Juni 73

ON THE INFLUENCE OF HUNTING MODE AND LINK WIRING ON THE LOSS OF LINK SYSTEMS

Dieter Bazlen, Gerald Kampe and Alfred Lotze
University of Stuttgart
Stuttgart, Federal Republic of Germany

ABSTRACT

The problems are studied how group selection link systems should be structured, wired and hunted to achieve optimal traffic characteristics. Furthermore a loss-and-load-equivalent mapping of large group selection link systems to smaller ones is presented.

A new approximate loss calculation method is developed.

All studies were supported by extensive Monte Carlo Simulations.

Beitrag des Instituts für Nachrichtenvermittlung und Datenverarbeitung der Universität Stuttgart zum 7th International Teletraffic Congress Stockholm vom 13.-20. Juni 1973

1. INTRODUCTION

In this paper the problems are studied how link systems should be structured, wired and hunted to get optimal traffic characteristics. These studies are performed by means of simulation and calculation resp. Monte Carlo Simulation/1,3/ was applied with a total of more than 300 million calls. About 200 group selection link systems with 2, 3, 4 and 6 stages have been considered. The total number of outlets per link system is in the range of 100 up to 1000 trunks. Of course, in this paper the most essential results can be reported only.

Poissonian traffic was offered to all considered link systems (cf. section 2.8). The holding times are negative exponentially distributed.

In Chapter 2 a survey is given on the investigated link system types, the selected structures considered here and two different modes to hunt the inlets.

In Chapter 3 the influence of different wiring and hunting modes on the probability of loss is investigated.

Chapter 4 deals with a new procedure of link system design to obtain a minimum requirement of crosspoints per Erlang for prescribed traffic and loss as well as for good overload characteristics.

In Chapter 5 a handy method is presented which facilitates a loss-and-load-equivalent mapping of large group selection link systems to smaller ones for the purpose of their economic full scale simulation on a digital computer.

Chapter 6 deals with a new approximate loss calculation for group selection link systems. Calculated and simulated results are compared.

2. SURVEY ON THE STRUCTURES STUDIED ; DEFINITION OF THE INPUT PROCESS

2.1 Summary

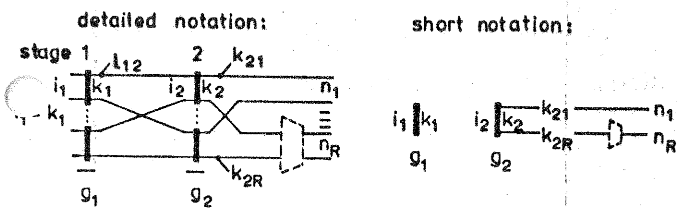
In this chapter a survey is given on the link system structures which are discussed in Chapters 3 through 6. Of course, only a s e l e c t i o n out of the total of more than 200 investigated structures is shown. All structures are designed for group selection and do not concentrate the traffic but possibly in the last stage.

The various link systems consist of 2,3,4 or 6 stages. 5-stage link systems were not considered by reason of additional computer time.

For a better understanding of link system structure parameters, the general considerations are given in Section 2.2 through 2.6. Each link system is signed by a code and listed in Section 2.7. Section 2.8 describes the inlet hunting.

2.2 Structures with two stages

Fig.1 shows the notations applied to 2-stage link systems. In all studied 2-stage link systems there exists at most one link between each multiple of stage 1 and stage 2 (2-stage fan out structure, cf. Section 2.3). 2-stage structures with more than one link connecting each multiple of stages 1 and 2 are not presented here.



The symbol ⌐¬ means a grading, i.e. $n_r < g_2 k_{2r}$.

Fig. 1: Notations for 2-stage link systems.

The following notations are applied:

i_j = inlets per multiple in stage j
k_j = outlets per multiple in stage j } (j=1,...,S)
g_j = number of multiples in stage j
S = number of stages
R = number of outgoing trunk groups
$l_{j,j+1}$ = average number of links from each multiple in stage j to each multiple in stage j+1
k_{sr} = outlets per multiple to group r
n_r = number of trunks per group r } (r=1,...,R)

2.3 Structures with three stages

Fig. 2 shows the notations for 3-stage link systems, applied to two distinct structure types:

- fan out structure: at most one path leads from a certain inlet of stage 1 to a certain multiple of the last stage.
- meshed structure: more than one path exists between a certain inlet of stage 1 and a certain multiple of the last stage.

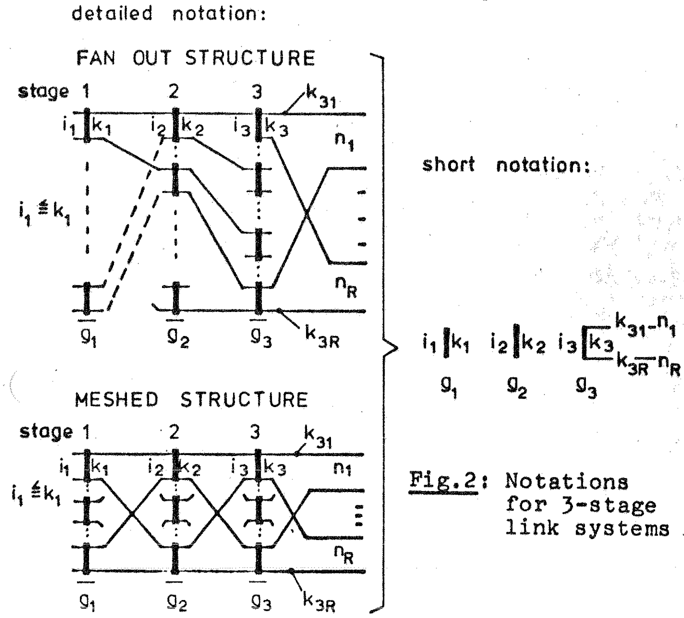


Fig.2: Notations for 3-stage link systems

The multiples of two successive stages can be wired such, that the link system is subdivided into linkblocks with regard to these stages.

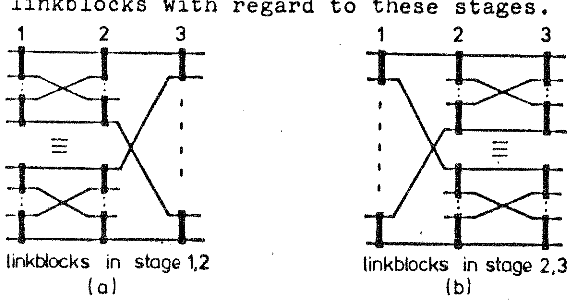
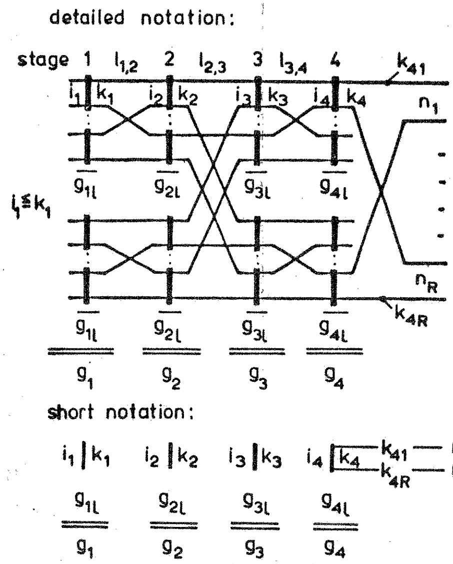


Fig.3: Linkblocks in 3-stage link systems

Fig.3 shows two examples of 3-stage link systems with linkblocks. General remarks on linkblocks are given in Section 3.5.1.

2.4 Structures with four stages

Fig.4 shows the notations for 4-stage meshed link systems with group selection. Link systems with linkblocks (Fig.4) and without linkblocks (Fig.5) are considered.



with g_{jl} = number of multiples per linkblock in stage j.

Fig.4: Notations for 4-stage link systems with linkblocks

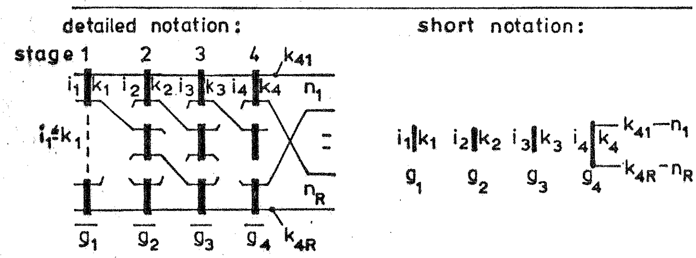


Fig.5: Notations for 4-stage link systems without linkblocks

2.5 Structures with six stages

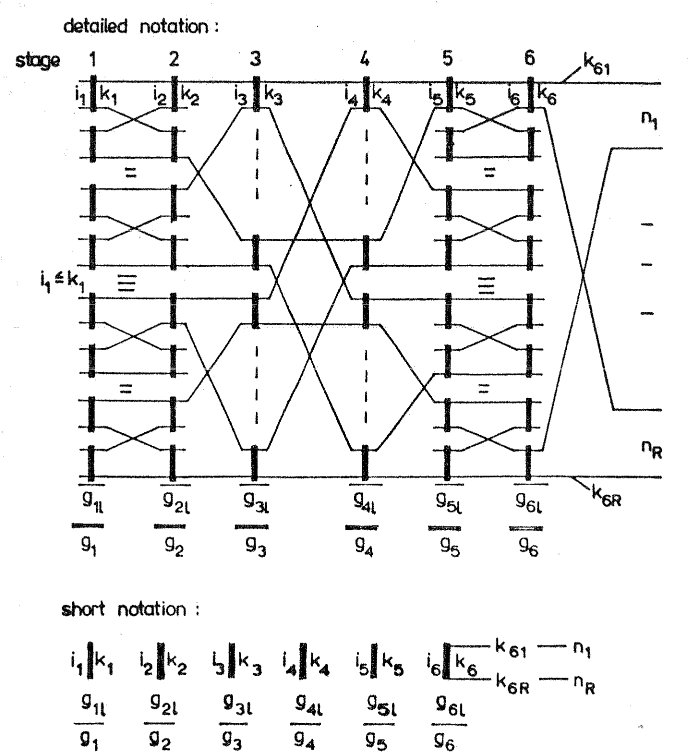


Fig.6: Notations for 6-stage link systems

Fig.6 shows the notations for 6-stage link systems with meshed structure. The studied structures are designed without linkblocks or with linkblocks e.g. in stages 1-2-3 and 4-5-6 (Fig.6).

