CALL CONGESTION IN LINK SYSTEMS WITH INTERNAL AND EXTERNAL TRAFFIC

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ABSTRACT

In modern telephone exchanges the subscriber selection network is generally a multistage link system with both-way traffic, i.e. not only calls are offered to the link system which occupy one path through the link system (external traffic) but also such calls are offered which occupy two paths through the link system (internal traffic). An internal traffic call originates from a certain inlet of the link system and is connected to another inlet (subscriber) of this link system, i.e. the call needs two paths through the link system, an outgoing and an incoming path. In private automatic branch exchanges (PABX) link systems with internal and external traffic can also be found.

For the approximate calculation of the call congestion of such link systems a method is presented for PCT 1 (pure chance traffic No. 1, i.e. Poisson input having constant call intensity and negative exponential holding time distribution). This calculation is based on the method of combined inlet and route blocking (CIRB) [4,5,7].

Furthermore, this method makes use of the distribution function of the probabilities of state which was derived in [1,2] for full available groups with internal and external traffic.

In Chapter 1 the principle of the calculation method is explained. In Chapter 2 the internal and external traffic is characterized. In Chapter 3 the different operation modes of link systems with internal and external traffic are discussed. The various formulae of the calculation method will be derived in Chapter 4. In Chapter 5 the calculation results are compared with the results of artificial traffic trials performed on a digital computer.

1. THE PRINCIPLE OF THE CALCULATION METHOD

The call congestion is divided into two parts [4,7]:

1. The call congestion, caused by inlet blocking of the 1st stage.
2. The call congestion, caused by route blocking, i.e. the part of the call congestion, caused either by the limited access to the considered outgoing group or - in case of full access - by the state "all lines of this group busy".

In the following this idea will be applied also to systems with internal and external traffic.

The various parts of the call congestion are calculated by means of the following probabilities of state:

- $w(x)$ probability, that x outlets of the considered multiple of the 1st stage are occupied.
- $p_j(x)$ probability, that x lines in the considered outgoing group No. j are busy.

These functions $w(x)$ and $p_j(x)$ are assumed to be independent of each other. The traffic Y_1 carried per multiple of the 1st stage as well as the traffic Y_{sj} carried on the outgoing group No. j are prescribed. Furthermore, as an approximation, the known functions for the probabilities of state for full access groups with internal and external traffic are applied [1,2]. Thus, the probability distributions $w(x)$ and $p_j(x)$ can be calculated iteratively by means of generating offered traffics A_{o1} and A_{osj} in such a way that the prescribed carried traffics Y_1 and Y_{sj} result [2,7].

2. DEFINITION OF "EXTERNAL" AND "INTERNAL" TRAFFIC

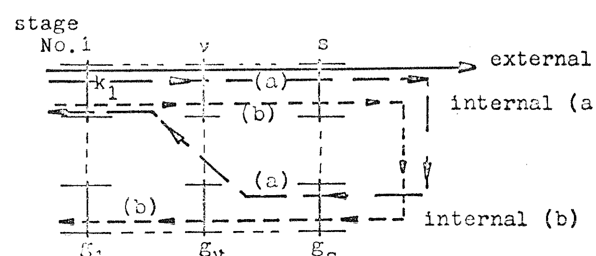

Fig. 1

External traffic: A call establishes o n e path through the link system.

Internal traffic: One call establishes t w o paths through the link system. Hereby two cases have to be distinguished:

a) The origin and the destination of the internal connection are situated on the same multiple of the 1st stage (cf. fig. 1, path (a)).
b) The origin and the destination of the internal connection are situated on different multiples of the 1st stage (cf. fig. 1, path (b)).

To calculate the probability distribution for a trunk group - i.e. either for a multiple in the 1st stage or for an outgoing group behind the last stage - the total internal traffic has to be divided into two parts:

- that part which requires two paths in the considered group is named i n t e r n a l traffic with regard to this group (cf. e.g. fig. 1: the connection (a) occupies two paths within the k_1 outlets of the considered multiple).
- that part which establishes only one path in a considered group is named e x t e r n a l traffic with regard to this group (cf. e.g. fig. 1: the connection (b) occupies one path only within the k_1 outlets of the considered multiple).

3. THE OPERATION MODES

3.1 Operation Mode 1

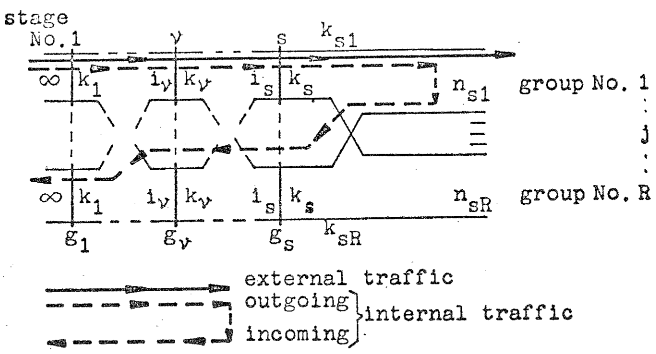

Fig. 2

Each outlet of the link system in fig. 2 (right hand side) can be used for traffic in outgoing or incoming direction. Therefore, internal traffic needs two lines of the same outgoing group.

