_{max}},\rho\right)}{r_{max}} \cdot \int_{0}^{\infty} h(s) \cdot ds = \frac{1}{r_{max}} \cdot f\left(\frac{C}{r_{max}},\rho\right) = \frac{E[T]}{E[S]}$$
(4.8)

using the expression

$$f(n,\rho) = 1 + \frac{1 + (n - n_g) \cdot (1 - \rho)}{n \cdot (1 - \rho)} \cdot \frac{\frac{(n\rho)^{n_g}}{n_g!}}{(1 - \rho) \cdot \sum_{i=0}^{n_g - 1} \frac{(n\rho)^i}{i!} + \frac{(n\rho)^{n_g}}{n_g!}}, \quad n_g = \lfloor n \rfloor \quad (4.9)$$

which was originally denoted as "delay factor" by Lindberger in [4] and which was extended to non-integer values of n in [1]. A simple approximation of  $f(n, \rho)$  is given by:

$$\tilde{f}(n,\rho) = max \left(1, \rho + \left(\frac{1}{1-\rho} - \rho\right) \cdot \frac{1}{n}\right)$$
(4.10)

## **Mean factors**

$$\Phi_{\Delta, PS} = \frac{1}{E\left[\frac{T}{S}\right] \cdot r_{max}} = \frac{E[S]}{E[T] \cdot r_{max}} = \frac{1}{f\left(\frac{C}{r_{max}}, \rho\right)},$$
  
$$\delta_{\Delta, PS} = E\left[\frac{T}{S}\right] \cdot r_{max} = \frac{E[T]}{E[S]} \cdot r_{max} = f\left(\frac{C}{r_{max}}, \rho\right)$$
(4.11)

$$\varphi_{T,PS} = \frac{\mathrm{E}[S]}{\mathrm{E}[T] \cdot r_{max}} = \varphi_{\Delta,PS}, \, \delta_{T,PS} = \frac{\mathrm{E}[T]}{\mathrm{E}[S]} \cdot r_{max} = f\left(\frac{C}{r_{max}},\rho\right) = \delta_{\Delta,PS} \quad (4.12)$$

$$\varphi_{\Phi, PS} = \mathbf{E}\left[\frac{S}{T}\right] \cdot \frac{1}{r_{max}}, \, \delta_{\Phi, PS} = \frac{r_{max}}{\mathbf{E}\left[\frac{S}{T}\right]} \tag{4.13}$$

$$\varphi_{G,PS} = \varphi_{\Phi,PS} \tag{4.14}$$

$$\delta_{\Delta, w, PS} = \frac{E\left[S \cdot \frac{T \cdot r_{max}}{S}\right]}{E[S]} = \frac{r_{max} \cdot E[T]}{E[S]} = f\left(\frac{C}{r_{max}}, \rho\right) = \delta_{\Delta, PS}$$
(4.15)

$$\delta_{T, w, PS} = \frac{\mathbb{E}[S \cdot T]}{\mathbb{E}[S] \cdot \mathbb{E}\left[\frac{S}{r_{max}}\right]} = \frac{r_{max} \cdot \mathbb{E}[S \cdot T]}{\mathbb{E}[S]^2}$$
(4.16)

$$\varphi_{\Phi, w, PS} = \frac{E\left[\frac{S^2}{T}\right]}{E[S] \cdot r_{max}}$$
(4.17)

$$\varphi_{G, w, PS} = \varphi_{\Phi, w, PS} \tag{4.18}$$

## **Conditional mean factors**

$$\varphi_{\Delta, PS}(s) = \frac{1}{E\left[\frac{T \cdot r_{max}}{S} \middle| S = s\right]} = \frac{s}{r_{max} \cdot E[T|S = s]} = \frac{1}{f\left(\frac{C}{r_{max}}, \rho\right)} = \varphi_{\Delta, PS},$$
  
$$\delta_{\Delta, PS}(s) = E\left[\frac{T \cdot r_{max}}{S} \middle| S = s\right] = \frac{E[T|S = s] \cdot r_{max}}{s} = f\left(\frac{C}{r_{max}}, \rho\right) = \delta_{\Delta, PS} \quad (4.19)$$

$$\varphi_{\Phi, PS}(s) = \mathbf{E}\left[\frac{S}{T \cdot r_{max}} \middle| S = s\right] = \frac{s}{r_{max}} \cdot \mathbf{E}\left[\frac{1}{T} \middle| S = s\right],$$
  

$$\delta_{\Phi, PS}(s) = \frac{1}{\mathbf{E}\left[\frac{S}{T \cdot r_{max}} \middle| S = s\right]} = \frac{r_{max}}{s \cdot \mathbf{E}\left[\frac{1}{T} \middle| S = s\right]}$$
(4.20)