2.6 The "linkwidth" of a group selection link system

The linkwidth of a link system is defined by the inlet linkwidth $LW_{in} = P/N_{in}$ and the outlet linkwidth $LW_{out} = P/N_{out}$ with

P = number of links between two successive stages (in the considered structures for group selection, P does not vary inside the link system)

N_{in} = total number of link system inlets

N_{out} = total number of link system outlets

Fig.7 shows the 4 basic structural features being studied in this paper. The influence of inlet linkwidth and outlet linkwidth on the probability of loss is studied in Chapter 4. The results presented there show clearly that only feature A or at least feature B should be applied. Features C and D yield remarkably higher probabilities of loss.

	Type of Link System	LW_{in}	LW_{out}	Feature of Link System
A	Wide	>1	>1	N_{in} — P — N_{out}
B	Wide	>1	≈1	N_{in} — P — N_{out}
C	Narrow	≈1	≈1	N_{in} — P — N_{out}
D	Narrow	≈1	>1	N_{in} — P — N_{out}
$N_{in}=g_1 \cdot i_1$; $N_{out}=g_S \cdot k_S$; $P=g_1 \cdot k_1 = \ldots = g_{S-1} \cdot k_{S-1}$				

Fig.7: Four basic structural features

2.7 List of presented structures

Fig.8 gives a survey on the structures. The following chapters refer to these structures by the indicated code only.

Structure (short notation)	Code	Cross-points	discussed in Chapter
8∣10 (12) 12∣10 (10) 10x10 (1—10)	L20	2160	3.3 ; 6.5
10∣10 (10) 10∣10 (10) 10x10 (1—10)	L21	2000	3.3 ; 6.5
8∣10 (12/48) 12∣10 (10/40) 10x20 (1—D—20); D Simplified Standard Grading	L22	8640	3.3 ; 6.5
10∣12 (10) 10∣10 (12) 6∣5 (20) 5x20 (1—20)	L30	3000	3.4 ; 6.5
9∣10 (10) 10∣10 (10) 5∣5 (20) 5x20 (1—20)	L31	2400	4.3 ; 6.5
10∣10 (10) 10∣10 (10) 5∣5 (20) 5x20 (1—20)	L32	2500	4.3 ; 6.5
10∣10 (20) 10∣10 (20) 20∣20 (10) 10x20 (2—20)	L33	8000	5.3 ; 6.5
10∣10 (10) 10∣10 (10) 10∣10 (10) 5x20 (2—20)	L34	3000	5.3 ; 6.5
10∣10 (4) 10∣10 (4) 4∣4 (10) 2x20 (2—20)	L35	960	5.3 ; 6.5
10∣10 (10) 10∣10 (10) 10∣10 (10) 10x10 (1—10)	L36	3000	6.5 ; 6.5
5∣10 (50) 5∣5 (100) 10∣5 (50) 5x50 (1—50)	L37	7500	4.3
5∣7 (50) 5∣5 (70) 7∣5 (50) 5x50 (1—50)	L38	5250	3.4 ; 6.5
5∣6 (50) 5∣5 (60) 6∣5 (50) 5x50 (1—50)	L39	4500	3.4 ; 6.5
5∣10 (50) 5∣5 (100) 5∣5 (100) 10∣5 (50) 5x50 (1—50)	L40	10000	4.3
5∣8 (5/50) 5∣5 (8/80) 5∣5 (8/80) 8∣5 (5/50) 5x50 (1—50) or without linkblocks	L41	8000	3.5 ; 4.3 ; 6.5
5∣8 (50) 5∣5 (80) 5∣5 (80) 8∣5 (50) 0.5—25, 0.5—25, 1—50, 1—50, 2—100	L42	8000	6.5
5∣7 (50) 5∣5 (70) 5∣5 (70) 7∣5 (50) 5x50 (1—50)	L43	7000	4.3 ; 6.5
7∣7 (50) 5∣5 (70) 5∣5 (70) 7∣5 (50) 5x50 (1—50)	L44	7700	4.3
10∣10 (25) 10∣10 (25) 10∣10 (25) 10∣10 (25) 5x50 (2—50)	L45	10000	4.3
20∣20 (50) 10∣10 (100) 10∣10 (100) 20∣20 (50) 20x50 (1—50)	L46	60000	5.3
20∣20 (10) 10∣10 (20) 10∣10 (20) 4∣4 (50) 4x50 (1—50)	L47	8800	5.3
10∣10 (5/25) 5∣5 (10/50) 5∣5 (10/50) 10∣10 (5/25) 5x50 (2—50) or without linkblocks	L48	7500	3.5 ; 4.3 ; 6.5
5∣6 (50) 5∣5 (60) 5∣5 (60) 5∣5 (60) 5∣5 (60) 6∣5 (50) 5x50 (1—50)	L60	9000	4.3
5∣7 (50) 3∣3 (116) 3∣3 (116) 3∣3 (116) 3∣3 (116) 7∣5 (50) 5x50 (1—50)	L61	7676	3.6 ; 6.5
5∣6 (50) 3∣3 (100) 3∣3 (100) 3∣3 (100) 3∣3 (100) 6∣5 (50) 5x50 (1—50)	L62	6600	5.3 ; 6.5
5∣6 (20) 3∣3 (40) 3∣3 (40) 3∣3 (40) 3∣3 (40) 2.4∣2 (50) 2x50 (1—50)	L63	2280	5.3
5∣5 (5/50) 5∣5 (5/50) 5∣5 (10/50) 5∣5 (10/50) 5∣5 (5/50) 5∣5 (5/50) 5x50 (1—50) or without linkblocks	L64	7500	3.6 ; 6.5

Fig. 8: List of presented structures.

2.8 The input process

It is assumed that Poissonian traffic $A' = \lambda \cdot h$ is offered, having a constant call rate λ. The holding times are negative exponentially distributed (mean holding time h).

Since link systems have a finite number of inlets, two methods can be distinguished regarding the offer of calls:

Method I : To each multiple of stage 1 an individual p a r t i a l traffic $A_p' = A'/g_1$ is offered. This partial traffic is clipped, if all i_1 inlets of the considered multiple are busy (probability of blocking b_I). Thus, the actual offered partial traffic becomes $A_p = A_p' \cdot (1-b_I)$ and the total offered traffic is $A = \sum_{i=1}^{g_1} A_{pi}$

Method II : To a l l multiples of stage 1 the t o t a l traffic A' is offered. To achieve balanced offered traffic per multiple, any offered call can hunt the total number of inlets in sequential order starting with the first inlet of a randomly selected first stage multiple. Without regard to further connection through the link system, the call occupies the first free inlet which is found in the first selected or subsequent first stage multiples. Therefore the total traffic is clipped only, if all $N_{in} = i_1 \cdot g_1$ inlets of the link system are busy (probability of inlet blocking b_{II}). Thus the actually offered total traffic becomes $A = A' \cdot (1-b_{II})$.

Because of $i_1 \ll N_{in}$ it holds $b_{II} \ll b_I$.

Method I can cause a more intensive smoothing of traffic than method II and can yield losses even lower than in full accessible trunk groups. To avoid this falsifying "Sub Erlang Loss"-effect as far as possible, in all presented tests method II was applied.

3. THE INFLUENCE OF LINK WIRING AND HUNTING MODES ON THE LOSS OF LINK SYSTEMS

3.1 Survey

The probability of loss in link systems depends not only on the offered traffic and the structure but also on the mode of link wiring and hunting (routing of an incoming call). These questions are studied in this chapter. Wide and narrow structures (cf. Section 2.6) consisting of 2,3,4 and 6 stages are investigated.

In 2-stage and 3-stage link systems s e q u e n t i a l wiring and s e q u e n t i a l hunting with home position is found to yield the lowest loss. In 4-stage and 6-stage link systems c y c l i c wiring and s e q u e n t i a l hunting with home position yields minimum loss. It is shown that link wiring with linkblocks results in increased loss compared with link systems wired without linkblocks.

