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Design Parameters and Loss Calculation of Link Systems

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Abstract—The problems of how group-selection link systems should be structured, wired, and hunted to achieve optimal traffic characteristics are studied. Furthermore, a loss-and-load-equivalent mapping of large group-selection link systems to smaller ones is presented.

A new approximate loss calculation method is developed.

All studies were supported by extensive Monte Carlo simulations.

I. INTRODUCTION

IN THIS PAPER the problems of how link systems should be structured, wired, and hunted to get optimal traffic characteristics are studied. These studies are performed by means of simulation and calculation, respectively. Monte Carlo simulation [1], [3] was applied with a total of more than 300 million calls. About 200 group-selection link systems with 2, 3, 4, and 6 stages have been considered. The total number of outlets per link system is in the range of 100 up to 1000 trunks. Of course, in this paper only the most essential results can be reported.

Poissonian traffic was offered to all considered link systems (cf. Section II-H). The holding times are negative exponentially distributed.

In each case, point-to-group selection was applied. Each call offered to an idle inlet in the first stage can hunt all accessible trunks of the desired group behind the last stage by means of the marker in order to occupy an idle one. This point-to-group selection yields for the same carried traffic extremely smaller losses than the point-to-point selection mode (see Fig. 1). In this latter mode, first an idle trunk of the desired group is selected. In the second step, the marker has to find a chain of idle links from the calling inlet to the *a priori* fixed idle trunk.

In Section II a survey is given on the investigated link-system types, the selected structures considered here, and two different modes to hunt the inlets.

In Section III the influence of different wiring and hunting modes on the probability of loss is investigated.

Section IV deals with a new procedure of link-system design to obtain a minimum requirement of crosspoints per Erlang for prescribed traffic and loss as well as for good overload characteristics.

In Section V a handy method is presented which facilitates a loss-and-load-equivalent mapping of large group-selection link systems to smaller ones for the purpose of their economic full-scale simulation on a digital computer.

Section VI deals with a new approximate loss calculation for group-selection link systems. Calculated and simulated results are compared.

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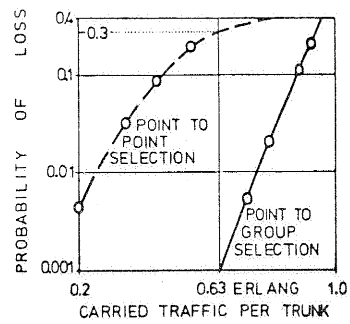


Fig. 1. Probability of loss versus carried traffic per trunk in case of point-to-point selection and point-to-group selection (simulation results for structure $L42$, Fig. 9).

II. SURVEY ON THE STRUCTURES STUDIED, DEFINITION OF THE INPUT PROCESS

A. Summary

In this section a survey is given of the link-system structures which are discussed in Sections III–VI. Of course, only a *selection* out of the total of more than 200 investigated structures is shown. All structures are designed for group selection and do not concentrate the traffic, except possibly in the last stage.

The various link systems consist of 2, 3, 4, or 6 stages. Five-stage link systems were not considered by reason of additional computer time.

For a better understanding of link-system structure parameters, the general considerations are given in Section II-B–II-F. Each link system is signed by a code and listed in Section II-G. Section II-H describes the inlet hunting.

B. Structures With Two Stages

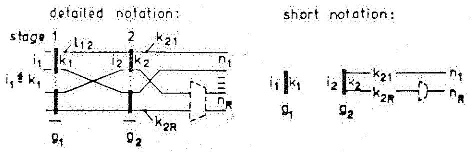
Fig. 2 shows the notations applied to 2-stage link systems. In all studied 2-stage link systems there exists at most one link between each multiple of stage 1 and stage 2 (2-stage fan-out structure, cf. Section II-C). Two-stage structures with more than one link connecting each multiple of stages 1 and 2 are not presented here.

The following notations are applied:

i_j inlets per multiple in stage j, $(j = 1,\cdots,S)$;

k_j outlets per multiple in stage j, $(j = 1,\cdots,S)$;

g_j number of multiples in stage j, $(j = 1,\cdots,S)$;



The symbol [ ] means a grading, i.e. $n_r < g_2 k_{2r}$.

Fig. 2. Notations for 2-stage link systems.

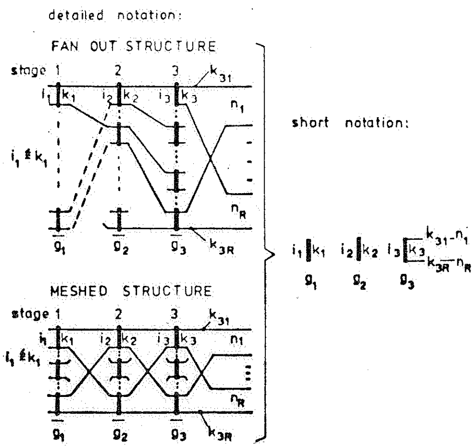


Fig. 3. Notations for 3-stage link systems.

S	number of stages;
R	number of outgoing trunk groups;
$l_{j,j+1}$	average number of links from each multiple in stage j to each multiple in stage $j+1$;
k_{Sr}	outlets per multiple to group r, $(r = 1,\cdots,R)$;
n_r	number of trunks per group r, $(r = 1,\cdots,R)$.

C. Structures With Three Stages

Fig. 3 shows the notations for 3-stage link systems, applied to two distinct structure types.

1) *fan-out structure:* at most one path leads from a certain inlet of stage 1 to a certain multiple of the last stage.
2) *meshed structure:* more than one path exists between a certain inlet of stage 1 and a certain multiple of the last stage.

The multiples of two successive stages can be wired such that the link system is subdivided into linkblocks with regard to these stages.

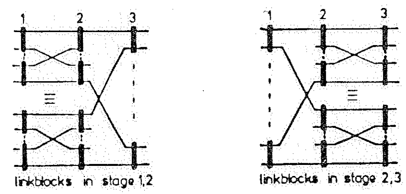
Fig. 4 shows two examples of 3-stage link systems with linkblocks. General remarks on linkblocks are given in Section III-E1.

D. Structures With Four Stages

Fig. 5 shows the notations for 4-stage meshed link systems with group selection. Link systems with linkblocks (Fig. 5) and without linkblocks (Fig. 6) are considered.

E. Structures With Six Stages

Fig. 7 shows the notations for 6-stage link systems with meshed structure. The studied structures are designed without linkblocks or with linkblocks, e.g., in stages 1–2–3 and 4–5–6 (Fig. 7).



(a) (b)

Fig. 4. Linkblocks in 3-stage link systems.

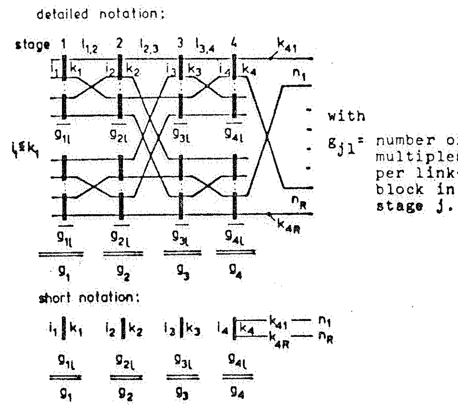


Fig. 5. Notations for 4-stage link systems *with* linkblocks.

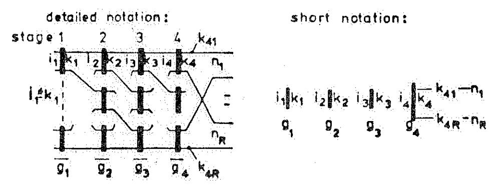


Fig. 6. Notations for 4-stage link systems *without* linkblocks.