## Distributions

$$F_{\Delta, PS}(x) = P\left(\frac{T}{S} \le \frac{x}{r_{max}}\right) = 1 - Q_{G, PS}\left(\frac{r_{max}}{x}\right) = 1 - F_{G, PS}\left(\frac{1}{x}\right) = 1 - F_{\Phi, PS}\left(\frac{1}{x}\right),$$
  
$$x \in [1, \infty)$$
(4.21)

$$F_{T, PS}(x) = P\left(T \le \frac{x \cdot E[S]}{r_{max}}\right) = Q_{T, PS}\left(\frac{x \cdot E[S]}{r_{max}}\right), \ x \in [0, \infty)$$
(4.22)

$$F_{\Phi, PS}(x) = P\left(\frac{S}{T} \le x \cdot r_{max}\right) = Q_{G, PS}(x \cdot r_{max}) = F_{G, PS}(x) = 1 - F_{\Delta, PS}\left(\frac{1}{x}\right),$$
  
$$x \in (0, 1]$$
(4.23)

$$F_{G,PS}(x) = P\left(\frac{S}{T} \le x \cdot r_{max}\right) = Q_{G,PS}(x \cdot r_{max}) = F_{\Phi,PS}(x) = 1 - F_{\Delta,PS}\left(\frac{1}{x}\right),$$
$$x \in (0,\infty)$$
(4.24)

$$F_{\Delta,w}(x) = \frac{1}{\mathrm{E}[S]} \cdot \int_{0}^{\infty} \left( s \cdot \mathrm{P}\left(\frac{T \cdot r_{max}}{s} \le x \middle| S = s\right) \cdot h(s) \right) ds$$
$$= \frac{1}{\mathrm{E}[S]} \cdot \int_{0}^{\infty} \left( s \cdot \left(1 - \mathrm{P}\left(\frac{s}{T} \le \frac{r_{max}}{x} \middle| S = s\right)\right) \cdot h(s) \right) ds$$
$$= 1 - \mathcal{Q}_{G,w}\left(\frac{r_{max}}{x}\right) = 1 - F_{\Phi,w}\left(\frac{1}{x}\right), \qquad x \in [1,\infty)$$
(4.25)

$$F_{\Phi,w}(x) = \frac{1}{\mathrm{E}[S]} \cdot \int_{0}^{\infty} \left( s \cdot \mathrm{P}\left(\frac{s}{T \cdot r_{max}} \le x \middle| S = s\right) \cdot h(s) \right) ds$$
$$= Q_{G,w}(x \cdot r_{max}) = 1 - F_{\Delta,w}\left(\frac{1}{x}\right), \qquad x \in (0,1]$$
(4.26)

## 4.2 TCP model

#### **Model parameters**

С	link rate
r <sub>max</sub>	maximal access rate
ρ	link utilization
MSS	maximum segment size
τ	propagation delay (on feedback channel)
h	MSS service time $h = \frac{MSS}{C}$
no delayed ACK	

#### **Random variables**

Using the random variable  $T_{offset}$  representing transfer time offset minimum transfer time and maximum delay can be described as follows:

$$T_{min, TCP} = T_{offset} + T_{min, PS} = T_{offset} + \frac{S}{r_{max}}$$
(4.27)

$$G_{max, TCP} = \frac{S}{T_{min, TCP}} = \frac{r_{max}}{1 + \frac{T_{offset}}{S} \cdot r_{max}}$$
(4.28)

#### **Known results**

In the case of infinitely large sender-side access buffer and ssthresh initial value as well as the delayed ACK option being turned off at the receiver side the delay offset can be expressed in terms of object size S, propagation delay  $\tau$ , and TCP maximum segment size (*MSS*):

$$T_{offset} = d(S) \tag{4.29}$$

using

$$d(s) = D(K(s)),$$
 (4.30)

$$D(i) = (\tau + h) \cdot (i - 1) - h(2^{i} - 2), \quad i = 1, 2, \dots$$
(4.31)

$$K(s) = \min\left(\left\lfloor \operatorname{ld}\left(\left\lceil \frac{s}{MSS} \right\rceil + 1\right)\right\rfloor, \kappa + 1\right)$$
(4.32)

$$\kappa = \left\lfloor \operatorname{ld}\left(\frac{\tau}{h} + 1\right) \right\rfloor \tag{4.33}$$