In the loss range of 10 per cent up to 100 per cent there exists no remarkable influence of link wiring and hunting mode on the loss. Wide optimum structures (Chapter 4) are by far less sensitive to link wiring and hunting modes than unfavourable narrow structures (Section 2.6).

3.2 The studied modes

As Fig.9 shows, 6 types of link wiring and 2 types of hunting mode are investigated (strictly random hunting yields approximately the same results as sequential hunting with random start position and is therefore not discussed here).

The code, indicated in Fig.9 is a short notation of link wiring and hunting mode, e.g. (SW-H)(CN-H) stands for a 3-stage link system: between stages 1 and 2 the links are sequentially wired within linkblocks (=SW) and sequentially hunted with home position (=H). Between stages 2 and 3 the links are cyclically wired without linkblocks (=CN) and sequentially hunted with home position.

3.3 Structures with two stages

In Fig.10 simulation results are compared of a w i d e structure (L20) and a n a r r o w one (L21). In both cases the following rule for 2-stage link systems is obvious: The mode (SN-H) is advantageous, if the loss does not exceed 10 per cent. For higher losses no remarkable difference between the various wiring and hunting

Code of link wiring		Type of link wiring	Example
Structures without linkblocks	SN	sequentially wired links, no linkblocks	
	CN	cyclically wired links, no linkblocks	
Structures with linkblocks	SW	sequentially wired links, within linkblocks	block 1 block 2
	CW	cyclically wired links, within linkblocks	block 1 block 2
	SB	sequentially wired links, between linkblocks	
	CB	cyclically wired links, between linkblocks	
Code of hunting mode		**Type of hunting mode**	
	H	Sequential hunting with home position	
	R	Sequential hunting with random start position	

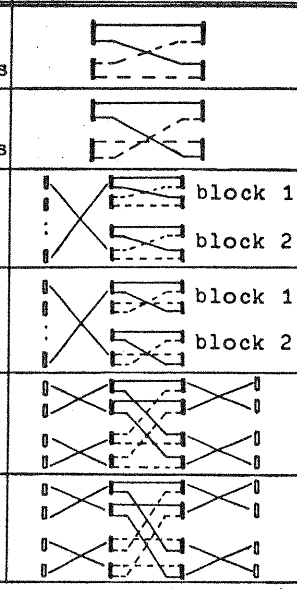


Fig.9: Types of link wiring and hunting modes

modes exists. In link systems with graded trunk groups (SN-H) is still more favourable (Fig.11), if these gradings are designed with progressive commoning which is suitable for sequential hunting with home position (cf. /10,11/).

In 2-stage link systems the modes (CN-H),(SN-R) and (CN-R) do not differ regarding the resulting loss /13/. Therefore, sequential hunting with random start position is not considered any more by the investigations of Section 3.4 through 3.6.

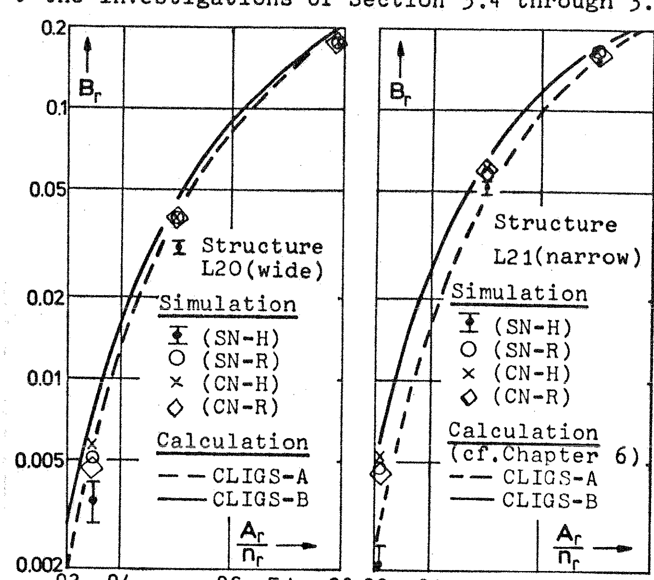


Fig. 10: Wiring and hunting in 2-stage link systems (here $A_r/n_r = A_{tot}/N_{out}$) (Simulation with 95% confidence interval.)

3.4 Structures with three stages

In Fig.12 and Fig.13 $B_r = f(A_r/n_r)$ of two m e s h e d 3-stage link systems is presented. As L30 has a wide structure (Section 2.6), the modes (SN-H)(SN-H) and (CN-H)(CN-H) give nearly the same loss with a slight advantage for (SN-H)(SN-H), see Fig.12.

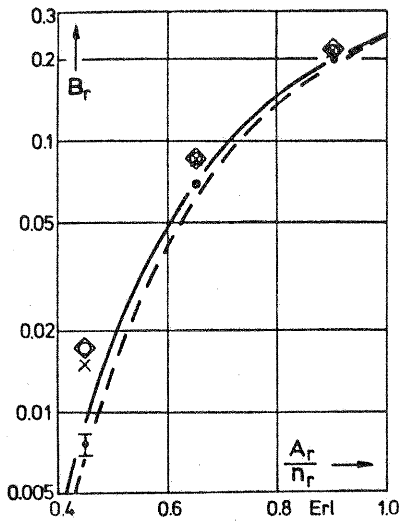
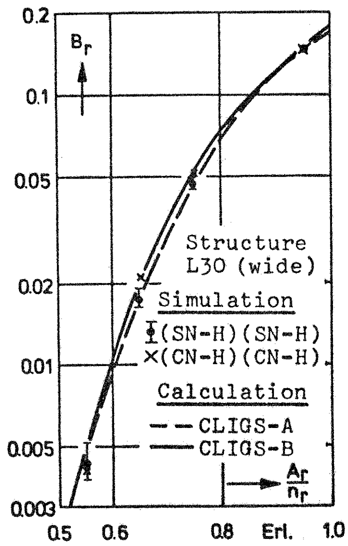


Fig.11: Advantage of (SN-H) in case of graded trunk groups (here $A_r/n_r = A_{tot}/N_{out}$)



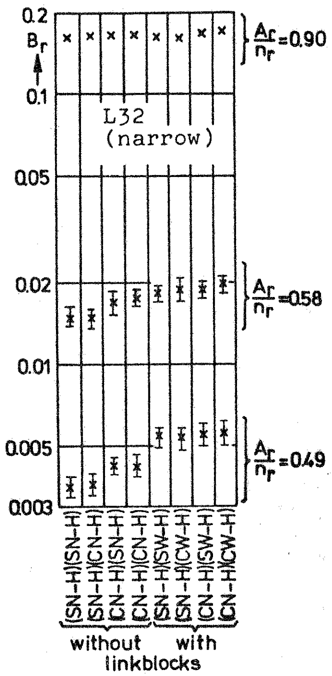


Fig. 12: Wiring and hunting in a wide 3-stage meshed link system

Fig. 13: Wiring and hunting in a narrow 3-stage meshed link system

(here $A_r/n_r = A_{tot}/N_{out}$)

An extensive study of system L32 is given in Fig.13 where 8 different wiring modes are investigated /14/. The considered narrow structure can be wired with or without linkblocks in stages 2 and 3 because each multiple of stage 2 hunts 10 out of 20 last stage multiples only. In case of linkblocks we get a structure similar to Fig.3(b). Fig.13 shows that (SN-H)(SN-H) is best. Cyclic wiring between stages 1 and 2 is not favourable. Linkblocks increase the loss even more. Like in 2-stage link systems, all studied modes are equivalent in case of high offered traffic.

3-stage f a n o u t structures show minimal losses in case of mode (SN-H)(SN-H), too. Fig.14 gives two examples which demonstrate that in case of fanout structures even wide structures are sensitive to wiring and hunting mode.

3.5 Structures with four stages

3.5.1 General remarks

In spite of their loss increasing properties, linkblocks are often favourable because of the following technical reasons:

- in case of increasing traffic the link system can easier be extended by attaching additional linkblocks,
- the common control may sometimes be organized simpler for blocks.

In Section 3.5.2 some block and nonblock configurations of link system L48 are compared.

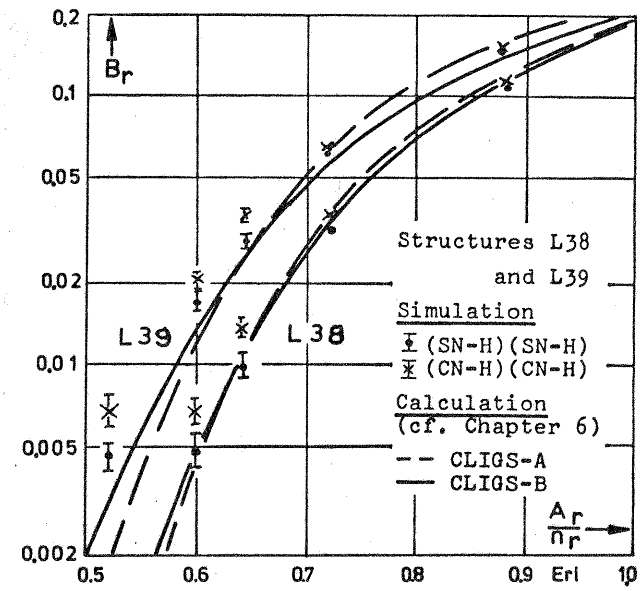


Fig. 14: Wiring and hunting in 3-stage fanout link systems (here $A_r/n_r = A_{tot}/N_{out}$).