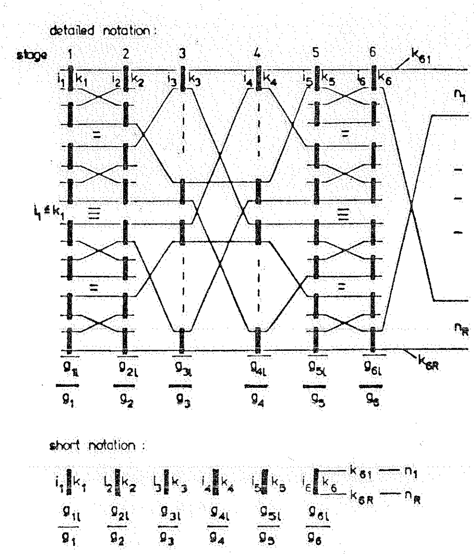


Fig. 7. Notations for 6-stage link systems.

F. The "Linkwidth" of a Group-Selection Link System

The linkwidth (LW) of a link system is defined by the inlet linkwidth $\text{LW}_{\text{in}} = P/N_{\text{in}}$ and the outlet linkwidth

	Type of Link System	LW_{in}	LW_{out}	Feature of Link System
A	Wide	>1	>1	N_{in} — P — N_{out}
B	Wide	>1	=1	N_{in} — P — N_{out}
C	Narrow	=1	=1	N_{in} — P — N_{out}
D	Narrow	=1	>1	N_{in} — P — N_{out}
$N_{in}=g_1 \cdot i_1$; $N_{out}=g_S \cdot k_S$; $P=g_1 \cdot k_1 = \ldots = g_{S-1} \cdot k_{S-1}$				

Fig. 8. Four basic structural features.

$LW_{out} = P/N_{out}$ with

- P number of links between two successive stages (in the considered structures for group selection, P does not vary inside the link system);
- N_{in} total number of link-system inlets;
- N_{out} total number of link-system outlets.

Fig. 8 shows the 4 basic structural features being studied in this paper. The influence of inlet linkwidth and outlet linkwidth on the probability of loss is studied in Section IV. The results presented there show clearly that only feature A or at least feature B should be applied. Features C and D yield remarkably higher probabilities of loss.

G. List of Presented Structures

Fig. 9 gives a survey on the structures. The following sections refer to these structures by the indicated code only.

Structure (short notation)	Code	Cross-points	discussed in Chapter
8‖10 (12); 12‖10 (10) 1—10; 10x10	L20	2160	3.3 ; 6.5
10‖10 (10); 10‖10 (10) 1—10; 10x10	L21	2000	3.3 ; 6.5
8‖10 (12/48); 12‖10 (10/40) 1—[SSG]—20; 10x20 — [SSG] Simplified Standard Grading	L22	8640	3.3 ; 6.5
10‖12 (10); 10‖10 (12); 6‖5 (20) 1—20; 5x20	L30	3000	3.4 ; 6.5
9‖10 (10); 10‖10 (10); 5‖5 (20) 1—20; 5x20	L31	2400	4.3 ; 6.5
10‖10 (10); 10‖10 (10); 5‖5 (20) 1—20; 5x20	L32	2500	4.3 ; 6.5
10‖10 (20); 10‖10 (20); 20‖20 (10) 2—20; 10x20	L33	8000	5.3 ; 6.5
10‖10 (10); 10‖10 (10); 10‖10 (10) 2—20; 5x20	L34	3000	5.3 ; 6.5
10‖10 (4); 10‖10 (4); 4‖4 (10) 2—20; 2x20	L35	960	5.3 ; 6.5
10‖10 (10); 10‖10 (10); 10‖10 (10) 1—10; 10x10	L36	3000	6.5 ; 6.5
5‖10 (50); 5‖5 (100); 10‖5 (50) 1—50; 5x50	L37	7500	4.3
5‖7 (50); 5‖5 (70); 7‖5 (50) 1—50; 5x50	L38	5250	3.4 ; 6.5
5‖6 (50); 5‖5 (60); 6‖5 (50) 1—50; 5x50	L39	4500	3.4 ; 6.5
5‖10 (50); 5‖5 (100); 5‖5 (100); 10‖5 (50) 1—50; 5x50	L40	10000	4.3
5‖8 (5/50); 5‖5 (8/80); 5‖5 (8/80); 8‖5 (5/50) 1—50; 5x50 — or without linkblocks	L41	8000	3.5 ; 4.3 ; 6.5
5‖8 (50); 5‖5 (80); 5‖5 (80); 8‖5 (50) 0.5—25, 0.5—25, 1—50, 1—50, 2—100	L42	8000	6.5
5‖7 (50); 5‖5 (70); 5‖5 (70); 7‖5 (50) 1—50; 5x50	L43	7000	4.3 ; 6.5
7‖7 (50); 5‖5 (70); 5‖5 (70); 7‖5 (50) 1—50; 5x50	L44	7700	4.3
10‖10 (25); 10‖10 (25); 10‖10 (25); 10‖10 (25) 2—50; 5x50	L45	10000	4.3
20‖20 (50); 10‖10 (100); 10‖10 (100); 20‖20 (50) 1—50; 20x50	L46	60000	5.3
20‖20 (10); 10‖10 (20); 10‖10 (20); 4‖4 (50) 1—50; 4x50	L47	8800	5.3
10‖10 (5/25); 5‖5 (10/50); 5‖5 (10/50); 10‖10 (5/25) 2—50; 5x50 — or without linkblocks	L48	7500	3.5 ; 4.3 ; 6.5
5‖6 (50); 5‖5 (60); 5‖5 (60); 5‖5 (60); 5‖5 (60); 6‖5 (50) 1—50; 5x50	L60	9000	4.3
5‖7 (50); 3‖3 (116); 3‖3 (116); 3‖3 (116); 3‖3 (116); 7‖5 (50) 1—50; 5x50	L61	7676	3.6 ; 6.5
5‖6 (50); 3‖3 (100); 3‖3 (100); 3‖3 (100); 3‖3 (100); 6‖5 (50) 1—50; 5x50	L62	6600	5.3 ; 6.5
5‖6 (20); 3‖3 (40); 3‖3 (40); 3‖3 (40); 3‖3 (40); 2.4‖2 (50) 1—50; 2x50	L63	2280	5.3
5‖5 (5/50); 5‖5 (5/50); 5‖5 (10/50); 5‖5 (10/50); 5‖5 (5/50); 5‖5 (5/50) 1—50; 5x50 — or without linkblocks	L64	7500	3.6 ; 6.5

Fig. 9. List of presented structures.

H. The Input Process

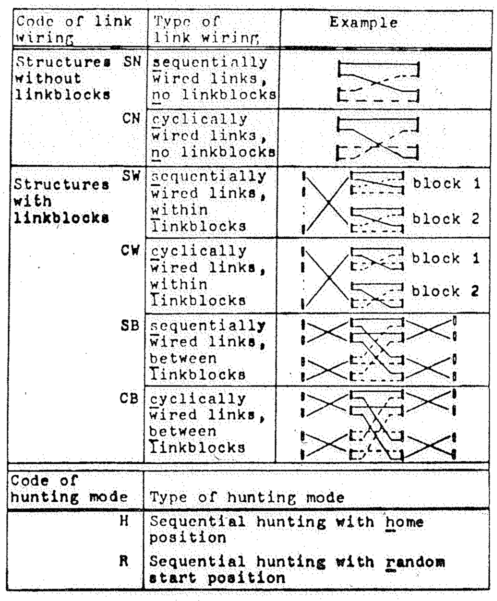
It is assumed that Poissonian traffic $A' = \lambda \cdot h$ is offered, having a constant call rate λ. The holding times are negative exponentially distributed (mean holding time h).

Since link systems have a finite number of inlets, two methods can be distinguished regarding the offer of calls.