The parameters $k_1,..,k_s,g_1,..,g_s$ etc. of the structure of the link system are given (cf. fig. 2). Let the carried traffics of a considered outgoing group No. j (j = 1,2,...,R) be prescribed, where:

Y_{sj} = total carried traffic,

${}_iY_{sj}$ = internal carried traffic,

${}_eY_{sj}$ = external carried traffic,

with

$$Y_{sj} = {}_iY_{sj} + {}_eY_{sj} \qquad (1)$$

The ratio of internal to total carried traffic is:

$$d_{sj} = \frac{{}_iY_{sj}}{Y_{sj}} \qquad (2)$$

The total carried traffic of the link lines is:

$$Y_{total} = \sum_{j=1}^{R} Y_{sj} \qquad (3)$$

Furthermore, the carried traffic of a multiple in stage No. ν (ν = 1,2,...,s) amounts to:

$$Y_ν = \frac{Y_{total}}{g_ν} \qquad (4)$$

The total internal carried traffic of the link system is given by:

$${}_iY_{total} = \sum_{j=1}^{R} {}_iY_{sj} \qquad (5)$$

With this total internal traffic of the link system we find the share per multiple in stage No. 1:

$${}_iY_1' = \frac{{}_iY_{total}}{g_1} \qquad (6)$$

We assume that the internal traffic is split into the g_1 multiples of the 1^{st} stage symmetrically. To calculate the probability distribution for the outlets of the considered multiple in the 1^{st} stage we have to calculate i t s o w n internal traffic. This internal traffic per multiple amounts to (cf. Chapter 2):

$${}_iY_1 = \frac{{}_iY_1'}{g_1} = \frac{1}{g_1} \cdot \frac{\sum_{j=1}^{R} {}_iY_{sj}}{g_1} \qquad (7)$$

The external carried traffic per multiple is:

$${}_eY_1 = Y_1 - {}_iY_1 \qquad (8)$$

The ratio of internal to total carried traffic per multiple of the 1^{st} stage is:

$$d_1 = \frac{{}_iY_1}{Y_1} \qquad (9)$$

3.2 Operation Mode 2

One of the two outgoing groups carries internal traffic only. The other group carries external traffic only (cf. fig. 3).

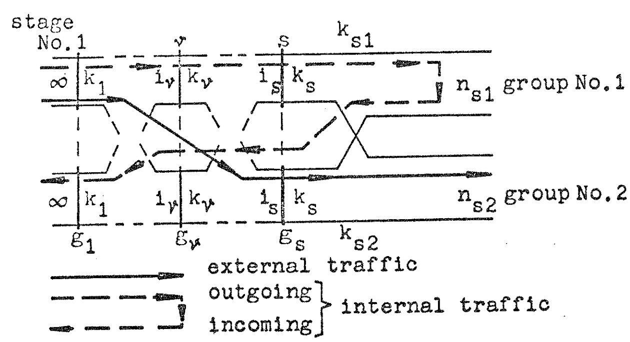

Fig. 3

Link systems having this operation mode can be found similarly in many telephone exchanges.

With given parameters of the structure as well as the prescribed carried traffics on the outgoing groups $Y_{s1} = {}_iY_{s1}$ and $Y_{s2} = {}_eY_{s2}$ we get:

$$\left. \begin{array}{l} d_{s1} = 1 \\ d_{s2} = 0 \end{array} \right\} \qquad (10)$$

(cf. eq. (2))

Furthermore, we calculate $Y_ν$ according to eq.(4), ${}_iY_1$ according to eq. (7), ${}_eY_1$ according to eq. (8), and d_1 according to eq. (9).

3.3 Operation Mode 3

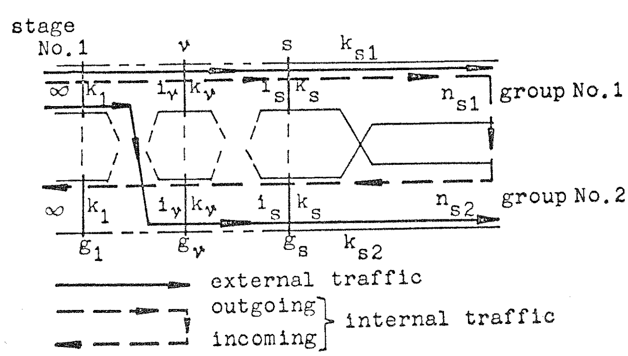

Fig. 4

Just as in Chapter 3.2 we have only two outgoing groups. Group No. 1 carries external as well as o u t g o i n g internal traffic. Whereas, group No. 2 carries external as well as i n - c o m i n g internal traffic (cf. fig. 4).

Link systems having this operation mode are also applied in many switching systems.