3.5.2 Simulation results

For w i d e 4-stage link systems Fig.15 gives an example by link system L41. In this case the subdivision into linkblocks has no remarkable influence on the loss. On the other hand n a r r o w (not recommendable) 4-stage link systems show a more distinct relation between wiring and hunting mode and probability of loss (Fig.15, link system L48). Examples of a more detailed study on block or nonblock design are given in Fig.16.: At A_r/n_r=0.56Erl. the loss varies from 0.098 to 0.237 per cent depending on the structure.

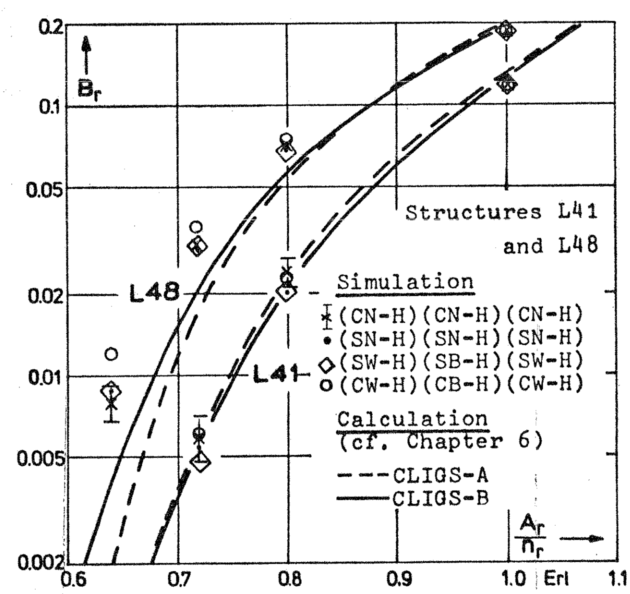


Fig. 15: Wiring and hunting in 4-stage link systems (here $A_r/n_r = A_{tot}/N_{out}$).

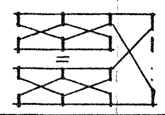
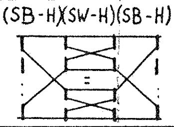
Wiring and hunting with linkblocks	Simulation $(B \pm \Delta B)/10^{-3}$	Wiring and hunting without linkblocks	Simulation $(B \pm \Delta B)/10^{-3}$
(SW-H)(CB-H)(SW-H)	1.97 ± 0.13	(SN-H)(CN-H)(CN-H)	0.98 ± 0.08
(SW-H)(SB-H)(SW-H)	2.34 ± 0.20	(CN-H)(CN-H)(CN-H)	1.05 ± 0.12
(CW-H)(CB-H)(CW-H)	2.26 ± 0.12	(SN-H)(SN-H)(SN-H)	2.11 ± 0.16
(SW-H)(SW-H)(SN-H) 	2.37 ± 0.30		
(SB-H)(SW-H)(SB-H) 	2.34 ± 0.20		

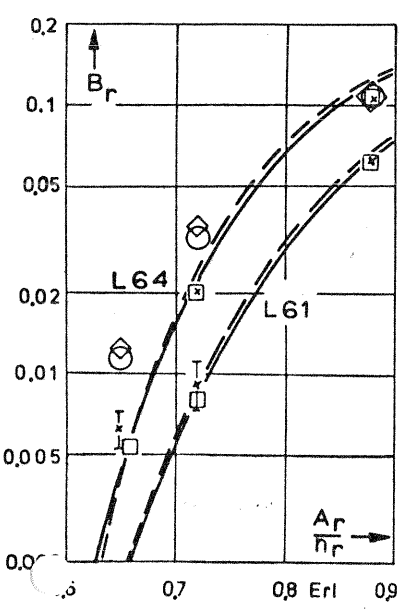
Fig. 16: Comparison of block and non-block configuration of system L 48.
$A_r/n_r = 0.56$ Erl.

In Section 3.3 and 3.4 it is demonstrated that in 2-stage and 3-stage link systems minimum loss is obtained for sequentially wired links combined with sequential hunting with home position. The reason may be, that in this mode the upper multiples of stage 2 get maximum load and the lower multiples have a certain "free path reserve". In contrary to this result, the mode (CN-H) per stage is favourable in 4-stage and 6-stage link systems (cf. Section 3.6). This may be explained by the fact, that in systems with more than 3 stages the outlets of multiples in a certain stage generally do not reach all multiples of the succeeding stage and therefore their possible free path reserve is not effective. In these systems it is more advantageous to distribute the traffic equally among all multiples per stage. This can be achieved in structures without linkblocks more consequently, because in this case the traffic flow is not divided into parts by a block structure.

3.6 Structures with six stages

In accordance with the results obtained in 3-stage and 4-stage meshed link systems (see Fig.12 and Fig.15) also the loss in w i d e 6-stage link systems does not depend on the applied wiring and hunting mode. In Fig.17 system L61 is given as an example.

An example of the studied n a r r o w structures is represented by system L64 (Fig. 17). As for narrow systems with S=4 cyclic wiring without linkblocks is most favourable here.



Structures L61 and L64

Simulation
- (SN-H) in all stages
- □ (CN-H) in all stages
- ○ (SW-H)(SW-H)(SB-H)(SW-H)(SW-H)
- ◇ (CW-H)(CW-H)(CB-H)(CW-H)(CW-H)

Calculation (cf. Chapter 6)
- - - - CLIGS-A
- —— CLIGS-B

Fig.17: Wiring and hunting in 6-stage link systems (here $A_r/n_r = A_{tot}/N_{out}$).

4. OPTIMAL DESIGN OF LINK SYSTEMS FOR GROUP SELECTION

4.1 Survey

The paper /9/ presented at the 5th ITC, New York, 1967, has already demonstrated how to calculate "optimum link systems" from the v i e w p o i n t of m i n i m u m c r o s s p o i n t requirement for prescribed traffic transparency.

The following outlines make use of these minimum crosspoint rules. Furthermore they pay special attention to a favourable loss vs. load characteristic in the range from the expected traffic to be carried up to significant overload.

The results shown in Chapter 3 and further results presented here, prove that a group selection link system is on no account permitted to have a "n a r r o w structure", i.e. $k_1/i_1 \leq 1$, because of its very unfavourable loss increasing property. This requires in any case expansion from inlets to outlets of the first stage multiples, even for a small load a_1 per inlet, e.g. $a_1 = 0.5$ Erlang.

It will be shown that "narrow" link systems can be replaced by more favourable "wide" link systems without increase of crosspoint requirement, sometimes even with a saving of crosspoints.

The optimum link method /9/ yields for a group selection link system with given numbers N_{in}, N_{out} of inlets and outlets resp., and with prescribed transparency T, an assortment of various different structures having nearly the same minimal crosspoint requirement for the same carried traffic.

It will be demonstrated that, nevertheless, the structures can have rather different and more or less suitable overload characteristics, which should be regarded for the design.

4.2 The tools for design

4.2.1 Minimum crosspoint structures

The most necessary ideas and formulae of /9/ be shortly repeated:
Looking for minimum crosspoint structures, one has to differentiate a formula, describing the total crosspoint requirement. This differentiation must regard the wanted carried traffic as well as (approximately) the desired grade of service (probability of loss). Instead of a more or less complicated (perhaps more or less accurate) approximate loss formula, the grade of service is characterized by means of the desired traffic-TRANSPARENCY T of the link system. T is a function of carried traffic and system parameters:

$$T = \prod_{j=1}^{S-1} (k_j - y_j) \cdot k_S \qquad (4.1)$$

where $y_j = Y_{tot} / g_j$ is the carried traffic per multiple in stage No. j, j = 1..S-1 . Balanced traffic input is provided.

The t r a n s p a r e n c y T according to eq.(4.1) means that average quantity of different idle paths (each consisting of (S-1) links in series), which lead from an arbitrary free inlet of a first stage multiple to the total of $N_{out} = g_S \cdot k_S$ outgoing trunks.

Meshed link systems have for normally carried traffic often $T > N_{out}$. For a "wide" group selection system, well designed by means of the outlines below, one obtains from T an approximate l o w e r bound of the effective accessibility $k_{eff,r}$ to any outgoing group No. r (r=1..R), having n_r trunks.

For $T \cdot \frac{k_{Sr}}{k_S} \geq n_r$ the n_r trunks are practically full accessible (compare traffic tests in fig. 20).

For $T \cdot \frac{k_{Sr}}{k_S} < n_r$ the n_r trunks have more or less limited accessibility;

where $T \cdot \frac{k_{Sr}}{k_S}$ is a l o w e r bound of $k_{eff,r}$ (cf. Chapter 6)

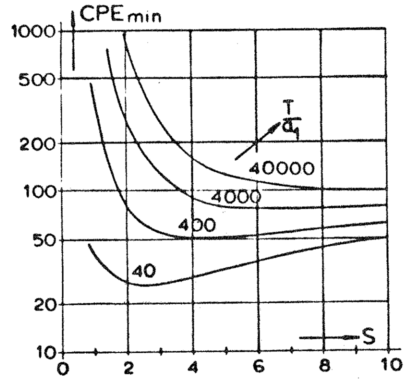


Fig. 18: The influence of the stage number S on the crosspoint requirement.