1) *Method I:* To each multiple of stage 1 an individual *partial* traffic $A_p' = A'/g_1$ is offered. This partial traffic is clipped, if all i_1 inlets of the considered multiple are busy (probability of blocking b_I). Thus the actual offered partial traffic becomes $A_p = A_p' \cdot (1 - b_I)$ and the total offered traffic is

$$A = \sum_{i=1}^{g_1} A_{pi}.$$

2) *Method II:* To *all* multiples of stage 1 the *total* traffic A' is offered. To achieve balanced offered traffic per multiple, any offered call can hunt the total number of inlets in sequential order starting with the first inlet of a *randomly selected* first-stage multiple. Without regard to further connection through the link system, the call occupies the first free inlet which is found in the first selected or subsequent first-stage multiples. Therefore, the total traffic is clipped only if all $N_{in} = i_1 \cdot g_1$ inlets of the link system are busy (probability of inlet blocking b_{II}). Thus the actually offered total traffic becomes $A = A' \cdot (1 - b_{II})$. Because of $i_1 \ll N_{in}$ it holds $b_{II} \ll b_I$.



Code of link wiring		Type of link wiring	Example
Structures without linkblocks	SN	sequentially wired links, no linkblocks	
	CN	cyclically wired links, no linkblocks	
Structures with linkblocks	SW	sequentially wired links, within linkblocks	block 1, block 2
	CW	cyclically wired links, within linkblocks	block 1, block 2
	SB	sequentially wired links, between linkblocks	
	CB	cyclically wired links, between linkblocks	
Code of hunting mode		**Type of hunting mode**	
	H	Sequential hunting with home position	
	R	Sequential hunting with random start position	

Fig. 10. Types of link wiring and hunting modes.

Method I can cause a more intensive smoothing of traffic than method II and can yield losses even lower than in full accessible trunk groups. To avoid this falsifying "sub-Erlang loss"-effect as far as possible, in all presented tests *method II* was applied.

III. THE INFLUENCE OF LINK WIRING AND HUNTING MODES ON THE LOSS OF LINK SYSTEMS

A. Survey

The probability of loss in link systems depends not only on the offered traffic and the structure but also on the mode of link wiring and hunting (routing of an incoming call). These questions are studied in this section. Wide and narrow structures (cf. Section II-F) consisting of 2, 3, 4, and 6 stages are investigated.

In 2-stage and 3-stage link systems *sequential* wiring and *sequential* hunting with home position is found to yield the lowest loss. In 4-stage and 6-stage link systems the difference between CN-H and SN-H (Fig. 16) is not significant but a slight tendency to smaller losses in case of *cyclical* wiring without linkblocks and *sequential* hunting with home position was found by many simulation runs. It is shown that link wiring with linkblocks results in increased loss compared with link systems wired without linkblocks.

In the loss range of 10 percent up to 100 percent there exists no remarkable influence of link wiring and hunting mode on the loss. Wide optimum structures (Section IV) are by far less sensitive to link wiring and hunting modes than unfavorable narrow structures (Section II-F).

B. The Studied Modes

As Fig. 10 shows, 6 types of link wiring and 2 types of hunting mode are investigated (strictly random hunting yields approximately the same results as sequential hunting with random start position and is therefore not discussed here).

The code indicated in Fig. 10 is a short notation of link wiring and hunting mode, e.g., (SW-H) (CN-H) stands for a 3-stage link system: between stages 1 and 2 the links are sequentially wired within linkblocks (=SW) and sequentially hunted with home position (=H). Between stages 2 and 3 the links are cyclically wired without linkblocks (=CN) and sequentially hunted with home position.

C. Structures with Two Stages

In Fig. 11 simulation results of a *wide* structure (*L*20) and a *narrow* one (*L*21) are compared. In both cases, the following rule for 2-stage link systems is obvious. The mode (SN-H) is advantageous if the loss does not exceed 10 percent. For higher losses no remarkable difference between the various wiring and hunting modes exists. In link systems with graded trunk groups (SN-H) is still more favorable (Fig. 12), if these gradings are designed with progressive commoning which is suitable for sequential hunting with home position (cf. [10], [11]).

In 2-stage link systems the modes (CN-H), (SN-R), and (CN-R) do not differ regarding the resulting loss [14]. Therefore, sequential hunting with random start

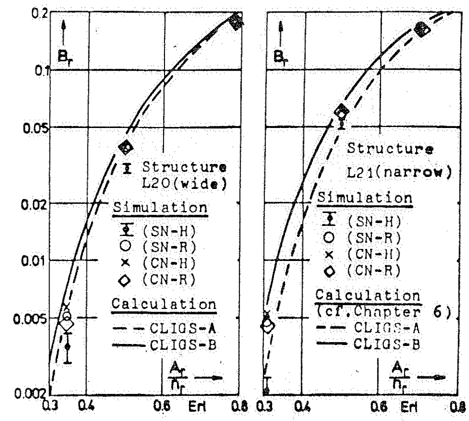


Fig. 11. Wiring and hunting in 2-stage link systems (here $A_r/n_r = A_{tot}/N_{out}$) (simulation with 95-percent confidence interval).

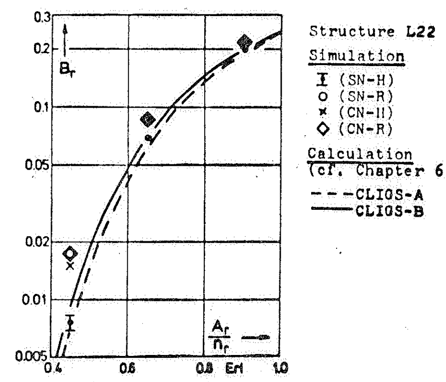


Fig. 12. Advantage of (SN-H) in case of graded trunk groups (here $A_r/n_r = A_{tot}/N_{out}$).

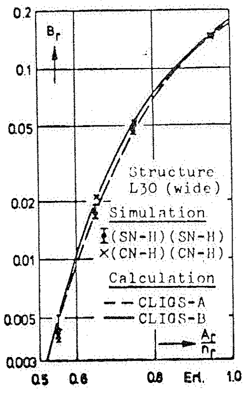


Fig. 13. Wiring and hunting in a wide 3-stage meshed link system (here $A_r/n_r = A_{tot}/N_{out}$).

position is not considered any more by the investigations of Section III-D–III-F.

D. Structures with Three Stages

In Figs. 13 and 14 $B_r = f(A_r/n_r)$ of two *meshed* 3-stage link systems is presented. As *L*30 has a wide structure (Section II-F), the modes (SN-H)(SN-H) and (CN-H)(CN-H) give nearly the same loss with a slight advantage for (SN-H)(SN-H) (see Fig. 13).

An extensive study of system *L*32 is given in Fig. 14 where 8 different wiring modes are investigated [15]. The considered narrow structure can be wired with or without linkblocks in stages 2 and 3 because each multiple of stage 2 hunts 10 out of 20 last-stage multiples only. In the case of linkblocks we get a structure similar to Fig. 4(b). Fig. 14 shows that (SN-H)(SN-H) is best. Cyclic wiring between stages 1 and 2 is not favorable. Linkblocks increase the loss even more. As in 2-stage link systems, all studied modes are equivalent in case of high offered traffic.

Three-stage *fan-out* structures show minimal losses in the case of mode (SN-H)(SN-H), too. Fig. 15 gives two examples which demonstrate that in the case of fan-out structures even wide structures are sensitive to wiring and hunting mode.

E. Structures with Four Stages

1) *General Remarks:* In spite of their loss-increasing properties, linkblocks are often favorable because of the following technical reasons: 1) in case of increasing traffic the link system can easier be extended by attaching additional linkblocks; and 2) the common control may sometimes be organized simpler for blocks.

In Section III-E2 some block and nonblock configurations of link system *L*48 are compared.

2) *Simulation Results:* For *wide* 4-stage link systems Fig. 16 gives an example by link system *L*41. In this case, the subdivision into linkblocks has no remarkable influence on the loss. On the other hand, *narrow* (not recommendable) 4-stage link systems show a more distinct relation between wiring and hunting mode and probability of loss (Fig. 16, link system *L*48). Examples of a more detailed study on block or nonblock design are given in Fig. 17: at $A_r/n_r = 0.56$ Erlang the loss varies

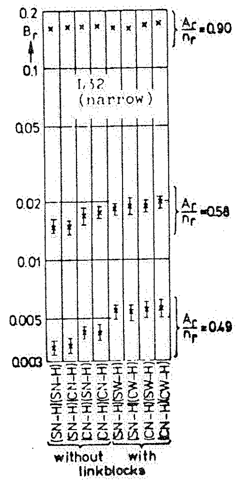


Fig. 14. Wiring and hunting in a narrow 3-stage meshed link system (here $A_r/n_r = A_{tot}/N_{out}$).