The parameters of the structure and the carried traffics Y_{s1} and Y_{s2} on the outgoing groups are given.

Since an internal connection establishes only one line in an outgoing group, we get (cf.Chapter 2):

$$d_{s1} = d_{s2} = 0 \tag{11}$$

In this case the ratio d_{z1} of internal to total carried traffic on the $g_1 \cdot k_1$ link lines between stage No. 1 and stage No. 2 must be known. Thus it follows that (cf. Chapter 3.1, assumption for eq.(7)):

$$d_1 = \frac{d_{z1}}{g_1} \tag{12}$$

Furthermore, we find the total carried traffic Y_ν per multiple of stage No. ν ($\nu = 1,2,\ldots,s$) according to eq.(4).

The internal traffic per multiple of stage No. 1 is:

$${}_iY_1 = d_1 \cdot Y_1 \tag{13}$$

The external traffic ${}_eY_1$ follows from eq.(8).

4. CALCULATION OF CALL CONGESTION AND OFFERED TRAFFICS

4.1 Assumptions

- The probabilities of state of the outlets of each multiple of the 1st stage and those of the outgoing trunk groups are independent of each other.
- An internal call of the considered link system can be connected - between its outgoing and incoming line of stage No. s (last stage) - via further selector stages of the exchange.

  Time delays and losses caused by these selector stages are neglected.
- The external traffic is produced either by sources which are connected directly to the link system (outgoing external traffic) or by calls which come from the outside (incoming external traffic).

  The calculation method makes no difference between outgoing and incoming e x t e r n a l traffic. The sum of both external traffics is handled as a Poisson traffic offered to the left hand side of the system (cf. fig. 4).

4.2 Mean Accessibility m_j

The calculation of route blocking makes use of the Mean Accessibility m_j from an inlet of the 1st stage to the considered outgoing group No. j. I.e. the quantity m_j can be interpreted as the average number of lines in the considered group No. j which can be hunted "free" or "busy" by the inlets of any multiple in stage No. 1 [4,5].

According to [4] we get:

$$m_j = \left\{ \prod_{\nu=1}^{s-1} (k_\nu - Y_\nu) \right\} k_{sj} + \eta_j \cdot Y_1 \quad , \tag{14}$$

where

η_j = ratio of the carried traffic on the outgoing group No. j to the total carried traffic of the system.

The following limitations have to be regarded for eq.(14):

$$\left.\begin{array}{ll} \prod_{\nu=1}^{\chi} (k_\nu - Y_\nu) \leq g_{\nu+1} & \chi = 1,2,\ldots,s-1 \\ \text{and} \quad m_j \leq n_{sj} & \end{array}\right\} \tag{15}$$

Example: If $\prod_{\nu=1}^{\chi}(k_\nu - Y_\nu) > g_{\nu+1}$ and $m_j > n_{sj}$ we set $\prod_{\nu=1}^{\chi}(k_\nu - Y_\nu) = g_{\nu+1}$ and $m_j = n_{sj}$, respectively.

4.3 Congestion Probability $\sigma_j(x)$

Route blocking b_{sj} is calculated as in the case of a one stage arrangement with constant limited accessibility m_j. Outgoing groups with external traffic only are calculated by means of the MPJ - formula [5,6]. For groups with internal and external traffic an adapted version of the MPJ-formula is applied [2].

For a constant and integer accessibility k and for the number n of lines $\sigma(x)$ holds:

$$\sigma(x) = \frac{\binom{x}{k}}{\binom{n}{k}}$$

In our case, however, the Mean Accessibility m_j is as a rule not an integer value. Therefore, we calculate $\sigma(x)$ by means of a linear interpolation between the two neighbouring integer values of m_j, m_{ja} and m_{jb} [3], where

m_{ja} = neighbouring integer value $\leq m_j$,
m_{jb} = neighbouring integer value $\geq m_j$.

We get

$$\sigma_j(x) = \frac{\binom{x}{m_{ja}}}{\binom{n_{sj}}{m_{ja}}}(m_{jb} - m_j) + \frac{\binom{x}{m_{jb}}}{\binom{n_{sj}}{m_{jb}}}(m_j - m_{ja}) \tag{16}$$

4.4 Probabilities of State

4.4.1 The Probability of State $p_j(x)$ for the Outgoing Group No. j

Analogously to [1,2] we get the recurrence formula:

$$\left.\begin{array}{l} p_j(x+2) = \dfrac{{}_eA_{osj}}{x+2} p_j(x+1) + \dfrac{2\,{}_iA_{osj}}{x+2} p_j(x) \\ \text{with} \quad \sum_{x=1}^{n_{sj}} p_j(x) = 1 \end{array}\right\} \tag{17}$$

,

where

${}_eA_{osj}$ = generating external traffic offered,
${}_iA_{osj}$ = generating internal traffic offered.