In addition to T one prescribes for the design also the total carried traffic Y_{tot}, the total number of inlets N_{in} and therewith the carried traffic per inlet

$$a_1 = Y_{tot} / N_{in} \qquad (4.2)$$

The theory /9/ yields the following formulae for the structural design of systems with minimum crosspoint requirement:

$$S_{opt} = \ln \frac{T}{4 \cdot a_1} \qquad (4.3)$$

is that number of stages which leads to the smallest crosspoint requirement.
The necessary number of crosspoints C referred to the total traffic Y_{tot} yields the Crosspoints per Erlang for a minimum crosspoint structure by (cf.Fig.18):

$$CPE_{min} = 4 \cdot S \sqrt[S]{\frac{T}{4 \cdot a_1}} \qquad (4.4)$$

Eq. (4.4) is true for $S \lesseqgtr S_{opt}$, if eq.(4.5) to (4.8) are observed:

$$k_j = 2 \cdot \sqrt[S]{\frac{T}{4 \cdot a_1}} \qquad j = 2,3 \ldots S-1 \quad (4.5)$$

$$i_1 = i_j = k_j \qquad j = 2,3 \ldots S-1 \quad (4.6)$$

$$k_1 = i_1 \cdot \frac{a_1}{0.5} = i_1 \cdot 2 \cdot a_1 \qquad (4.7)$$

$$k_S = i_1 \qquad (4.8)$$

The minimum total number of required crosspoints becomes with eq. (4.4)

$$C = Y_{tot} \cdot CPE \qquad (4.9)$$

From eq. (4.4) and (4.9) follows for prescribed quantity C , regarding eq.(4.5) to (4.8):

$$T_{max} = \left(\frac{C}{N_{in} \cdot a_1 \cdot 4 \cdot S} \right)^S \cdot 4 a_1 \qquad (4.10)$$

The formulae (4.3) and (4.5) to (4.8) still disregard the fact, that the quantitites S,k,i can only be realized as integers. Rounding up or down does, however, not influence significantly the crosspoint requirements CPE or C resp. The same is true with respect to some variations of the rounded integer values of k and i, to attain suitable integer ratios, such as N_{in}/i_1, being the number of multiples in stage No. 1, etc.

Of course, the designer has to check, whether the realized final structure approximates well enough the prescribed transparency T_{op} at the "operating point" (normally carried traffic).

4.2.2 Transparency vs. carried traffic

The transparency T is a parabolic function, which decreases with increasing carried traffic. Acc. to eq. (4.1) this decrease dT/dY_{tot} or dT/da_1 resp. depends not only on the traffic, but also significantly on the number S of stages. Fig.19 gives a clear picture of this property by means of 3 different link systems (S=3 with C=21166 crosspoints, S=4 with C=17814 crosspoints and S=6 with C=16866 crosspoints). All 3 link systems are designed acc. to section 4.2.1 for an operation point transparency $T_{op}/N_{out} = 1.75$ at $a_1 = 0.7$ Erl. and with $N_{in} = N_{out} = 400$.

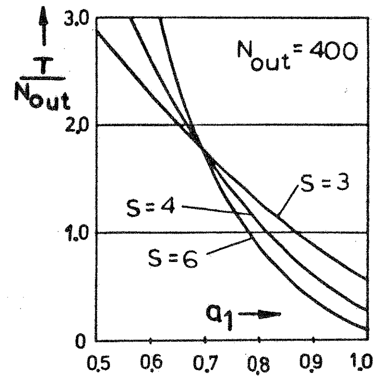


Fig. 19: The influence of the stage number S on $T = f(a_1)$.

The design has to take into account this overload characteristic (Section 4.3):

The smallest number S of stages which yields a wide system, having the desired operating transparency T_{op}, guarantees the smallest overload sensitivity.

4.2.3 The introduction of a "virtual" carried traffic a_1^* on a first stage inlet

As it has been checked by many hundred artificial traffic trials and as demonstrated in Chapter 3 (see Fig.10,12,13,15,17) a link system for group selection should have on no account a "narrow structure": Intermediate link stages No.2,... S-1 should have $k_j = i_j$. The first stage should have a ratio $k_1/i_1 \gtrsim 1.2$, e v e n t h e n if eq. (4.7) yields a smaller value. By this BASIC RULE of design the link system is in any case a "wide" one, i.e. it has an inlet linkwidth $LW_{in} = P/N_{in} > 1$, where $P = g_1 \cdot k_1$. This avoids to a great extent unfavourable loss increasing patterns of momentarily established paths, because the probability "all k_1 outlets blocked" is highly reduced.

The regard of this standard is the more important the greater the number S of stages is. To make this necessity compatible with the crosspoint saving design described in Section 4.2.1, a "virtual inlet load" $a_1^* > a_1$ is introduced.

It will be demonstrated that any "narrow system" can be improved by widening w i t h o u t a d d i t i o n a l crosspoint requirement.

Moreover, a certain "virtual inlet load a_1^*" can also be applied (for any structure) as a mean to design a link system less sensitive against overload.

4.3 Outlines for the design

4.3.1 General remarks

From the considerations in Section 4.2.2 and 4.2.3 follows that the design of group selection link systems mainly has to regard the following points:

1) The prescribed transparency T_{op} at the operating point i.e. for the traffic Y_{tot} to be n o r m a l l y carried, should be attained (at least approximately).

By means of Y_{tot} and the various partial traffics Y_r per outgoing group No. r, one calculates the probabilities B_r of loss acc. to Chapter 6.

If necessary one has to introduce a "virtual" inlet load a_1^* to ensure a "wide" system or to protect a system in particular against overload (see also 4.2.2).

3) The structure should be designed such that for prescribed operating transparency T_{op}, and for a corresponding crosspoint requirement in the order of CPE_{min} the decreased transparency T_{ov} (in case of short traffic peaks as well as of a longer lasting overload period) remains as bearable as possible (see 4.2.2).

In the following sections the design is explained in detail by means of various examples.

4.3.2 Example No. 1

System design for the following prescribed parameters:

N_{in} = 250, N_{out} = 250 (e.g. R=5, n_r=50(r=1..R) or R=3, n_1=50, n_2=n_3=100 etc.)

Y_{tot} = 160 Erlang, a_1 = 0.64 Erlang

T = 250 = 1.0·N_{out} .

With T/a_1 = 250/0.64 = 391 follows from Fig.18 that CPE_{min} is nearly the same one for S=3,4,5 and 6 stages:

With eq. (4.4) one obtains for S=3/4/5/6/ the values CPE = 55.3/50.3/50/51.5 .

The crosspoint requirement regarding S=4 and S=5 resp. does not differ remarkably. Therefore, S=4 be chosen.

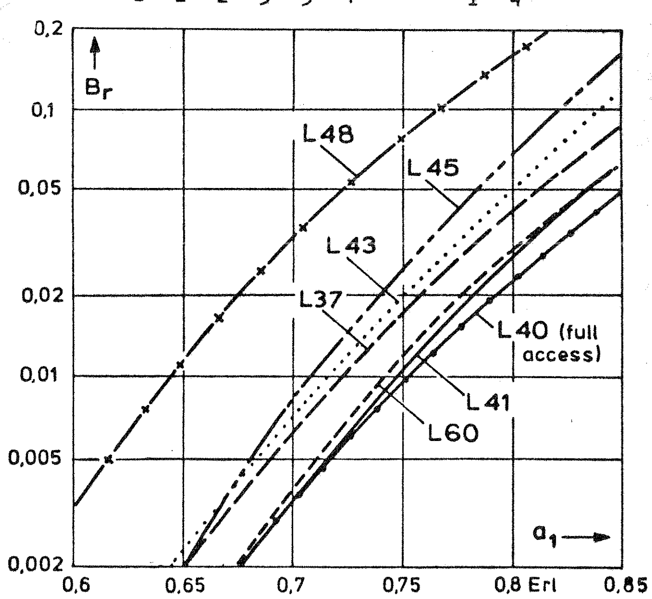
With S=4, T=250 and a_1=0.64 one finds with eq. (4.5) up to (4.8):

$i_1=\ldots i_4 = k_2=\ldots=k_4=6.29$ and

$k_1 = i_4 = 8.05$.

Appropriately one realizes a (slightly less expensive) structure which regards that the number $N_{in}=N_{out}$=250, i.e. N_{in}/i_1 should be integer.

Be chosen $i_1=i_2=k_2=i_3=k_3=k_4=5$ and $k_1=i_4=8$.



[illegible].20: Loss in link systems with about 8000 crosspoints (simulation results only)

Therewith we get the link system L41:

5|8 5|5 5|5 8|5 with C=8000.

The transparency becomes $T(a_1=0.64)$ = 216. Because $T\cdot k_{Sr}/k_S < n_r$ yields lower bounds for $k_{eff,r}$, the loss differs not remarkably from the loss in case of full accessibility (see Fig.20 and cf. Chapter 6).

4.3.3 Example No.2 for a_1=0.5 and a_1^* > 0.5

a) For the sake of simplicity we take also N_{in}=250, N_{out}=250 (R=5, n_r=50 (r=1,....R)) and here Y_{tot}=0.5·250 = 125 Erlang, i.e. a_1=0.5. Desired be T_{op}/N_{out}≈1.6, i.e. T ≈ 400. Eq. (4.5) up to (4.8) yield $i_1=k_1=i_2=..=k_4$= 7.5.