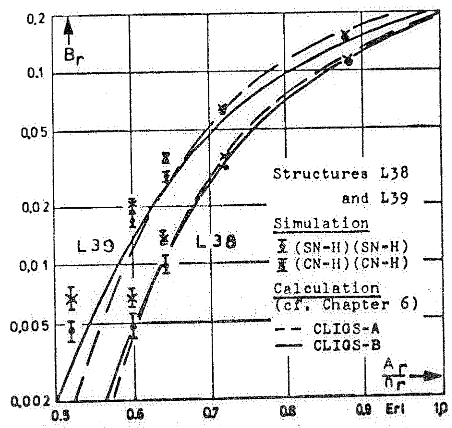


Fig. 15. Wiring and hunting in 3-stage fan-out link systems (here $A_r/n_r = A_{tot}/N_{out}$).

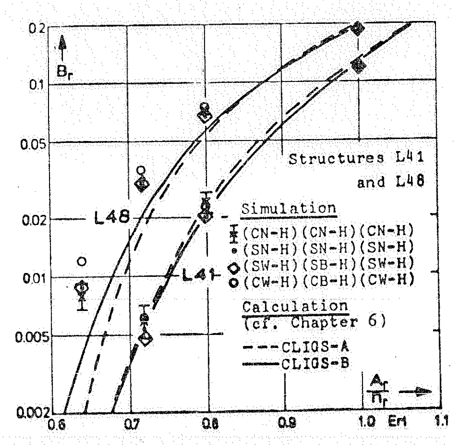


Fig. 16. Wiring and hunting in 4-stage link systems (here $A_r/n_r = A_{tot}/N_{out}$).

from 0.098 to 0.237 percent depending on the structure.

In Sections III-C and III-D it is demonstrated that in 2-stage and 3-stage link systems minimum loss is obtained for sequentially wired links combined with sequential hunting with home position. The reason may be that in this mode the upper multiples of stage 2 get maximum load and the lower multiples have a certain "free-path reserve." Contrary to this result, the mode (CN-H) per stage is favorable in 4-stage and 6-stage link systems

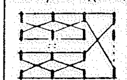
Wiring and hunting <u>with</u> linkblocks	Simulation $(B \pm \Delta B) \cdot 10^{-3}$	Wiring and hunting <u>without</u> linkblocks	Simulation $(B \pm \Delta B) \cdot 10^{-3}$
(SW-H)(CB-H)(SW-H)	1.97 ± 0.13	(SN-H)(CN-H)(CN-H)	0.98 ± 0.08
(SW-H)(SB-H)(SW-H)	2.34 ± 0.20	(CN-H)(CN-H)(CN-H)	1.05 ± 0.12
(CW-H)(CB-H)(CW-H)	2.26 ± 0.12	(SN-H)(SN-H)(SN-H)	2.11 ± 0.16
(SW-H)(SW-H)(SN-H) 	2.37 ± 0.30		
(SB-H)(SW-H)(SB-H) 	2.34 ± 0.20		

Fig. 17. Comparison of block and nonblock configuration of system $L48$ (A_r/n_r = 0.56 Erlang).

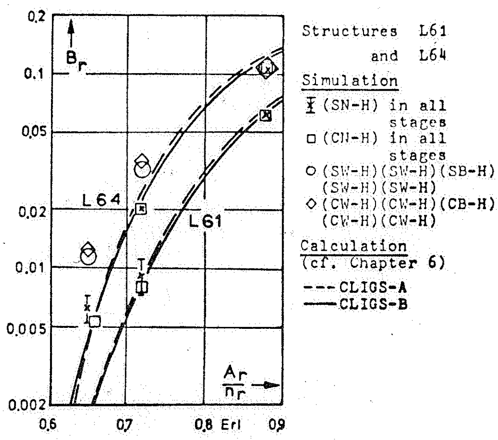


Fig. 18. Wiring and hunting in 6-stage link systems (here $A_r/n_r = A_{tot}/N_{out}$).

(cf. Section III-F). This may be explained by the fact that in systems with more than 3 stages the outlets of multiples in a certain stage generally do not reach all multiples of the succeeding stage, and therefore their possible free-path reserve is not effective. In these systems it is more advantageous to distribute the traffic equally among all multiples per stage. This can be achieved in structures without linkblocks more consequently, because in this case the traffic flow is not divided into parts by a block structure.

F. Structures with Six Stages

In accordance with the results obtained in 3-stage and 4-stage meshed link systems (see Figs. 13 and 16) also the loss in *wide* 6-stage link systems does not depend on the applied wiring and hunting mode. In Fig. 18 system $L61$ is given as an example.

An example of the studied *narrow* structures is represented by system $L64$ (Fig. 18). As for narrow systems with $S = 4$ cyclic wiring without linkblocks is most favorably here.

IV. OPTIMAL DESIGN OF LINK SYSTEMS FOR GROUP SELECTION

A. Survey

Lotze [9] has already demonstrated how to calculate "optimum link systems" from the *viewpoint of the minimum crosspoint requirement* for prescribed traffic transparency.

The following outlines make use of these minimum crosspoint rules. Furthermore, they pay special attention to a favorable loss versus load characteristic in the range from the expected traffic to be carried up to significant overload.

The results shown in Section III and further results presented here prove that a group-selection link system is *on no account* permitted to have a "narrow structure," i.e., $k_1/i_1 \leq 1$, because of its very unfavorable loss-increasing property. This requires in any case expansion from inlets to outlets of the first-stage multiples, even for a small load a_1 per inlet, e.g., $a_1 = 0.5$ Erlang.

It will be shown that "narrow" link systems can be replaced by more favorable "wide" link systems without increase of crosspoint requirement, sometimes even with a saving of crosspoints (cf. Section IV-B3b).

The optimum link method [9] yields for a group-selection link system with given numbers N_{in}, N_{out} of inlets and outlets, respectively, and with prescribed transparency T, an assortment of various different structures having nearly the same minimal crosspoint requirement for the same carried traffic.

It will be demonstrated that, nevertheless, the structures can have rather different and more or less suitable overload characteristics, which should be regarded for the design.

B. The Tools for Design

1) Minimum Crosspoint Structures: The most necessary ideas and formulas of [9] are briefly reviewed.

Looking for minimum crosspoint structures, one has to differentiate a formula describing the total crosspoint requirement. This differentiation must regard the wanted carried traffic as well as (approximately) the desired grade of service (probability of loss). Instead of a more or less complicated (perhaps more or less accurate) approximate loss formula, the grade of service is characterized by means of the desired traffic-transparency T of the link system. T is a function of carried traffic and system parameters

$$T = \prod_{j=1}^{S-1} (k_j - y_j) \cdot k_S \qquad (1)$$

where $y_j = Y_{tot}/g_j$ is the carried traffic per multiple in stage number j, $j = 1, \cdots, S - 1$. Balanced traffic input is provided.

The *transparency* T according to (1) means that average quantity of different idle paths [each consisting of $(S - 1)$ links in series], which lead from an arbitrary free inlet of a first-stage multiple to the total of $N_{out} = g_S \cdot k_S$ outgoing trunks.

Meshed link systems have for normally carried traffic often $T > N_{out}$. For a "wide" group-selection system, well designed by means of the outlines below, one obtains from T an approximate *lower* bound of the effective accessibility $k_{\text{eff},r}$ to any outgoing group number r ($r = 1,\cdots,R$) having n_r trunks.

For $T\cdot\frac{k_{Sr}}{k_S} \geqq n_r$ the n_r trunks are practically full accessible (compare traffic tests in Fig. 21).