Whereas the carried traffics ${}_eY_{sj}$ and ${}_iY_{sj}$ are prescribed, the quantities ${}_eA_{osj}$ and ${}_iA_{osj}$ have to be determined (by iteration) such that the following equations (18),(19), and (20B) are fulfilled:

$${}_eY_{sj} = {}_eA_{osj}\,(1 - p_j(n_{sj})) \tag{18}$$

$${}_iY_{sj} = 2\,{}_iA_{osj}\{1 - (p_j(n_{sj}) + p_j(n_{sj} - 1))\} \tag{19}$$

Remark to eq.(19): The factor 2 depends on the fact that an internal connection establishes <u>two</u> paths in the considered outgoing group No. j. Internal call congestion arises, if $\geq (n_{sj} - 1)$ lines are busy. Therefore, we get $p_j(n_{sj}) + p_j(n_{sj}-1)$ (cf. Chapter 4.5.1, eq.(28)).

Next, we calculate the ratio of internal to total traffic offered. It is defined by

$$c_{osj} = \frac{{}_iA_{osj}}{{}_eA_{osj} + {}_iA_{osj}} \tag{20A}$$

$$c_{osj} = \frac{d_{sj}}{d_{sj} + 2(1-d_{sj})\dfrac{1 - p_j(n_{sj}) - p_j(n_{sj}-1)}{1 - p_j(n_{sj})}} \tag{20B}$$

4.4.2 The Probability of State w(x) for a Multiple of Stage No. 1

Analogously to eq.(17),(18),(19),and (20) we get:

$$w(x+2) = \frac{{}_eA_{o1}}{x+2}\, w(x+1) + \frac{2\,{}_iA_{o1}}{x+2}\, w(x) \qquad (21)$$

$${}_eY_1 = {}_eA_{o1}\cdot(1 - w(k_1)) \qquad (22)$$

$${}_iY_1 = 2\,{}_iA_{o1}\cdot(1 - (w(k_1) - w(k_1-1))) \qquad (23)$$

$$c_{o1} = \frac{{}_iA_{o1}}{{}_eA_{o1} + {}_iA_{o1}} = \frac{d_1}{d_1 + 2(1-d_1)\dfrac{1-w(k_1)-w(k_1-1)}{1 - w(k_1)}} \qquad (24)$$

where

${}_eA_{o1}$ = generating external traffic offered,
${}_iA_{o1}$ = generating internal traffic offered,
c_{o1} = ratio of internal to total offered traffic.

The probabilities of state w(x) can be calculated iteratively in the same way as $p_j(x)$ in Chapter 4.4.1.

4.5 Inlet Blocking

The following equations exist for the different parts of inlet blocking:

- E x t e r n a l inlet blocking, referred to the external traffic offered:

  $${}_eb_1 = w(k_1) \qquad (25)$$

  (Loss occurs if all k_1 outlets of the considered multiple are busy.)

- O u t g o i n g internal inlet blocking, referred to the internal traffic offered:

  $${}_{ig}b_1 = {}_eb_1 = w(k_1) \qquad (26)$$

  (Internal outgoing loss occurs also if all k_1 outlets are busy.)

- I n c o m i n g internal inlet blocking, referred to the internal traffic offered:

  $${}_{ic}b_1 = w(k_1 - 1) \qquad (27)$$

  (In the state $\{k_1-1\}$ an outgoing internal call occupies the last free outlet, i.e. the incoming part of the internal call is lost.)

- T o t a l internal inlet blocking, referred to the internal traffic offered:

  $${}_ib_1 = {}_{ig}b_1 + {}_{ic}b_1 \qquad (28)$$

  (Outgoing internal inlet blocking and incoming internal inlet blocking are mutually exclusive events. Therefore, the two probabilities can be added.)

4.6 Route Blocking

4.6.1 Operation Mode 1

The following equations exist for the different parts of the route blocking:

- E x t e r n a l route blocking, referred to the external traffic offered:

  $${}_eb_{sj} = (m_{jb} - m_j)\cdot p_j(m_{ja})\cdot \sigma_j(m_{ja}) + \sum_{x=m_{jb}}^{n_{sj}} p_j(x)\cdot \sigma_j(x) \qquad (29)$$

  Remark to eq.(29): The call congestion of a trunk group with limited access and external traffic only is given by:

  $$b = \sum_{x=k}^{n} p(x)\cdot \sigma(x)$$

  In our case the Mean Accessibility m_j is not an integer. Therefore, we calculate ${}_eb_{sj}$ by linear interpolation (cf. Chapter 4.3). After some transformations we get eq.(29).

- O u t g o i n g internal route blocking, referred to the internal traffic offered:

  $${}_{ig}b_{sj} = {}_eb_{sj} \qquad (30)$$

- I n c o m i n g internal route blocking, referred to the internal traffic offered:

  $${}_{ic}b_{sj} = (m_{jb} - m_j)\cdot p_j(m_{ja} - 1)\cdot \sigma_j(m_{ja}) + \sum_{x=m_{ja}}^{n_{sj}-1} p_j(x)\cdot(1 - \sigma_j(x))\cdot \sigma_j(x+1) \qquad (31)$$

  Remark to eq.(31): The call congestion of a trunk group with limited access and internal traffic only is given by [2]:

  $$b = \sum_{x=k-1}^{n-1} p(x)\cdot(1-\sigma(x))\cdot \sigma(x+1)$$

  Because m_j is not an integer, we have also to interpolate linearly (cf. remark to eq.(29)).