The mean value of the delay offset further depends on the object size distribution:

$$E[T_{offset}] = \int_{0}^{\infty} D(K(s)) \cdot h(s) ds$$
  
=  $\sum_{i=1}^{\kappa} \left( D(i) \cdot \left( \int_{(2^{i+1}-2) \cdot MSS}^{(2^{i+1}-2) \cdot MSS} h(s) ds \right) + D(\kappa+1) \cdot \int_{(2^{\kappa+1}-2) \cdot MSS}^{\infty} h(s) ds$   
=  $\sum_{i=1}^{\kappa} \left( D(i) \cdot \left( H((2^{i+1}-2) \cdot MSS) - H((2^{i}-2) \cdot MSS) \right) \right) + D(\kappa+1) \cdot \left( 1 - H((2^{\kappa+1}-2) \cdot MSS) \right)$  (4.34)

#### **Relative measures**

Fun and delay factors may be related to either  $T_{min, TCP}$  and  $G_{max, TCP}$  or to  $T_{min, PS}$  and  $G_{max, PS}$ , respectively. In the first case the derived measured are tagged with subscript TCP while in the latter case TCP/PS is used as subscript.

### **4.3 TCP model with simple connection setup**

In [2], a special fun factor was introduced that considers that the transmission of the requested object does not start immediately (due to delay caused by a connection setup phase) and that a user may also be satisfied with his observed transfer delay, even if he cannot use the whole access bandwidth. This especially holds in the case of short requests where the full access speed cannot be utilized because of special TCP mechanisms (e.g. slow start).

For the definition of this fun factor, the following parameters have to be introduced:

- $d_t$  target delay (for pure waiting)
- $t_t$  target transfer time
- $T_D$  initial delay for connection setup
- $T_L$  observed loading time

Fun factor 4 according to [2] is defined as follows:

$$\Phi_4 = \min\left\{1, \frac{d_t + S/r_t}{T_D + T_L}\right\}$$
(4.35)

The expectation of fun factor 4 is given by

$$\varphi_4 = \mathbb{E}[\Phi_4] = \mathbb{E}\left[\min\left\{1, \frac{d_t + S/r_t}{T_D + T_L}\right\}\right]$$
(4.36)

and the distribution is denoted as

$$F_{\Phi_4}(x) = P(\Phi_4 \le x), \qquad x \in [0, 1]$$
 (4.37)

As the connection setup phase is not considered in our model (and also in all previously introduced performance measures), we simply take  $T_D$  from an arbitrary distribution. This certainly does not reflect the reality properly as the correlation between network state and duration of the setup phase is neglected. However, first results to indicate the impact of considering connection setup are obtained.

# **5** Performance comparison

During all simulations we assume that the bottleneck link has a capacity of C = 10 Mbit/s and that interarrival time of TCP flows has an exponential distribution.

## 5.1 PS model

## Mean values

Fig. 5.1 - Fig. 5.10 depict mean fun and delay factors defined in Section 4.1. In all figures, the objects size is exponentially distributed with mean 10 kByte.

From these figure, the following effects can be observed:

- φ<sub>Δ</sub> (known as reciprocal of delay factor *f* according to Lindberger's definition [4]) and φ<sub>Φ</sub> (corresponding to the mean fun factor φ as originally defined by Charzinski [1]) significantly differ for greater access rates. This can especially be seen in Fig. 5.7 where the ratio φ<sub>Δ</sub>/φ<sub>Φ</sub> is depicted.
- The mean fun factors get worse with increasing access rate.
- $\phi_{\Phi}$  and  $\phi_{\Phi,w}$  only slightly differ and are thus not depicted in the overview in Fig. 5.5.
- φ<sub>Δ</sub>(s) does not depend on the actual object size whereas φ<sub>Φ</sub>(s) slightly decreases for greater objects, see Fig. 5.8.
- The approximation of  $\delta_{\Delta}$  presented in Section 4.1 fits the exact analysis quite well for medium to large values of  $R_{\text{max}}/C$  especially in the heavy load case.