For $T\cdot\frac{k_{Sr}}{k_S} < n_r$ the n_r trunks have more or less limited accessibility,

where $T\cdot\frac{k_{Sr}}{k_S}$ is a lower bound of $k_{\text{eff},r}$ (cf. Section VI).

In addition to T one prescribes for the design also the total carried traffic Y_{tot}, the total number of inlets N_{in}, and therewith the carried traffic per inlet

$$a_1 = Y_{\text{tot}}/N_{\text{in}}. \tag{2}$$

The theory [9] yields the following formulas for the structural design of systems with minimum crosspoint requirement.

$$S_{\text{opt}} = \ln (T/4\cdot a_1) \tag{3}$$

is that number of stages which leads to the smallest crosspoint requirement.

The necessary number of crosspoints C referred to the total traffic Y_{tot} yields the crosspoints per Erlang (CPE) for a minimum crosspoint structure by (cf. Fig. 19)

$$\text{CPE}_{\min} = 4\cdot S(T/4\cdot a_1)^{1/S}. \tag{4}$$

Equation (4) is true for $S \lesseqgtr S_{\text{opt}}$, if (5)–(8) are observed:

$$k_j = 2\cdot(T/4\cdot a_1)^{1/S}, \qquad j = 2,3,\cdots,S-1 \tag{5}$$

$$i_1 = i_j = k_j, \qquad j = 2,3,\cdots,S-1 \tag{6}$$

$$k_1 = i_1\cdot(a_1/0.5) = i_1\cdot 2\cdot a_1 \tag{7}$$

$$k_S = i_1. \tag{8}$$

The minimum total number of required crosspoints becomes with (4)

$$C = Y_{\text{tot}}\cdot\text{CPE}. \tag{9}$$

From (4) and (9) follows for prescribed quantity C, regarding (5)–(8):

$$T_{\max} = \left(\frac{C}{N_{\text{in}}\cdot a_1\cdot 4\cdot S}\right)^S\cdot 4a_1. \tag{10}$$

Formulas (3) and (5)–(8) still disregard the fact that the quantities S, k, i can only be realized as integers. Rounding up or down does, however, not influence significantly the crosspoint requirements CPE or C, respectively. The same is true with respect to some variations of the rounded integer values of k and i, to attain suitable integer ratios, such as N_{in}/i_1, being the number of multiples in stage number 1, etc.

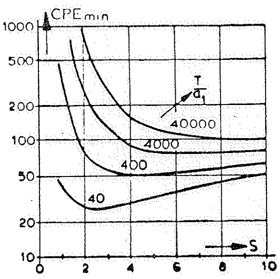


Fig. 19. The influence of the stage number S on the crosspoint requirement.

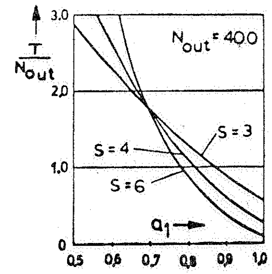


Fig. 20. The influence of the stage number S on $T = f(a_1)$.

Of course, the designer has to check whether the realized final structure approximates well enough the prescribed transparency T_{op} at the "operating point" (normally carried traffic).

2) Transparency Versus Carried Traffic: The transparency T is a parabolic function, which decreases with increasing carried traffic. According to (1) this decrease dT/dY_{tot} or dT/da_1, respectively, depends not only on the traffic but also significantly on the number S of stages. Fig. 20 gives a clear picture of this property by means of 3 different link systems ($S = 3$ with $C = 21166$ crosspoints, $S = 4$ with $C = 17814$ crosspoints, and $S = 6$ with $C = 16866$ crosspoints). All 3 link systems are designed according to section IV-B1 for an operation-point transparency $T_{\text{op}}/N_{\text{out}} = 1.75$ at $a_1 = 0.7$ Erlang and with $N_{\text{in}} = N_{\text{out}} = 400$. The design has to take into account this overload characteristic (Section IV-C).

The smallest number S of stages which yields a wide system, having the desired operating transparency T_{op}, guarantees the smallest overload sensitivity.

C. Outlines for the Design

1) General Remarks: From the considerations in Section IV-B2 it follows that the design of group-selection link systems mainly has to regard the following points.

1) The prescribed transparency T_{op} at the operating point, i.e., for the traffic Y_{tot} to be *normally* carried, should be attained (at least approximately). By means of Y_{tot} and the various partial traffics Y_r per outgoing group number r, one calculates the probabilities B_r of loss according to Section VI.

2) The structure should be designed such that for prescribed operating transparency T_{op}, and for a corresponding crosspoint requirement in the order of $\text{CPE}_{\min}$, the decreased transparency T_{ov} (in case of short traffic peaks as well as of a longer lasting overload period) remains as bearable as possible (see Section IV-B2).

In the following sections the design is explained in detail by means of various examples.

2) Example Number 1: System design for the following prescribed parameters:

$$N_{\text{in}} = 250$$

$$N_{\text{out}} = 250 \text{ [e.g., } R = 5,\ n_r = 50 (r = 1,\cdots,R)$$

$$\text{or } R = 3,\ n_1 = 50,\ n_2 = n_3 = 100, \text{ etc.]}$$

$$Y_{\text{tot}} = 160 \text{ Erlang}$$

$$a_1 = 0.64 \text{ Erlang}$$

$$T_{\text{op}} = 250 = 1.0 \cdot N_{\text{out}}.$$

With $T/a_1 = 250/0.64 = 391$, it follows from Fig. 19 that $\text{CPE}_{\min}$ is nearly the same one for $S = 3, 4, 5$, and 6 stages.

With (4) one obtains for $S = 3/4/5/6/$ the values CPE = 55.3/50.3/50/51.5.

The crosspoint requirement regarding $S = 4$ and $S = 5$, respectively, does not differ remarkably. Therefore, $S = 4$ is chosen.

With $S = 4$, $T = 250$, and $a_1 = 0.64$ one finds with (5)–(8):

$$i_1 = \cdots = i_4 = k_2 = \cdots = k_4 = 6.29$$

$$k_1 = i_4 = 8.05.$$

Appropriately, one realizes a (slightly less expensive) structure which regards that the number $N_{\text{in}} = N_{\text{out}} = 250$, i.e., N_{in}/i_1 should be an integer.

Be chosen $i_1 = i_2 = k_2 = i_3 = k_3 = k_4 = 5$ and $k_1 = i_4 = 8$. Therewith we get the link system $L41$:

$$5 \mid 8 \qquad 5 \mid 5 \qquad 5 \mid 5 \qquad 8 \mid 5 \qquad \text{with } C = 8000.$$

The transparency becomes $T(a_1 = 0.64) = 216$. Because $T \cdot k_{Sr}/k_S < n_r$ yields lower bounds for $k_{\text{eff},r}$, the loss differs not remarkably from the loss in case of full accessibility (see Fig. 21 and cf. Section VI).

3) Example Number 2 for $a_1 = 0.5$ and $a_1^* > 0.5$: a) For the sake of simplicity we take also $N_{\text{in}} = 250$, $N_{\text{out}} = 250$ ($R = 5$, $n_r = 50 (r = 1,\cdots,R)$) and here $Y_{\text{tot}} = 0.5 \cdot 250 = 125$ Erlang, i.e., $a_1 = 0.5$. Desired be $T_{\text{op}}/N_{\text{out}} \approx 1.6$, i.e., $T \approx 400$. Equations (5)–(8) yield $i_1 = k_1 = i_2 = \cdots = k_4 = 7.5$.

With regard to $N_{\text{in}} = N_{\text{out}} = 250$, the following structure is realized (whose transparency is no more very close to the optimum):

$$L48{:}10 \mid 10 \qquad 5 \mid 5 \qquad 5 \mid 5 \qquad 10 \mid 10 \qquad \text{with } C = 7500.$$

One finds $T(a_1 = 0.5) = 313$ (instead of 400, being the theoretical optimum for $k_j = 7.5$). For $a_1 = 0.6$ one still obtains $T(0.6) = 160$.