- T o t a l internal route blocking, referred to the internal traffic offered:

  $${}_ib_{sj} = {}_{ig}b_{sj} + {}_{ic}b_{sj} \qquad (32)$$

4.6.2 Operation Mode 2

$d_{s1} = 1$ and $d_{s2} = 0$ are given (cf. Chapter 3.2). With regard to $d_{s1} = 1$ we get ${}_eb_{s1} = 0$. The internal route blocking of the group No. 1 can be calculated according to eq.(30),(31), and (32).

With regard to $d_{s2} = 0$ we get ${}_ib_{s2} = 0$. The external route blocking of the outgoing group No. 2 can be calculated according to eq.(29).

4.6.3 Operation Mode 3

With regard to $d_{s1} = d_{s2} = 0$ (cf. Chapter 3.3) we get ${}_ib_{s1} = {}_ib_{s2} = 0$. The external route blocking of the outgoing group No. 1 and No. 2 can be calculated according to eq.(29).

4.7 Call Congestions, Resulting from Inlet and Route Blocking

4.7.1 Call Congestion of External and Outgoing Internal Calls

The e x t e r n a l call congestion, referred to the external traffic offered to group No.j is:

$$\boxed{{}_eb_{(j)} = {}_eb_1 + (1 - {}_eb_1)\,{}_eb_{sj}} \qquad (33)$$

(${}_eb_1$ and ${}_eb_{sj}$ are assumed to be independent.)

The o u t g o i n g internal call congestion, referred to the internal traffic offered is:

$${}_{ig}b_{(j)} = {}_eb_{(j)} \qquad (34)$$

4.7.2 Call Congestion of Incoming Internal Calls

4.7.2.1 General Remarks

The i n c o m i n g internal call congestion, referred to the internal traffic offered has to be calculated as follows:

We have to distinguish between 4 cases of blocking (cf. fig. 5):

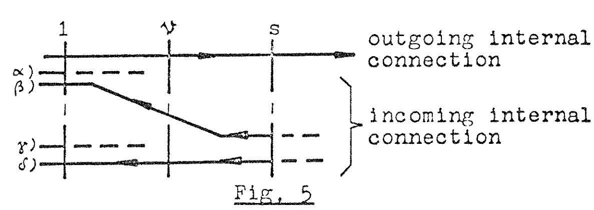

Fig. 5

Case α) The destination multiple is identical to the origin multiple; the incoming internal call cannot find an idle link path from stage No. s to the destination multiple in stage No. 1.

Case β) The destination multiple is identical to the origin multiple; all free lines of the incoming group are connected with multiples of the stage No. s, whose paths to the destination multiple are blocked.

Case γ) The destination multiple is not identical to the origin multiple; the incoming internal call cannot find an idle link path from stage No. s to the destination multiple in stage No. 1.

Case δ) The destination multiple is not identical to the origin multiple; all free lines of the incoming group are connected with multiples of the stage No. s, whose paths to the destination multiple are blocked.

For each of the 3 operation modes these 4 cases have to be considered!

4.7.2.2 Operation Mode 1

Case α) $${}^{\alpha}_{ic}b_{(j)} = \frac{1}{g_1}\,{}_{ic}b_1\,(1 - {}_eb_{sj}) \quad , \qquad (35)$$

where

$\frac{1}{g_1}$ = probability {destination multiple identical to origin multiple},

$(1-{}_eb_{sj})$ = prob. {no outgoing route blocking},

${}_{ic}b_1$ = prob. {destination multiple blocked}.

Case β) $${}^{\beta}_{ic}b_{(j)} = \frac{1}{g_1}\,{}_{ic}b_{sj}\,(1 - {}_ib_1) \quad , \qquad (36)$$

where

$\frac{1}{g_1}$ as in case α),

$(1-{}_ib_1)$ = prob. {destination multiple not blocked}

${}_{ic}b_{sj}$ = prob. {incoming route has no idle path to the destination multiple}.

Case γ) $${}^{\gamma}_{ic}b_{(j)} = (1-\frac{1}{g_1})\,{}_eb_1(1-{}_eb_1)(1-{}_eb_{sj}) \;, \qquad (37)$$

where

$(1-\frac{1}{g_1})$ = prob. {destination multiple not identical to origin multiple},

$(1-{}_eb_1)$ = prob. {origin multiple not blocked},

$(1-{}_eb_{sj})$ as in case α),

${}_eb_1$ = prob. {destination multiple blocked}.