The performance results for other object size distributions (with identical mean) are rather unchanged. In the case of a Pareto distribution only the weighted fun/delay factors proved to be slightly different.

## Distributions

Fig. 5.11 - Fig. 5.12 depict fun and delay factor distributions for exponentially distributed object sizes whereas Fig. 5.13 and also Fig. 5.14 consider Pareto distributed object sizes with shape parameter  $\alpha = 1.6$  (corresponding to Hurst parameter H = 0.7). It can be seen that slightly smaller values can be observed for the weighted fun factor in both cases. The main proposition of these figures is that only the transfer time distribution (and therefore  $F_{\Delta, PS}(x)$ ) and  $F_{\Phi, PS}(x)$ ) significantly change (Fig. 5.14).



Fig. 5.1: Mean fun factor ( $r_{max} = 10 \text{ Mbit/s}$ )



Fig. 5.3: Mean fun factor ( $r_{max} = 768 \text{ kbit/s}$ )



Fig. 5.5: Mean fun factors



Fig. 5.2: Mean fun factor ( $r_{max} = 2$  Mbit/s)



Fig. 5.4: Mean fun factor ( $r_{max} = 64$  kbit/s)



Fig. 5.6: Mean delay factors



Fig. 5.7: Ratio of fun factors



Fig. 5.9: Fun factor over  $r_{max}/C$ 







Fig. 5.8: Conditional mean fun factors ( $\rho = 0.8$ )











# 5.2 TCP model

In the simulation study based on the TCP model implemented with support of the IND simulation library [5], the following parameters are unchanged:

- MSS = 1460 byte
- TCP/IP header length = 40 byte
- delayed ACK is turned on

## 5.2.1 Goodput

#### Mean values

In Fig. 5.15 - Fig. 5.17,  $\phi_G$  is depicted against the propagation delay for different object size distributions. Here, the following effects can be observed:

- $\phi_G$  is smaller for larger access rates due to higher user exectations.
- For smaller propagation delay, the objects size distribution has only a minor impact on  $\varphi_G$ .
- Decreasing the load to values smaller than 0.8 causes only a slight increase of  $\varphi_G$ .

#### **Conditional mean values**

Fig. 5.18 - Fig. 5.21 depict  $\varphi_G(s)$  and thus shows the dependence of  $\varphi_G$  on the actual request size. The following effects can be seen:

- The window mechanism of TCP is clearly visible indicated be the peaked shape of the curves. Here, the shape of the first peak is independent of  $\tau$  because the window mechanism is only effective for larger requests.
- Decreasing the load to values smaller than 0.8 only slightly increases  $\varphi_G(s)$ .
- For larger values of the propagation delay, the objects size hardly influences the mean goodput. Thus most of the time is spent to wait for acknowledgements in order to continue sending compared to the time the transmission of the data takes.



Fig. 5.15: Mean relative goodput (constant object size)



Fig. 5.17: Mean relative goodput (Pareto distributed object size)

#### 5.2.2 Transfer time

Fig. 5.22 and Fig. 5.23 depict the conditional mean transfer time for different access rates. From Fig. 5.22, it can be seen that the conditional mean transfer time increases almost linearly for requests with increasing size. The steps in the curves with higher propagation delay clearly indicate the influence of the window mechanism and thus indicate where (and how long) the request waits for acknowledges in order to continue being transmitted.

For greater access speeds, Fig. 5.23 shows that the effect of the window mechanism is much higher. This is indicated by the height of the steps compared to the slope of the curves.

#### 5.2.3 Fun factor considering connection setup

Fig. 5.24 - Fig. 5.31 depict  $\varphi_4$  and thus not only consider traffic parameters and performance results, but also target values for delay and rate. For our evaluations, the target values suggested in [2] are applied, i.e. target delay of 500 ms and target rate of 50 kpbs for 64 kpbs and 500 kbps for 768 kbps access rate, respectively. However, it should be remarked here, that the results strongly depend on these target values.