For the designed traffic $a_1 = 0.5$ system $L48$ has obviously a negligible loss. The same holds for the other structures discussed below. But in the case of (unexpectedly) higher values of a_1 one recognizes that wide structures having about the same number of crosspoints are significantly more favorable.

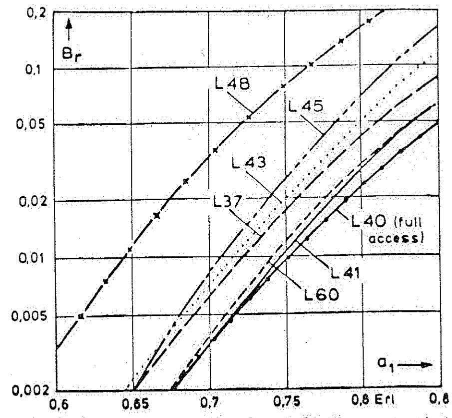


Fig. 21. Loss in link systems with about 8000 crosspoints (simulation results only).

b) Now, we try to find a wide system instead of that one designed in Section IV-B3a, with $k_1 \gtrsim 1.2 \cdot i_1$ and without an additional crosspoint requirement ($C \approx 7500$).

We insert into $(k_1/i_1) = 1.4 = 2a_1^*$ the auxiliary parameter a_1^* (not identical with the traffic $a_1 = 0.5 < a_1^*$) and obtain according to (10) with constant C and $k_1 \gtrsim 1.2 \cdot i_1$ a reduced value

$$T_{\max}^* = \left(\frac{7500}{250 \cdot 0.7 \cdot 4 \cdot 4}\right)^4 \cdot 4 \cdot 0.7 = 144.14.$$

From (5)–(8) we get with a_1^* and $T = 144.14$ the parameters of the *wide* structure $i_1 = i_2 = k_2 = i_3 = k_3 = k_4 = 5.36$ and $k_1 = i_4 = 7.5$.

Be realized $i_1 = 5$, $k_1 = 7$, etc. This "improved" structure $L43$ looks now:

$$5 \mid 7 \quad 5 \mid 5 \quad 5 \mid 5 \quad 7 \mid 5 \quad \text{with } C = 7000 < 7500.$$

We get $T(0.5) = 233$ which is remarkably below $T(0.5) = 313$ of the above system $L48$, however, still fairly close to $T = N_{\text{out}} = 250$. Further, we get $T(0.6) = 163 \approx 160$ for the system $L48$. Nevertheless, Fig. 21 shows that $B_r = f(A_r/n_r)$ is by far better for the wide system $L43$ than for the narrow system $L48$.

c) Of course, one can find still other "improved" structures, e.g., with $a_1^* = 0.6,\cdots,0.8$ Erlang and sometimes slightly increased crosspoint requirements. [Another suitable structure would be $L41$ with $C = 8000$ and $T(0.6) = 244$.] The losses of all these structures have been measured by artificial traffic tests and are drawn in Fig. 21. The difference to the system $L48$ using $k_1 = i_1$ is striking!

4) Further Examples: In the following examples further structures are considered to show the influence of the ratio k_1/i_1 and the number of stages S on the loss of link systems.

1) In Figs. 21 and 22, among others, the curves for loss and transparency, respectively, for the 3-stage link system $L37$ and the 4-stage system $L48$ are drawn. The wide 3-stage link system is more advantageous than the narrow 4-stage system, both having the same number of crosspoints $C = 7500$.

2) Figs. 21 and 22 contain furthermore two 4-stage link systems having $C = 10\,000$. The inferiority of the

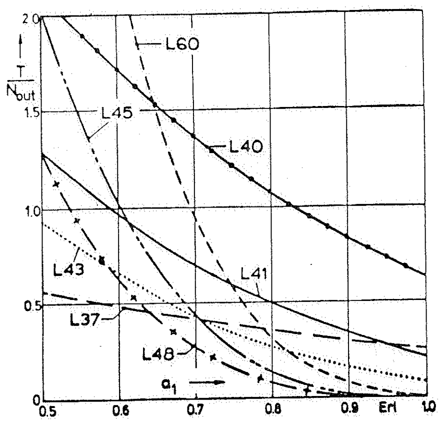


Fig. 22. Transparency of link systems having about 8000 crosspoints each.

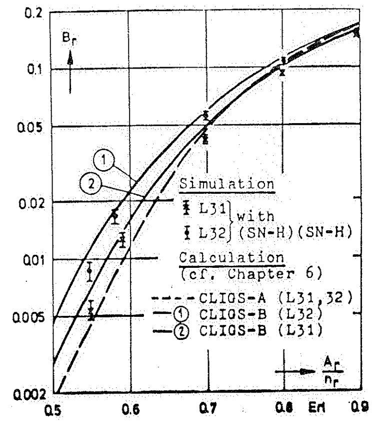


Fig. 23. The influence of $i_1 = k_1$ in a 3-stage link system (here $A_r/n_r = A_{tot}/N_{out}$).

narrow system $L45$ with $k_1/i_1 = 1$ is obvious comparing with the wide and optimal system $L40$.

3) The 6-stage link system $L60$ with $C = 9000$ has in the range of $a_1 = 0.7$–0.8 Erlang slightly higher losses than $L41$ with $C = 8000$ (cf. Fig. 21). This is caused by the steeper slope dT/da_1 of $L60$ (cf. Fig. 22).

4) In Fig. 23 the loss-increasing effect of $k_1/i_1 = 1$ is shown by means of two 3-stage link systems. The two structures $L31$ and $L32$ differ only in the first stage. $L31$ has 9 | 10 and $L32$ has 10 | 10 first-stage multiples. This results in remarkably increased losses for the narrow system $L32$. ($L31$ represents a structure according to feature B in Section II (Fig. 8) with $LW_{in} > 1$ and $LW_{out} = 1$).

5) A similar case to that of Fig. 23 is considered in Fig. 24. Again both structures differ only in the first stage. $L43$ has 5 | 7 and $L44$ has 7 | 7 first-stage multiples. For the narrow system $L44$ the loss is increased by a factor 3–1.6 in the range of $A_r/n_r \approx 0.65$–0.8 Erlang. ($L44$ represents a narrow structure according to feature D in Section II Fig. 8 with $LW_{in} = 1$ and $LW_{out} > 1$.)

As to the suitable design of link systems with point-to-point selection, which is not handled here, the authors refer to the extensive investigations in [13].

V. MAPPING OF LARGE LINK SYSTEMS

A. Survey

Up to now large link systems can often not be full-scale simulated. Instead, a partial graph of the link system is investigated by artificial traffic trials (e.g., [8]).

In this section a new method is presented which facilitates a loss-and-load-equivalent (LLE) single-valued mapping of large group-selection link systems to smaller ones (the number S of stages remains constant). These small systems can be full-scale simulated requiring significantly less computer time and less storage capacity. The accuracy of this LLE mapping is shown by simulation results.

B. Outline of the Method

In Section IV it was shown that the loss characteristics of link systems depend directly on the link-system transparency. Therefore, the presented LLE method is based on the idea to design a smaller link system (parameters marked by an asterisk) with the same average number of "visible" last-stage multiples as in the large system (cf. Section IV-B).

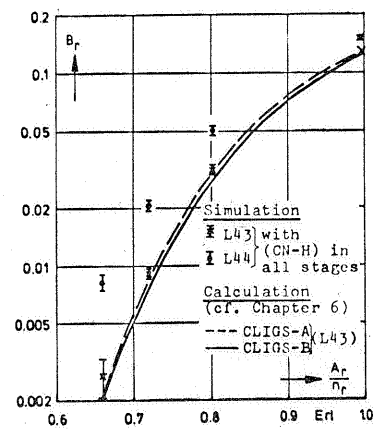


Fig. 24. The influence of $i_1 = k_1$ in a 4-stage link system (here $A_r/n_r = A_{tot}/N_{out}$).