Case δ) $${}^{\delta}_{ic}b_{(j)} = (1-\frac{1}{g_1})\;{}_{ic}b_{sj}\,(1-{}_eb_1)^2 \quad , \qquad (38)$$

where

$(1-\frac{1}{g_1})$ as in case γ),

$(1-{}_eb_1)^2$ = prob. {origin and destination multiple not blocked},

${}_{ic}b_{sj}$ as in case β).

With eq.(35),(36),(37), and (38) the total call congestion for incoming internal calls holds:

$${}_{ic}b_{(j)} = {}^{\alpha}_{ic}b_{(j)} + {}^{\beta}_{ic}b_{(j)} + {}^{\gamma}_{ic}b_{(j)} + {}^{\delta}_{ic}b_{(j)} \qquad (39)$$

4.7.2.3 Operation Mode 2

For group No. 1 (internal traffic only) we find ${}_eb_{(1)} = 0$. Therefore, only eq.(35),(36),(37),(38), and (39) are relevant.

In group No. 2 we have external traffic only, i.e. ${}_ib_{(2)} = 0$. Equation (33) is valid.

4.7.2.4 Operation Mode 3

The internal traffic is offered to group No. 1 only (cf. Chapter 3.3), i.e. ${}_ib_{(2)} = 0$.

The 4 parts of the call congestion for incoming internal calls are:

Case α) ${}^{\alpha}_{ic}b_{(1)}$ according to eq.(35)

Case β) $${}^{\beta}_{ic}b_{(1)} = \frac{1}{g_1}\,{}_eb_{s2}\,(1-{}_ib_1)(1-{}_eb_{s1}) \quad , \qquad (40)$$

where

$(1-{}_eb_{s1})$ = prob. {no outgoing route blocking},

${}_eb_{s2}$ = prob. {incoming route has no idle path to the destination multiple},

$1/g_1$ and $(1-{}_ib_1)$ as in eq.(36).

Case γ) ${}^{\gamma}_{ic}b_{(1)}$ according to eq.(37)

Case δ) $${}^{\delta}_{ic}b_{(1)} = (1-\frac{1}{g_1})\,{}_eb_{s2}\,(1-{}_eb_1)^2\;(1-{}_eb_{s1}) \qquad (41)$$

(cf. eq.(38) and (40), respectively.)

The total call congestion ${}_{ic}b_{(1)}$ is calculated according to eq.(39).

4.7.3 Call Congestion for Internal Traffic

Generally, we get the t o t a l i n t e r n a l call congestion, referred to the internal traffic offered from:

$$\boxed{{}_ib_{(j)} = {}_{ig}b_{(j)} + {}_{ic}b_{(j)}} \qquad (42)$$

4.8 Offered Traffics

4.8.1 Operation Modes 1 and 2

External Traffic offered is:

$$\boxed{{}_eA_{sj} = \frac{{}_eY_{sj}}{1 - {}_eb_{(j)}}} \qquad (43)$$

Internal Traffic offered is:

$$\boxed{{}_iA_{sj} = \frac{{}_iY_{sj}}{2\,(1 - {}_ib_{(j)})}} \qquad (44)$$

From eq.(43) and (44) we obtain the total traffic A_{sj} offered to the group No. j:

$$\boxed{A_{sj} = {}_eA_{sj} + {}_iA_{sj}} \qquad (45)$$

The ratio of internal to total traffic offered becomes:

$$\boxed{c_{sj} = \frac{{}_iA_{sj}}{A_{sj}}} \qquad (46)$$

4.8.2 Operation Mode 3

The prescribed traffic parameters Y_{s1}, Y_{s2} and d_{z1} (cf. Chapter 3.3) yield the total traffic

$Y_{total} = Y_{s1} + Y_{s2}$, according to eq. (3)

and its internal share:

$${}_iY_{total} = d_{z1} \cdot Y_{total} \qquad (47)$$

This internal share of the total traffic carried is divided equally into the outgoing direction No. 1 and the incoming direction No. 2:

$${}_iY^*_{s1} = {}_iY^*_{s2} = \frac{{}_iY_{total}}{2} \qquad (48)$$

(Remark to index *: In the operation mode 3 internal calls occupy o n e line per outgoing and incoming group, respectively.)

Outgoing group No. 1:

We obtain: $$\boxed{{}_eA_{s1} = \frac{Y_{s1} - {}_iY^*_{s1}}{1 - {}_eb_{(1)}}} \qquad (49)$$

and

$$ {}_iA^*_{s1} = \frac{{}_iY^*_{s1}}{1 - {}_ib_{(1)}} \quad (50) $$

Furthermore,

$$ A_{s1} = {}_eA_{s1} + {}_iA^*_{s1} \quad (51) $$

and

$$ c^*_{s1} = \frac{{}_iA^*_{s1}}{A_{s1}} \quad (52) $$

Outgoing group No. 2:

We get

$$ A_{s2} = {}_eA_{s2} = \frac{Y_{s2} - {}_iY^*_{s2}}{1 - {}_eb_{(2)}} \quad (53) $$

and

$$ c^*_{s2} = 0 \quad (54) $$

4.9 Call Congestions, Referred to the Total Traffic Offered

4.9.1 Operation Modes 1 and 2

We obtain:

External call congestion:

$$ {}_eB_j = (1 - c_{sj}) \cdot {}_eb_{(j)} \quad (55) $$

Internal call congestion:

$$ {}_iB_j = c_{sj} \cdot {}_ib_{(j)} \quad (56) $$

Total call congestion:

$$ B_j = {}_eB_j + {}_iB_j \quad (57) $$

4.9.2 Operation Mode 3

Outgoing group No. 1:

In eq.(55),(56),(57) the traffic ratio c_{s1} (according to eq.(46)) has to be replaced by c^*_{s1} (according to eq.(52)).

Outgoing group No. 2:

Since $c^*_{s2} = 0$ (cf. eq.(54)) we obtain:

$$ B_2 = {}_eB_2 = {}_eb_{(2)} \quad (58) $$

4.10 Mean Call Congestion, with Respect to All Outgoing Groups

We obtain:

$$ B_{total} = \frac{\sum_{j=1}^{R} A_{sj} \cdot B_j}{\sum_{j=1}^{R} A_{sj}} \quad (59) $$

5. COMPARISON BETWEEN CALCULATION AND ARTIFICIAL TRAFFIC TRIALS

In all diagrams I means the test results with a confidence interval of 95%. The solid lines are obtained with the presented calculation method.

5.1 Two-Stage Link System with Operation Mode 1

Diagram 1: Total call congestion B_{total} as function of the carried traffic per outlet Y_{s1}/n_{s1}, parameter d_{s1}.

1: $d_{s1} = 1$, internal traffic only
2: $d_{s1} = 0.5$, mixed internal and external traffic
3: $d_{s1} = 0$, external traffic only.

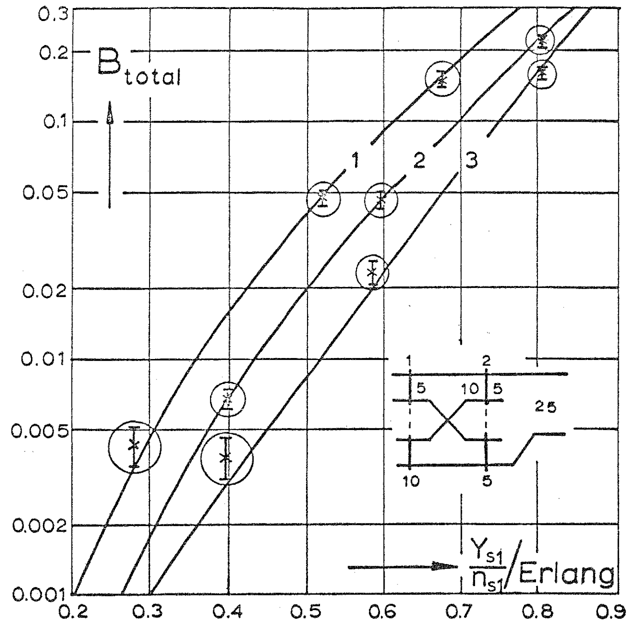

Diagram 1

Diagram 2 : 1 : Internal call congestion, referred to the internal traffic offered as function of Y_{s1}/n_{s1} ($d_{s1} = 0.5$).
2 : External call congestion, referred to the external traffic offered as function of Y_{s1}/n_{s1} ($d_{s1} = 0.5$).

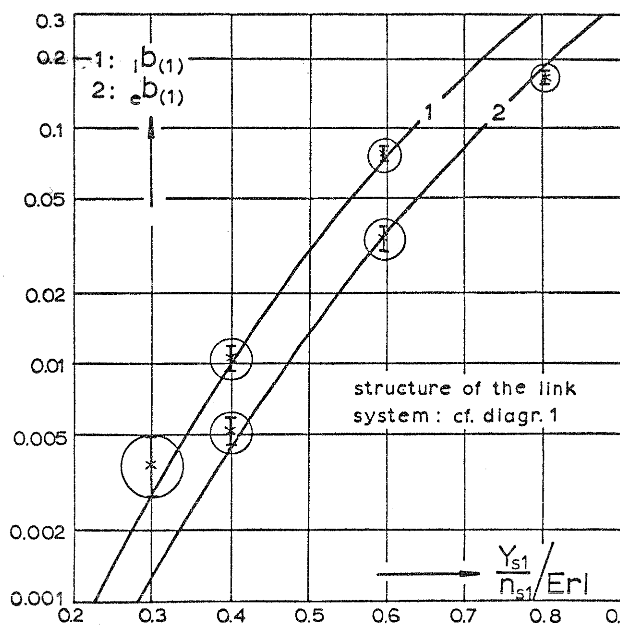

Diagram 2

5.2 Three-Stage Link System with Operation Mode 2

Diagram 3: 1 : Internal call congestion, referred to the internal traffic offered as function of the carried traffic per outlet $Y_{s1}/n_{s1} = Y_{s2}/n_{s2} = Y_s/n_s$ ($d_{s1} = 1$).
2 : External call congestion, referred to the external traffic offered as function of Y_s/n_s ($d_{s2} = 0$).