With regard to $N_{in}=N_{out}$=250 the following structure is realized (whose transparency is no more very close to the optimum):

L48 : 10|10 5|5 5|5 10|10 with C=7500.

One finds $T(a_1=0.5)$ = 313 (instead of 400, being the theoretical optimum for k_j=7.5). For a_1=0.6 one obtains still T(0.6) = 160.

This structure is, however, evidently unfavourable, because the standard rule $k_1 \gtrapprox 1.2\cdot i_1$ is neglected !

b) Let us therefore take a virtual a_1^*=0.7 Erl., but constant C≈7500. For that virtual value one finds again the most crosspoint saving structure. By means of eq. (4.10) T_{max}=144.14 .

From eq. (4.5) up to (4.8) one gets with T=144.14 the parameters $i_1=i_2=k_2=i_3=k_3=k_4$=5.36 and $k_1=i_4$=7.5.

Be realized i_1=5, k_1=7 etc. This "improved" structure L43 looks now:

5|7 5|5 5|5 7|5 with C=7000<7500

We get T(0.5)=233 which is remarkably below T(0.5) = 313 of the above system L48, however still fairly close to T≈N_{out} = 250. Further we get T(0.6)=163≈160 as for the system L48. Nevertheless, Fig.20 shows that $B_r=f(A_r/n_r)$ is by far better for the wide system L43 than for the narrow system L48.

c) Of course one can find still other "improved" structures e.g. with a_1^* =0.6...0.8 Erlang and sometimes slightly increased crosspoint requirements. (Another suitable structure would be L41 with C=8000 and T(0.6)=244.) The losses of all these structures have been measured by artificial traffic tests and are drawn in Fig.20. The inferiority of the system L48 using $k_1=i_1$ is striking !

4.3.4 Further examples

In the following examples further structures are considered to show the influence of the ratio k_1/i_1 and the number of stages S on the loss of link systems.

1. In Fig.20 and Fig. 21, among others, the curves for loss and transparency resp. for the 3-stage link system L37 and the 4-stage system L48 are drawn. The wide 3-stage link system is more advantageous than the narrow 4-stage system, both having the same number of crosspoints C=7500.
2. Fig. 20 and Fig. 21 contain furthermore two 4-stage link systems having C=10000. The inferiority of the narrow system L45 with k_1/i_1=1 is obvious comparing it with the wide and optimal system L40.
3. The 6-stage link system L60 with C=9000 has in the range of a_1=0.7 to 0.8 Erl. slightly higher losses than L41 with C=8000 (cf. Fig. 20). This is caused by the steeper slope dT/da_1 of L60 (cf. Fig.21).

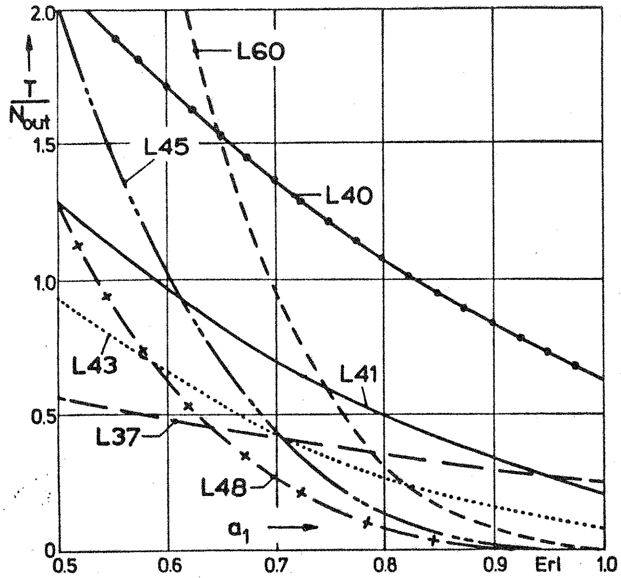


Fig.21: Transparency of link systems having about 8000 crosspoints each.

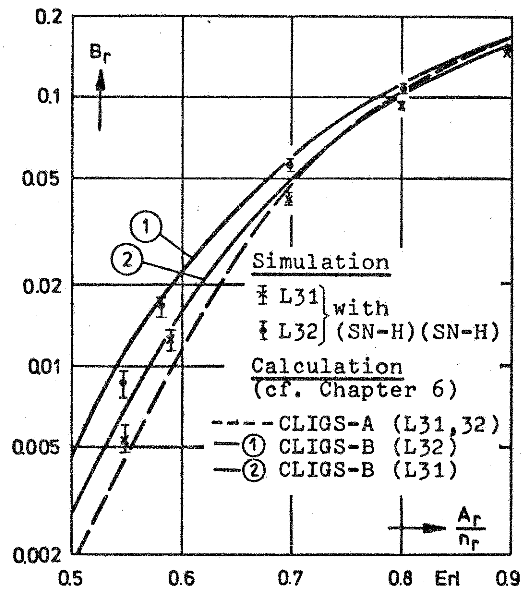


Fig. 22: The influence of $i_1=k_1$ in a 3-stage link system (here $A_r/n_r=A_{tot}/N_{out}$).

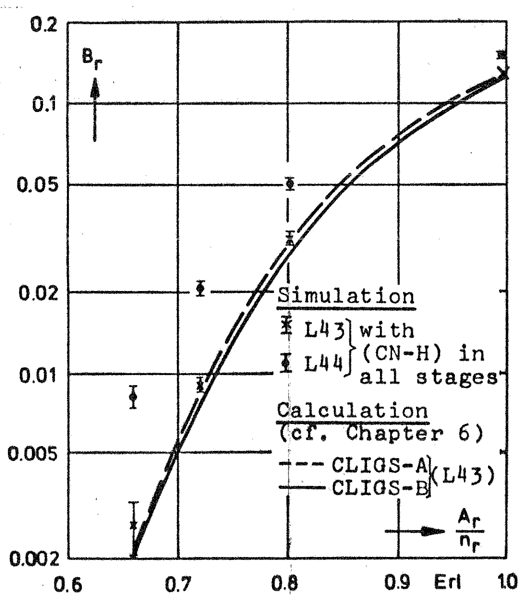


Fig. 23: The influence of $i_1=k_1$ in a 4-stage link system (here $A_r/n_r=A_{tot}/N_{out}$).

4. In Fig. 22 the loss-increasing effect of $k_1/i_1=1$ is shown by means of two 3-stage link systems. The two structures L31 and L32 differ only in the first stage. L31 has 9|10 and L32 has 10|10 first stage multiples. This results in remarkably increased losses for the narrow system L32. (L31 represents a structure according to the feature B in Chapter 2 (Fig.7) with $LW_{in}>1$ and $LW_{out}=1$).

5. A similar case as in Fig. 22 is considered in Fig. 23. Again, both structures differ only in the first stage. L43 has 5|7 and L44 has 7|7 first stage multiples. For the narrow system L44 the loss is increased by a factor 3 to 1.6 in the range of $A_r/n_r \approx 0.65$ to 0.8 Erl. (L44 represents a narrow structure acc. to feature D in Chapter 2 (Fig.7) with $LW_{in}=1$ and $LW_{out}>1$.)

5. MAPPING OF LARGE LINK SYSTEMS

5.1 Survey

Up to now large link systems can often not be full scale simulated. Instead, a partial graph of the link system is investigated by artificial traffic trials (e.g. /8/).

In this chapter a new method is presented which facilitates a loss-and-load-equivalent (LLE) single valued mapping of large group selection link systems to smaller ones (the number S of stages remains constant). These small systems can be full scale simulated requiring significantly less computer time and less storage capacity. The accuracy of this LLE-mapping is shown by simulation results.

5.2 Outline of the method

In Chapter 4 it was shown that the loss characteristics of link systems depend directly on the link system transparency. Therefore, the presented LLE-method is based on the idea to design a smaller link system (parameters marked by an asterix) with the same average number of "visible" last stage multiples as in the large system (cf. Section 4.2):

It holds

$$\frac{T^*}{k_S^*}\cdot\frac{1}{g_S^*} = \frac{T}{k_S}\cdot\frac{1}{g_S} \qquad (5.1)$$

with T^*, T = transparency

k_S^*, k_S = outlets per last stage multiple

g_S^*, g_S = number of last stage multiples

The smaller structure consists of m times less inlets, link lines and outlets resp., compared with the large structure (m<1, mapping factor). To achieve the same loss and load characteristics of a certain outgoing trunk group, the designer regards eq. (5.1) and proceeds as follows:

a) in stages 1,2,.j.,S-1 the multiple parameters remain constant ($i_j^*=i_j$, $k_j^*=k_j$), but the number of multiples per stage j is reduced by the factor m, i.e. $g_j^*=m\cdot g_j$;

b) in the last stage the number of multiples remains constant, ($g_S^*=g_S$), but their parameters are reduced by $i_S^*=m\cdot i_S$, $k_S^*=m\cdot k_S$;

c) the considered outgoing trunk group has the same number of trunks. Only the total number of outgoing trunks is reduced by $N_{out}^*=m\cdot N_{out}$.