Fig. 5.16: Mean relative goodput (exponentially distributed object size)



Fig. 5.18: Conditional mean goodput ( $\tau = 100 \text{ ms}$ )



Fig. 5.20: Conditional mean goodput ( $\tau = 500 \text{ ms}$ )



Fig. 5.22: Conditional mean transfer time  $(r_{max} = 128 \text{ kbit/s})$ 



Fig. 5.19: Conditional mean goodput( $\tau = 300 \text{ ms}$ )



Fig. 5.21: Conditional mean goodput ( $\tau = 700 \text{ ms}$ )



Fig. 5.23: Conditional mean transfer time  $(r_{max} = 768 \text{ kbit/s}))$ 



Fig. 5.26: Mean fun factor (different setup distributions,  $r_{max} = 128$  kbit/s)

Fig. 5.27: Mean fun factor (no setup delay,  $r_{max} = 128$  kbit/s)

#### Mean fun factor

From Fig. 5.24 it can be seen that different object size distributions hardly influence the principle shape of  $\varphi_4$ . For greater access rate, the difference is larger than for smaller access rate. Furthermore, it can be seen that an increase in the propagation delay has minor impact on  $\varphi_4$  in case of low access bandwidth whereas  $\varphi_4$  quickly drops down for increasing propagation delay in case of larger access bandwidth.

As already mentioned for  $\varphi_G$ , Fig. 5.25 indicates that  $\varphi_4$  remains almost constant for a carried traffic decreased below 0.8.

Fig. 5.26 depicts the shape of  $\varphi_4$  for different setup delay distributions and varying load against the propagation delay. It can be seen that the load is the critical parameter whereas the curves are similar for different setup delay distributions. In order to show the influence of setup delay, Fig. 5.27 depicts  $\varphi_4$  in case of zero setup time. Here,  $\varphi_4 = 1$  holds even for larger propagation delays



Fig. 5.28: Fun factor distribution (neg.-exp. distr. object size,  $r_{max} = 128$  kbit/s,  $\rho = 0.9$ )



Fig. 5.30: Fun factor distribution (constant object size,  $r_{max} = 768$  kbit/s,  $\rho = 0.9$ )



Fig. 5.29: Fun factor distribution (Pareto distributed object size,  $r_{max} = 128$  kbit/s,  $\rho = 0.9$ )



Fig. 5.31: Fun factor distribution (neg.-exp/Pareto. distr. object size,  $r_{max} = 768$  kbit/s,  $\rho = 0.9$ )

#### Fun factor distribution

Fig. 5.28 and Fig. 5.29 depict the complementary distribution of  $\Phi_4$  for an access rate of 128 kbps. It can be seen that the complementary distributions are rather similar for different request size distributions and propagation delays smaller than 700 ms. All these curves have in common that they start with a small slope and drop down slowly.

Fig. 5.30 and Fig. 5.31 show the complementary distribution in case of 768 kbps access rate. Like the curves with smaller access rate, the distributions start with a small slope. But, however, in case of greater access rate, the complementary distributions drop down very quickly and in case of constant and Pareto distributed objects sizes,  $\phi_4$  is always smaller than 1.

## 5.3 Comparison of PS and TCP model

In this section, results obtained with the processor sharing model and the TCP model are compared. For the TCP model, the following parameters are applied:

- MSS = 1000 bytes
- TCP/IP header length = 0
- no delayed ACK
- shared link buffer size and access link buffer size = 100 000 bytes
- propagation delay (on feedback channel):  $\tau = 100 \text{ ms}$
- initial congestion window (cwnd) = 2 MSS
- upper bound of the congestion window (cwnd) = 1000 MSS (approximately infinity)
- initial value of slow-start threshold (ssthresh) = 1000 MSS (approximately infinity)

Fig. 5.32 - Fig. 5.35 show the conditional mean transfer time delay for different access rates and load 0.8. In these figures, not only the results obtained for the PS model according to (4.6) and the TCP model (obtained by simulation) under the given load conditions are depicted, but also the curves for an empty PS and TCP model according to equations (4.1) and (4.27) using (4.29) - (4.33), respectively. The latter ones are lower bounds for the transfer time in the corresponding models and can be used as a basis for relative measures.

For larger access rates, the results obtained for the loaded PS and TCP model differ significantly as the curve of the TCP model is dominated by the slow start behaviour of TCP. This is visible through the stair step shape in the corresponding curves of the TCP simulation.