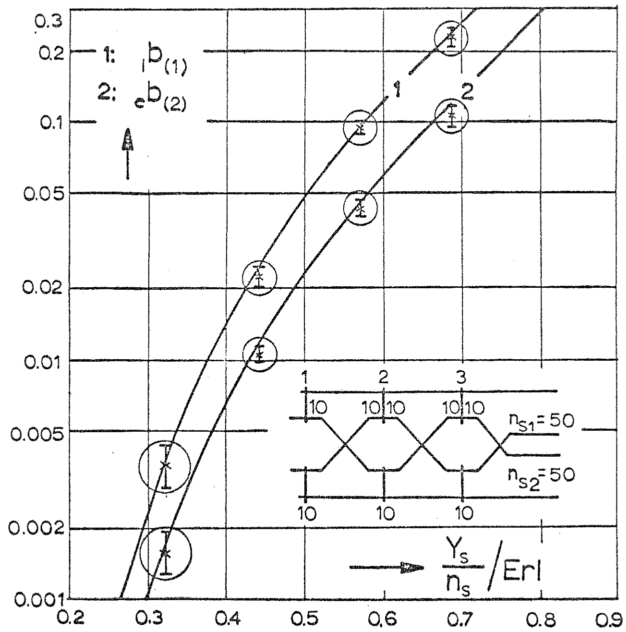

Diagram 3

5.3 Four-Stage Link System with Operation Mode 3

Diagram 4: Total call congestion as function of the carried traffic per outlet $Y_{s1}/n_{s1} = Y_{s2}/n_{s2} = Y_s/n_s$, parameter d_{z1}.

1 : d_{z1} = 0.5, mixed internal and external traffic.

2 : d_{z1} = 0, external traffic only.

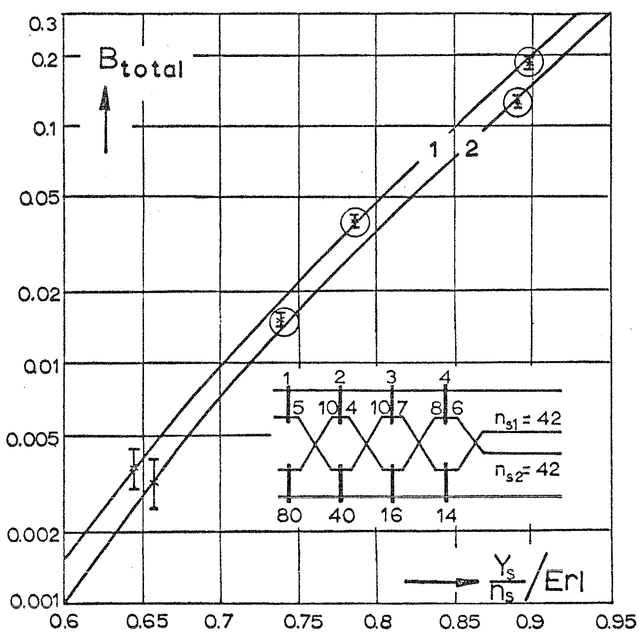

Diagram 4

Diagram 5: 1 : Internal call congestion, referred to the internal traffic offered as function of Y_s/n_s.

2 : External call congestion, referred to the external traffic offered as function of Y_s/n_s.

(d_{z1} = 0.5)

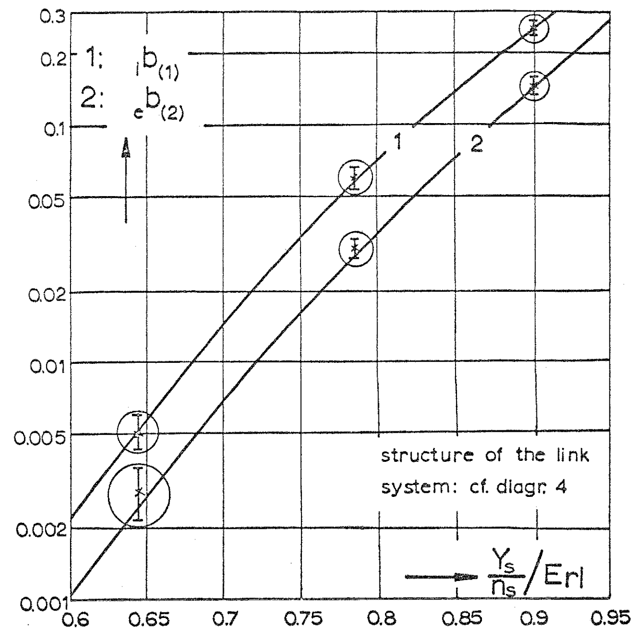

Diagram 5

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