Determination of the mapping factor m:
The designer must take into account that m should not lead to less then $R^* = 2$ outgoing groups to get a group selection link system. Furthermore, m is to be chosen such that the parameters g_j^* and R^* are integer numbers (cf. Section 5.3, example 1: $i_S^* = 2.4$).

Of course, the traffic is in case of a high usage operation more smoothed within the smaller system. This can, for very high utilization, reduce the probability of loss (cf. Section 5.3, ex. 2). Therefore, the total offered traffic A^* to the mapped small system should not exceed a value for which the probability of "all N_{in} inlets busy" is higher than about 1 per cent, according to $E_{1,N_{in}^*}(A^*)$ /4/.

5.3 Examples of mapping

Example 1: Fig.24 shows the comparison between a 250 trunk 6-stage link system (L62) and a 100 trunk 6-stage link system (L63). The applied mapping factor is m=0.4. The losses of both systems agree within the whole range of offered traffic.

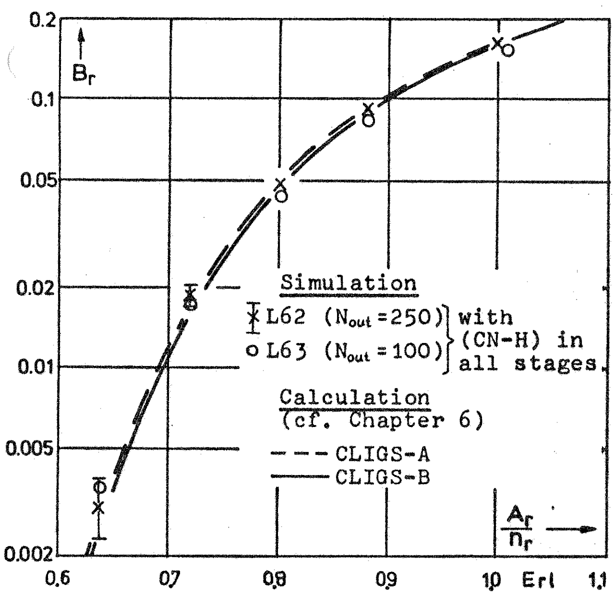


Fig. 24: LLE-mapping of a 6-stage link system.

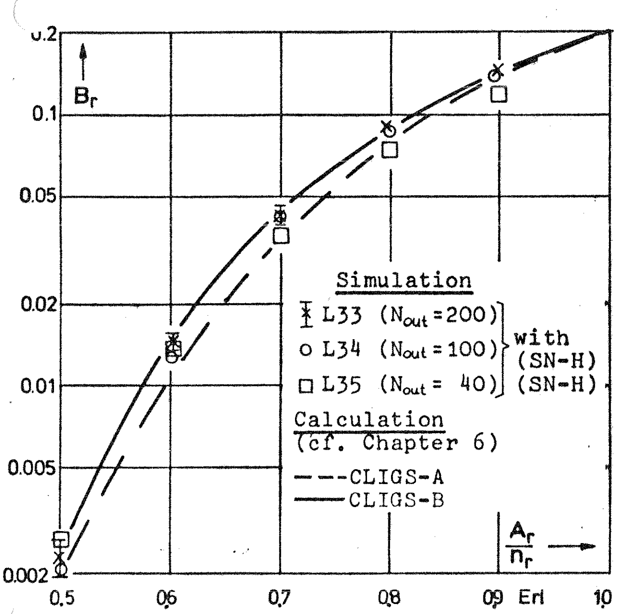


Fig. 25: LLE-mapping of a 3-stage link system.

Example 2: In Fig.25 LLE-mapping is demonstrated by three 3-stage link systems: structure L33 with 200 trunks is mapped to structure L34 with 100 trunks (m=0.5) and to structure L35 with 40 trunks (m=0.2) resp. At $A_r/n_r \lessapprox 0.7$ the mapping factor m=0.2 (40 inlets) yields equivalent loss whereas in case of $A_r/n_r > 0.7$ the loss of the smallest system (L35) increases less than for L33 because the prob. $E_{1,40}(A_r)$ becomes > 1 per cent. Thus, a mapping factor m=0.5 is more suitable for those offered traffics.

Example 3: Fig.26 gives an example for LLE-mapping of 4-stage link systems: structure L46 with 1000 trunks is mapped to the structure L47 with 400 trunks (mapping factor m=0.4). This saves about 30 per cent of storage capacity (data only). Furthermore, 64 per cent of expensive computer time was saved, provided, the simulation results were obtained in both cases with approximately the same confidence interval.

link system	number of offered calls	computer time (cycle time 0.6μs)	storage capacity: program + data	offered traffic (Erlang)	$B_r \pm \Delta B_r$ (per cent)
L 46	540000	≈38.7min	14k + 10k	39.98	4.13 ± 0.39
L 47	240000	≈14 min	14k + 7k	39.88	3.43 ± 0.22

Fig. 26: LLE-mapping of a 4-stage link system.

6. APPROXIMATE LOSS CALCULATION

6.1 Survey

This chapter deals with loss calculation by a new approximate formula for the effective accessibility regarding link systems with group selection, which have equal number of inlets and outlets per multiple except an expansion in the first and possibly concentration in the last stage resp. The abbreviation of the method be CLIGS.

Two versions CLIGS-A and CLIGS-B are derived. CLIGS-A is applicable for a quick manual evaluation, CLIGS-B is for computer evaluation only. Artificial traffic trials have proved the validity of these two methods.

6.2 The effective accessibility k_{eff}

The approximate effective accessibility k_{eff} is calculated from two terms (see Sections 6.2.1 and 6.2.2).

6.2.1 The "Free Fan"

The so-called "Free Fan" FF is defined by

$$FF = \prod_{j=1}^{S-1} (k_j - y_j) \qquad (6.1)$$

where k_j, y_j are the outlets and the carried traffic resp. per multiple in stage j (j=1..S-1). In eq. (6.1) the limitations hold:

$$\prod_{j=1}^{i} (k_j - y_j) \leq g_{i+1} \quad (i=1..S-1) \qquad (6.2)$$

This Free Fan FF represents that number of multiples in the last stage which are, on the average, accessible via free link paths from any free inlet of the first stage. The corresponding Free Fan-Accessibility to a certain outgoing group No. r is therewith

$$k_{FF,r} = FF \cdot k_{Sr} \qquad (6.3)$$

Eq. (6.3) equals to the average number of outlets to the considered outgoing group No. r, which can be hunted w i t h i n the Free Fan regardless whether they are idle or busy.

6.2.2 The "Busy Fan"

By means of the "Busy Fan" BF a second term of k_{eff} is calculated. The Busy Fan is defined by

$$BF = \prod_{j=1}^{S-1} k_j - FF \qquad (6.4)$$

where $\prod_{j=1}^{S-1} k_j$ is limited by g_S and means the "Maximum Fan", i.e. the maximum number of S-stage multiples being accessible from an inlet of the first stage (at least for traffic $Y_{tot}=0$).

The Busy Fan BF is equal to the average number of S-stage multiples w i t h i n this maximum fan, but o u t s i d e the Free Fan. From the BF follows

$$b_r = BF \cdot k_{Sr} \cdot \frac{y_r}{n_r} \qquad (6.5)$$

which is the average number of busy trunks in the considered group No. r w i t h i n the Busy Fan. The Free Fan does not include this number b_r of busy trunks.

Now it is known that "accessibility" is defined by
1) all accessible idle trunks of a group, and additionally
2) all busy trunks of this group which can be accessed and occupied once more, a s s o o n , a s t h e y b e c o m e i d l e.

The second point includes obviously not o n l y busy trunks of a group No. r w i t h i n the Free Fan. Additionally, it contains also that part of the b_r busy trunks (Eq. (6.5)) to which a free path from the considered first stage multiple to their own S-stage multiples gets open, as soon as their established connection terminates. This still unknown share of b_r forms also a part of the "effective accessibility". By one and the same termination also further idle outlets could get accessible.

How many trunks out of the b_r busy trunks within the Busy Fan do fulfill this condition "increase of access via the Free Fan as soon as an established call terminates ?"

The more links to the last stage (inlets of stage S) are idle, the greater is the probability that a further newly released S-stage inlet opens one or more paths via free links to those multiples in preceding stages S-1, S-2,..., 3,2 which are already part of the Free Fan. A first characterizing figure for this access-increasing effect is the average number f of all free links referred to all N_{out} trunks (for given total carried traffic Y_{tot}).

$$f = \frac{P - Y_{tot}}{N_{out}} \qquad (6.6)$$

with $P = g_S \cdot i_S$ and $N_{out} = g_S \cdot k_S$

Using this ratio f as an approximate factor to calculate from eq. (6.5) the "Busy Fan contribution" $k_{BF,r}$, one obtains:

$$k_{BF,r} = BF \cdot k_{Sr} \cdot \frac{y_r}{n_r} \cdot f \qquad (6.7)$$

The fitting of eq. (6.7) had, of course, to be checked and confirmed on the basis of many thousand artificial traffic trials, performed for this paper (see Chapter 2 to 5).

With eq. (6.3) and (6.7) the approximate formula for the effective accessibility to a group No. r holds

$$k_{eff,r} = k_{FF,r} + k_{BF,r} \qquad (6.8)$$

This formula is directly applied to the loss calculation in the following section (method CLIGS-A).