The minimum transfer times in the TCP model and the PS model also differ significantly for greater access rates. The impact of relating  $\phi_{\Delta}(s)$  to different minimum values is depicted in Fig. 5.36 and Fig. 5.37. In Fig. 5.36, the minimum transfer time obtained for the PS model is taken as reference, whereas the minimum obtained for the TCP model is taken as reference in Fig. 5.37. The huge difference of the curves indicates that – at least for greater access rates – it is not reasonable to relate the transfer time to the minimum obtained for the PS model as this



Fig. 5.32: Conditional mean transfer time  $(\rho = 0.8, r_{max} = 10 \text{ Mbit/s})$ 







Fig. 5.36: Conditional mean fun factors  $\varphi_{\Delta}(s)$ related to PS minimum ( $\rho = 0.8$ )



can never be reached. This would result in a  $\varphi_{\Delta}(s)$  that is bounded by a rather small interval. For  $r_{max} = 10$  Mbit/s, e.g., the maximum mean fun factor that can be reached is 0.2.

On the contrary, a relation of the transfer time to a more realistic minimum obtained for the TCP model assuming an unloaded backbone link yields a  $\varphi_{\Delta}(s)$  that is rather close to 1 as can be seen in Fig. 5.37. That means a load of 0.8 does not significantly reduce the mean fun factor. Moreover, the fact that  $\varphi_{\Delta}(s)$  is nearly independent of the object size *s* over a wide range is an appealing feature of that approach.

In Fig. 5.38 - Fig. 5.43, results for an increased load of 0.9 with the other parameters being unchanged are depicted. The same effect as described above can be observed. For the TCP model, the delay component which is caused by congestion on the bottleneck link (this component is responsible for the linear increase of  $\varphi_{\Delta}(s)$ ) is now larger. The linear slope of the curve for t(s) is, however, significantly smaller in the case of the TCP model as compared to the PS model. As clearly visible in Fig. 5.39 and Fig. 5.40 this leads to an intersection of the curves. Fig. 5.43, finally, shows that the mean fun factor related to the  $T_{min, TCP}$  is now significantly smaller than 1 due to the increased load.



Fig. 5.38: Conditional mean transfer time  $(\rho = 0.9, r_{max} = 10 \text{ Mbit/s})$ 



Fig. 5.40: Conditional mean transfer time  $(\rho = 0.9, r_{max} = 768 \text{ kbit/s})$ 



Fig. 5.42: Conditional mean fun factors  $\varphi_{\Delta}(s)$ related to PS minimum ( $\rho = 0.9$ )







Fig. 5.41: Conditional mean transfer time  $(\rho = 0.9, r_{max} = 128 \text{ kbit/s})$ 





# **6** Conclusions

The comparison of different fun factor definitions has shown that  $\phi_{\Delta}$  and  $\phi_{\Phi}$  significantly differ in the processor sharing model as well as in the TCP model. In the processor sharing case  $\phi_{\Delta}$  seems to be a promising QoS measure as it can be calculated (straight line) in case the access rate and the link rate are the same. Moreover,  $\phi_{\Lambda}(s)$  hardly depends on the request size.

For greater access rates, the PS model is not appropriate to describe the perceived QoS. The TCP window mechanism (especially slow start) influences the statistics significantly in many cases and thus cannot be generally neglected. In comparison to that, the effect of increasing the load from 0.8 to 0.9 is not as large as originally expected. Decreasing the load below a value of 0.8 results in a fun factor increase that is even hardly visible.

In order to take the influence of slow start into account,  $\phi_{\Delta}$  should be related to the minimum transfer time that is obtained for the TCP model instead of using the the minimum obtained for the PS model.

Case studies using a fun factor definition that considers connection setup delay have shown that the setup delay distribution has only minor influence on the mean fun factor in most cases.

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# References

- [1] J. Charzinski: "Fun factor dimensioning for elastic traffic." *Proceedings of the 13th ITC Specialist Seminar on IP Traffic Measurement, Modeling and Management*, Monterey, CA, September 2000, pp. 11-1-11-9.
- [2] J. Charzinski: "Measured HTTP performance and fun factors." Accepted for publication in *Proceedings of the 17th International Teletraffic Congress (ITC 17)*, Salvador da Bahia, Brazil, September 2001.
- [3] L. Kleinrock: *Queueing systems Volume II: computer applications*, New York, USA, John Wiley and Sons, 1976.
- [4] K. Lindberger: "Balancing quality of service, pricing and utilisation in multiservice networks with stream and elastic traffic." *Proceedings of the 16th International Teletraffic Congress (ITC 16)*, Edinburgh, June 1999, pp. 1127-1136.
- [5] IND Simulation Library, http://www.ind.uni-stuttgart.de/INDSimLib.