It holds

$$\frac{T^*}{k_S{}^*} \cdot \frac{1}{g_S{}^*} = \frac{T}{k_S} \cdot \frac{1}{g_S} \tag{11}$$

with

T^*, T transparency;
$k_S{}^*, k_S$ outlets per last-stage multiple;
$g_S{}^*, g_S$ number of last-stage multiples.

The smaller structure consists of m times less inlets, link lines, and outlets, respectively, compared with the large structure ($m < 1$, *mapping factor*). To achieve the same loss and load characteristics of a certain outgoing trunk group, the designer regards (11) and proceeds as follows:

a) in stages $1,2,\cdot j \cdot, S-1$ the multiple parameters remain constant ($i_j{}^* = i_j$, $k_j{}^* = k_j$), but the number of multiples per stage j is reduced by the factor m, i.e., $g_j{}^* = m \cdot g_j$;

b) in the last stage the number of multiples remains constant, ($g_S{}^* = g_S$), but their parameters are reduced by $i_S{}^* = m \cdot i_S$, $k_S{}^* = m \cdot k_S$;

c) the considered outgoing trunk group has the same number of trunks; only the total number of outgoing trunks is reduced by $N_{\text{out}}{}^* = m \cdot N_{\text{out}}$.

Determination of the mapping factor m: The designer must take into account that m should not lead to less than $R^* = 2$ outgoing groups to get a group-selection link system. Furthermore, m is to be chosen such that the parameters $g_j{}^*$ and R^* are integer numbers (cf. Section V-C, example 1: $i_S{}^* = 2.4$).

Of course, the traffic is, in the case of a high usage operation, more smoothed within the smaller system. This can, for very high utilization, reduce the probability of loss (cf. Section V-C, example 2). Therefore, the total offered traffic A^* to the mapped small system should not exceed a value for which the probability of "all N_{in} inlets busy" is higher than about 1 percent, according to $E_{1,N_{\text{in}}^*}(A^*)$ [4].

C. Examples of Mapping

1) Example 1: Fig. 25 shows the comparison between a 250-trunk 6-stage link system ($L62$) and a 100-trunk 6-stage link system ($L63$). The applied mapping factor is $m = 0.4$. The losses of both systems agree within the whole range of offered traffic.

2) Example 2: In Fig. 26 LLE-mapping is demonstrated by three 3-stage link systems: structure $L33$ with 200 trunks is mapped to structure $L34$ with 100 trunks ($m = 0.5$) and to structure $L35$ with 40 trunks ($m = 0.2$), respectively. At $A_r/n_r \lesssim 0.7$ the mapping factor $m = 0.2$ (40 inlets) yields equivalent loss, whereas in case of $A_r/n_r > 0.7$ the loss of the smallest system ($L35$) increases less than for $L33$ because the probability $E_{1,40}(A_r)$ becomes >1 percent. Thus a mapping factor $m = 0.5$ is more suitable for those offered traffics.

3) Example 3: Fig. 27 gives an example for LLE mapping of 4-stage link systems: structure $L46$ with 1000 trunks is mapped to the structure $L47$ with 400 trunks (mapping factor $m = 0.4$). This saves about 30 percent of storage capacity (data only). Furthermore, 64 percent of expensive computer time was saved, provided the simulation results were obtained in both cases with approximately the same confidence interval.

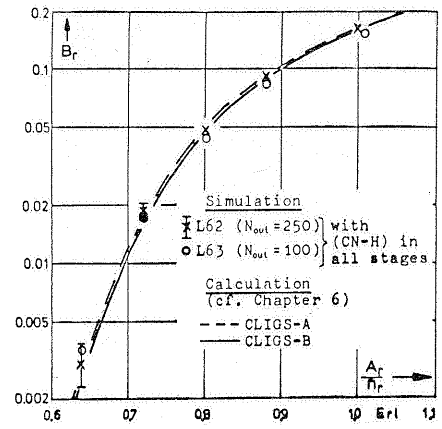


Fig. 25. LLE mapping of a 6-stage link system.

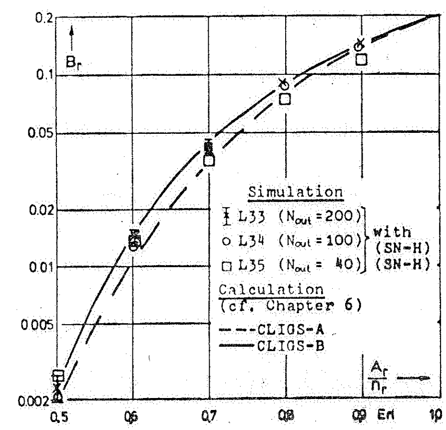


Fig. 26. LLE mapping of a 3-stage link system.

link system	number of offered calls	computer time (cycle time 0.6 μs)	storage capacity: program + data	offered traffic (Erlang)	$B_r \pm \Delta B_r$ (per cent)
L 46	540000	≈38.7min	14k + 10k	39.98	4.13 ± 0.39
L 47	240000	≈14 min	14k + 7k	39.88	3.43 ± 0.22

Fig. 27. LLE mapping of a 4-stage link system.

VI. APPROXIMATE LOSS CALCULATION

A. Survey

This section deals with loss calculation by a new approximate formula for the effective accessibility regarding link systems with group selection, which have an equal number of inlets and outlets per multiple except an expansion in the first and possibly concentration in the last stage, respectively. The abbreviation of the method is CLIGS.

Two versions CLIGS-A and CLIGS-B are derived. CLIGS-A is applicable for a quick manual evaluation; CLIGS-B is for computer evaluation only. Artificial traffic trials have proved the validity of these two methods.

B. The Effective Accessibility k_{eff}

The approximate effective accessibility k_{eff} is calculated from two terms (see Sections VI-B1 and VI-B2).

1) The "Free Fan:" The so-called "Free Fan" (FF) is defined by

$$\mathrm{FF} = \prod_{j=1}^{S-1} (k_j - y_j) \tag{12}$$

where k_j, y_j are the outlets and the carried traffic, respectively, per multiple in stage j ($j = 1,\cdots,S-1$). In (12) the limitations hold

$$\prod_{j=1}^{i} (k_j - y_j) \leq g_{i+1} (i = 1,\cdots,S-1). \tag{13}$$

This FF represents that number of multiples in the last stage which are, on the average, accessible via free link paths from any free inlet of the first stage. The corresponding FF accessibility to a certain outgoing group number r is therewith

$$k_{FF,r} = FF \cdot k_{Sr}. \quad (14)$$

Equation (14) equals the average number of outlets to the considered outgoing group number r which can be hunted within the FF regardless of whether they are idle or busy.

2) The "Busy Fan:" By means of the "Busy Fan" (BF) a second term of k_{eff} is calculated. The BF is defined by

$$BF = \prod_{j=1}^{S-1} k_j - FF \quad (15)$$

where $\prod_{j=1}^{S-1}$ is limited by g_S and means the "Maximum Fan," i.e., the maximum number of S-stage multiples being accessible from an inlet of the first stage (at least for traffic $Y_{tot} = 0$).

The BF is equal to the average number of S-stage multiples within this maximum fan, but outside the FF. From the BF follows

$$b_r = BF \cdot k_{Sr} \cdot (Y_r/n_r) \quad (16)$$

which is the average number of busy trunks in the considered group number r within the BF. The FF does not include this number b_r of busy trunks.

Now it is known that "accessibility" is defined by 1) all accessible idle trunks of a group, and additionally; 2) all busy trunks of this group which can be accessed and occupied once more, *as soon as they become idle.*

The second point includes obviously not only busy trunks of a group number r *within* the FF. Additionally, it contains also that part of the b_r busy trunks [(16)] to which a free path from the considered first-stage multiple to their own S-stage multiples gets open as soon as their established connection terminates. This still-unknown share of b_r also forms a part of the "effective accessibility." By one and the same termination also further idle outlets could become accessible.

How many trunks out of the b_r busy trunks within the BF do fulfill this condition "increase of access via the FF as soon as an established call terminates?"