6.3 Loss calculation by method CLIGS-A

By means of the effective accessibility $k_{eff,r}$ (according to eq. (6.8)) the considered group selection link system is replaced, for a certain carried traffic, by an equivalent one stage array with a constant accessibility $k=k_{eff,r}$. Then the well known MPJ-Loss Formula /5,7/ is applied

$$B_r = \frac{E_{1,n_r}(A_{or})}{E_{1,n_r-k_{eff,r}}(A_{or})} \qquad (6.9)$$

From the prescribed carried traffic Y_r of the considered trunk group No. r the parameter A_{or} has to be calculated iteratively /5/ to fulfill the condition

$$Y_r = A_{or}(1 - E_{1,n_r}(A_{or})) \qquad (6.10)$$

The actually offered traffic to group No. r becomes

$$A_r = \frac{Y_r}{1 - B_r} \qquad (6.11)$$

For given $(Y_r, n_r, k_{eff,r})$ existing loss tables /7/ can also easily be applied.

The probability of loss, calculated with eq. (6.9), is in good accordance with the results of artificial traffic tests for any crosspoint-saving "wide" structure (cf. Section 2.6).

6.4 Loss calculation by method CLIGS-B

The version CLIGS-B uses the same expectation value $k_{BF,r}$ as given in eq. (6.7), but it regards the statistical traffic variations on the inlets of the considered first stage multiple and therewith partially the statistical variations of the Free Fan size.

For the probability "x inlets of a first stage multiple are busy" the Erlang distribution is assumed /2,4/.

$$w(x) = \frac{A_{01}^x / x!}{\sum_{j=0}^{i_1} (A_{01}^j / j!)}, \quad x = 0,1..i_1 \qquad (6.12)$$

with

$$A_{01} = \frac{y_1}{1 - E_{1,i_1}(A_{01})} \qquad (6.13)$$

and prescribed traffic y_1 per first stage multiple.

Then one can derive (analogously to eq. (6.1), (6.3) and (6.8)):

$$FF(x) = (k_1 - x) \cdot \prod_{j=2}^{g-1} (k_j - y_j) \qquad (6.14)$$

limited by $(k_1 - x) \cdot \prod_{j=2}^{i} (k_j - y_j) \leq g_{i+1}, \quad i = 2..S-1$

$$k_{FF,r}(x) = FF(x) \cdot k_{Sr} \qquad (6.15)$$

$$k_{eff,r}(x) = (FF(x) + BF \cdot \frac{y_r}{n_r} f) k_{Sr} \qquad (6.16)$$

For each value of x ($x=0,1,..,i_1-1$) by means of $k_{eff,r}(x)$ a probability of loss is calculated, analogously to eq. (6.9) :

$$B_r(x) = \frac{E_{1,n_r}(A_{or})}{E_{1,n_r-k_{eff,r}(x)}(A_{or})} \qquad (6.17)$$

Finally the probability of loss for the considered group No. r is evaluated with eq. (6.12) and (6.17):

$$B_r = \sum_{x=0}^{i_1-1} B_r(x) \cdot \frac{w(x)}{1 - w(i_1)} \qquad (6.18)$$

6.5 Comparison between loss formulae and artificial traffic tests

6.5.1 Examples taken from Chapters 4 and 5

Tests and calculation for various link systems are compared in Fig. 10,12,14,15,17,24,25 in Chapters 4 and 5. It is obvious that the loss formulae hold very well for all "wide", crosspoint-saving link systems.

"Narrow", i.e. crosspoint wasting link systems with $S \geqq 3$ stages mostly have higher losses than calculated, in particular for losses $B_r \lessapprox 5$ per cent (cf. Fig.14,15).

The method CLIGS-B yields for narrow systems still more realistic values of loss because it takes into account the probability w(x) on the first stage inlets. This property can in particular be seen in Fig. 22 regarding the systems L31 and L32. The method CLIGS-A yields, however, fairly good results for the wide system L31, whereas the loss of the narrow structure L32 is underestimated. For $S \geqq 4$ the loss of narrow systems is also underestimated by CLIGS-B, because the statistical traffic variations are approximately regarded between stage 1 and 2 only.

6.5.2 Examples with unequal group sizes and/or nonuniform offered traffics per group

For reasons of simplicity, the link systems discussed in the previous chapters were designed with equal group sizes and uniform offered traffics per group. This is, however, on no account a condition for the accuracy of the calculation methods CLIGS-A and CLIGS-B.

Fig.27 shows further comparisons between simulation and calculation (selection out of a large number of concerning investigations):

Example 1: 10 equal-sized groups with 10 trunks each and with nonuniform offered traffic per group

Example 2: 5 equal-sized groups with 50 trunks each and with nonuniform offered traffic per group

Example 3: 5 unequal groups with 25 up to 100 trunks and with nonuniform offered traffic per group

link system	n_r	offered traffic A_{tot}/Erl	$\frac{A_r}{A_{tot}}$	Simulation B ± ΔB	Calculation CLIGS- A	Calculation CLIGS- B
L 36	$n_{1,\dots,6}$ =10		0.05	0.008 ± 0.003	0.005	0.007
(SN-H)	$n_{7,8}$ = 10	79.83	0.10	0.125 ± 0.006	0.119	0.132
in all	n_9 = 10		0.20	0.436 ± 0.003	0.434	0.452
stages	n_{10} = 10		0.30	0.609 ± 0.002	0.599	0.614
L 60	n_1 = 50		0.05	0	0	0
	n_2 = 50		0.10	0	0	0
(CN-H)	n_3 = 50	189.77	0.20	0.010 ± 0.002	0.012	0.012
in all	n_4 = 50		0.30	0.185 ± 0.004	0.194	0.194
stages	n_5 = 50		0.35	0.270 ± 0.002	0.263	0.263
L 42	n_1 = 25		0.06	0.008 ± 0.002	0.006	0.005
	n_2 = 25		0.12	0.251 ± 0.002	0.219	0.206
(CN-H)	n_3 = 50	248.92	0.12	0.0004 ± 0.0003	0.0002	0.0002
in all	n_4 = 50		0.35	0.452 ± 0.001	0.412	0.398
stages	n_5 = 100		0.35	0.023 ± 0.001	0.022	0.020

Fig. 27: Comparison of simulation and calculation in case of unequal group sizes and/or nonuniform offered traffic per group.

The good accordance between simulation and calculation is apparent.

Acknowledgement

The authors wish to express their thanks to Messrs. S. Becker (PTT Tel Aviv, Israel) and U. Herzog (University of Stuttgart) for many valuable discussions.

REFERENCES

/1/ KOSTEN,L.: On the Measurement of Congestion Quantities by Means of Fictious Traffic. PTT Bedrijf 1948/49,15-25.

/2/ JACOBAEUS,C.: A Study on Congestion in Link-Systems. Ericsson Technics (1950), No.48, 1-68.

/3/ NEOVIUS,G.: Artificial Traffic Trials Using Digital Computers. Ericsson Technics 11 (1955), 279-291.

/4/ BROCKMEYER,E.;HALSTRØM,H.L.;JENSEN,A.: The Life and Works of A.K. Erlang. Copenhagen 1960.

/5/ LOTZE,A.:Loss Formula, Artificial Traffic Checks and Quality Standards for Characterizing One Stage Gradings. 3rd ITC Paris 1961, Doc. 28.

/6/ LOTZE,A.: 3rd Report on Studies in Congestion Theory (CIRB). Institute for Switching and Data Technics, University of Stuttgart, 1963.

/7/ LOTZE,A.; WAGNER,W.: Tables of the Modified Palm-Jacobaeus-Loss-Formula. Institute for Switching and Data Technics, University of Stuttgart, 1963.

/8/ GRANTGES,R.S.;SINOWITZ,N.R.: NEASIM, A General Purpose Computer Simulation Program for Load Loss Analysis of Multi-Stage Central Office Switching Networks. Bell System Techn.J.43(1964),965-1004.

/9/ LOTZE,A.: Optimum Link Systems. 5th ITC New York 1967,Pre-Book, 83-89 (and literature cited there).

/10/ LOTZE,A.: History and Development of Grading Theory. 5th ITC New York 1967, Pre-Book, 148-161.

/11/ HEINRICH,G.; TRAUTMANN,K.: Vereinfachte Normmischungen hinter einstufigen Koppelanordnungen in der Vermittlungstechnik. NTZ 21 (1968), 141-146.

/12/ a) KÜMMERLE,K.: An Analysis of Loss Approximations for Link Systems. 5th ITC New York 1967, Pre-Book, 327-336.
b) KÜMMERLE,K.: Berechnungsverfahren für mehrstufige Koppelanordnungen mit konjugierter Durchschaltung. Systematik und Analyse. 9th Report on Congestion Theory. Institute for Switching and Data Technics, University of Stuttgart, 1969.
(with a large number of references on link system analysis).

/13/ KAMPE,G.; KOCH,U.: Auflegungsart und Absuchmodus bei mehrstufigen Linksystemen. Institute for Switching and Data Technics, University of Stuttgart, Monograph 1972.

/14/ BAZLEN,D.;KAMPE,G.;LE MINH,T.: Linksysteme mit ungleichen Richtungsbündeln. Institute for Switching and Data Technics, University of Stuttgart, Monograph 1972.