The more links to the last stage (inlets of stage S) are idle, the greater the probability that a further newly released S-stage inlet opens one or more paths via free links to those multiples in preceding stages $S-1, S-2, \cdots, 3, 2$ which are already part of the FF. A first characterizing figure for this access-increasing effect is the average number f of all free links referred to all N_{out} trunks (for given total carried traffic Y_{tot}).

It holds

$f = (P - Y_{tot})/N_{out}$, for meshed structures with

$$P = g_S \cdot i_S \text{ and } N_{out} = g_S \cdot k_S$$

or

$f = (k_S)^{-1}$, for fan-out structures.

Using this ratio f as an approximate factor to calculate from (16) the "BF contribution" $k_{BF,r}$, one obtains

$$k_{BF,r} = BF \cdot k_{Sr} \cdot (Y_r/n_r) \cdot f. \quad (17)$$

The fitting of (17) had, of course, to be checked and confirmed on the basis of many thousands of artificial traffic trials, performed for this paper (see Sections II–V).

With (14) and (17) the approximate formula for the effective accessibility to a group number r holds

$$k_{eff,r} = k_{FF,r} + k_{BF,r}. \quad (18)$$

This formula is directly applied to the loss calculation in the following section (method CLIGS-A).

C. Loss Calculation by Method CLIGS-A

By means of the effective accessibility $k_{eff,r}$ [according to (18)] the considered group-selection link system is replaced, for a certain carried traffic, by an equivalent *one-stage* array with a constant accessibility $k = k_{eff,r}$.

Then the well-known MPJ loss formula [5], [7] is applied

$$B_r = \frac{E_{1,n_r}(A_{0r})}{E_{1,n_r - k_{eff,r}}(A_{0r})}. \quad (19)$$

From the prescribed carried traffic Y_r of the considered trunk group number r the parameter A_{0r} has to be calculated iteratively [5] to fulfill the condition

$$Y_r = A_{0r}(1 - E_{1,n_r}(A_{0r})). \quad (20)$$

The actually offered traffic to group number r becomes

$$A_r = \frac{Y_r}{1 - B_r}. \quad (21)$$

For given $(Y_r, n_r, k_{eff,r})$ existing loss tables [7] can also easily be applied.

The probability of loss, calculated with (19), is in good agreement with the results of artificial traffic tests for any crosspoint-saving *wide* structure (cf. Section II-F).

D. Loss Calculation by Method CLIGS-B

The version CLIGS-B uses the same expectation value $k_{BF,r}$ as given in (17), but it regards the statistical traffic variations on the inlets of the considered first-stage multiple and therewith partially the statistical variations of the FF size.

For the probability "x inlets of a first-stage multiple are busy" the Erlang distribution is assumed [2], [4]

$$w(x) = \frac{A_{01}^x}{x!} \Big/ \sum_{j=0}^{i_1} \left(\frac{A_{01}^j}{j!}\right), \quad x = 0, 1, \cdots, i_1 \quad (22)$$

with

$$A_{01} = \frac{y_1}{1 - E_{1,i_1}(A_{01})} \quad (23)$$

link system	n_r	offered traffic A_{tot}/Erl	A_r/A_{tot}	Simulation B ± ΔB	Calculation CLIGS- A	Calculation CLIGS- B
L 36	$n_{1,\ldots,6}=10$		0.05	0.008 ± 0.003	0.005	0.007
(SN-H)	$n_{7,8}=10$	79.83	0.10	0.125 ± 0.006	0.119	0.132
in all	$n_9=10$		0.20	0.436 ± 0.003	0.434	0.452
stages	$n_{10}=10$		0.30	0.609 ± 0.002	0.599	0.614
L 60	$n_1=50$		0.05	0	0	0
	$n_2=50$		0.10	0	0	0
(CN-H)	$n_3=50$	189.77	0.20	0.010 ± 0.002	0.012	0.012
in all	$n_4=50$		0.30	0.185 ± 0.004	0.194	0.194
stages	$n_5=50$		0.35	0.270 ± 0.002	0.263	0.263
L 42	$n_1=25$		0.06	0.008 ± 0.002	0.006	0.005
	$n_2=25$		0.12	0.251 ± 0.002	0.219	0.206
(CN-H)	$n_3=50$	248.92	0.12	0.0004 ± 0.0003	0.0002	0.0002
in all	$n_4=50$		0.35	0.452 ± 0.001	0.412	0.398
stages	$n_5=100$		0.35	0.023 ± 0.001	0.022	0.020

Fig. 28. Comparison of simulation and calculation in case of unequal group sizes and/or nonuniform offered traffic per group.

and prescribed traffic y_1 per first-stage multiple.

Then one can derive [analogously to (12), (13), and (18)]

$$\mathrm{FF}(x) = (k_1 - x)\cdot\prod_{j=2}^{S-1}(k_j - y_j) \tag{24}$$

limited by

$$(k_1 - x)\cdot\prod_{j=2}^{i}(k_j - y_j) \le g_{i+1}, \quad i = 2,\cdots,S-1$$

$$k_{\mathrm{FF},r}(x) = \mathrm{FF}(x)\cdot k_{Sr} \tag{25}$$

$$k_{\mathrm{eff},r}(x) = [\mathrm{FF}(x) + \mathrm{BF}\cdot(Y_r/n_r)\cdot f]\cdot k_{Sr}. \tag{26}$$

For each value of $x(x = 0,1,\cdots,i_1 - 1)$ by means of $k_{\mathrm{eff},r}(x)$ a probability of loss is calculated analogously to (19)

$$B_r(x) = \frac{E_{1,n_r}(A_{0r})}{E_{1,n_r-k_{\mathrm{eff},r}(x)}(A_{0r})}. \tag{27}$$

Finally, the probability of loss for the considered group number r is evaluated with (22) and (27)

$$B_r = \sum_{x=0}^{i_1-1} B_r(x)\cdot\frac{w(x)}{1 - w(i_1)}. \tag{28}$$

E. Comparison Between Loss Formulas and Artificial Traffic Tests

1) *Examples Taken from Sections IV and V:* Tests and calculation for various link systems are compared in Fig. 11, 13, 15–17, 25, and 26 in Sections IV and V. It is obvious that the loss formulas hold very well for all *wide* crosspoint-saving link systems.

Narrow, i.e., crosspoint-wasting link systems with $S \ge 3$ stages, mostly have higher losses than calculated, in particular for losses $B_r \lessapprox 5$ percent (cf. Figs. 15 and 16).

The method CLIGS-*B* yields for narrow systems still more realistic values of loss because it takes into account the probability $w(x)$ on the first-stage inlets. This property can in particular be seen in Fig. 23 regarding the systems *L*31 and *L*32. The method CLIGS-*A* yields, however, fairly good results for the wide system *L*31, whereas the loss of the narrow structure *L*32 is underestimated. For $S \ge 4$ the loss of narrow systems is also underestimated by CLIGS-*B*, because the statistical traffic variations are approximately regarded between stage 1 and 2 only.

2) *Examples with Unequal Group Sizes and/or Nonuniform Offered Traffics per Group:* For reasons of simplicity, the link systems discussed in the previous sections were designed with equal group sizes and uniform offered traffics per group. This is, however, on no account a condition for the accuracy of calculation methods CLIGS-*A* and CLIGS-*B*.

Fig. 28 shows further comparisons between simulation and calculation (selection out of a large number of concerned investigations).

a) *Example 1:* Ten equal-sized groups with 10 trunks each and with nonuniform offered traffic per group.

b) *Example 2:* Five equal-sized groups with 50 trunks each and with nonuniform offered traffic per group.

c) *Example 3:* Five unequal groups with 25–100 trunks and with nonuniform offered traffic per group.

Good agreement between simulation and calculation is apparent.

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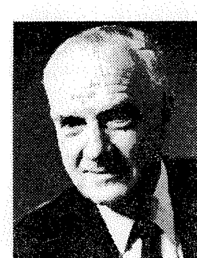

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