Link Systems with Both-Way Connections and Outgoing Finite-Source Traffic

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1. Introduction

In modern public and private exchanges the subscriber connection network (SCN) consists mostly of multi-stage link systems to which the q subscribers (traffic sources) are connected. Such link systems carry different types of traffic:

1. Internal traffic which is performed by connections between two subscribers of the same SCN.

2. Outgoing and incoming external traffic to and from other SCN's, which belong to the same or to any other exchange.

This paper deals with the approximate calculation of the probabilities of loss for such link systems in which a finite number of subscribers (traffic sources) produces the offered traffic (Pure Chance Traffic of type 2 (PCT 2), [7]).

According to the various structures of public and private exchanges the calculation has to distinguish several operation modes according to the number and use of the trunk groups which are connected to one side of the SCN (the subscribers are connected to the other side of the SCN).

After the definition of the three types of traffic (Section 2) these operation modes are discussed in Section 3. In Section 4 the path-selection mode for internal and external calls resp. in the link system is regarded. As the result definitions of the offered traffics and the probabilities of loss are given.

The basic idea for the calculation of the probability of loss is the method CIRB, which was developed for uni-directional traffic [3, 4, 5].

Furthermore, the distribution function of the probabilities of state for fully available groups with internal and external traffic and finite number of traffic sources is used, which is derived in [7] and represents an extension of [1, 2].

The new calculation method is shown in Section 5. In Section 6 results are compared with artificial traffic trials.

2. The three Types of Traffic

The three types of traffic are *(Fig. 1)*:

(1) Internal traffic (I-traffic): This traffic is generated by $q < \infty$ subscribers and therefore considered as offered finite source traffic with uniform and constant arrival rate α_i per idle source (PCT 2). This traffic leads to the same q subscribers of the considered link system. I.e. an internal connection occupies two subscribers and two paths in the link system, an outgoing and an incoming path.

(2) Outgoing external traffic (outgoing E-traffic): This traffic is also generated by the $q < \infty$ subscribers and leads via a trunk group (right side of the link system) e.g. to another exchange. This traffic is also

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PCT 2 with uniform and constant arrival rate α_{eg} per idle source. An outgoing external connection occupies only one subscriber and one path in the link system.

(3) Incoming external traffic (incoming E-traffic): This traffic is generated by subscribers of other SCN-s. Their number of subscribers q^* (traffic sources) is greater than the number of subscribers q of the considered link system: $q^* \gg q$. Therefore, this incoming E-traffic is PCT 1 (Pure Chance Traffic of type 1) with the arrival rate λ_{ec}. It leads via the considered link system to the q subscribers (left side of the link system). An incoming external connection occupies only one path and one subscriber in the link system.

3. Operation Modes

In public and private exchanges link systems with various structures and operation modes are realized. According to the number and use of the trunk groups three operation modes will be distinguished. (Besides these three most important operation modes further operation modes are possible [7]).

The three operation modes are characterized in *Table 1* (R = number of trunk groups).

4. Definition of the Offered Traffic and the Probabilities of Loss

4.1. *The Path Selection Mode and the Definition of the Traffic Rates*

To define the offered traffics it is necessary to regard the switching mode of I- and E-calls in the link system (SCN).

Mostly in modern exchanges a call produced by a special subscriber of the regarded SCN is connected via the SCN with a free Relay Set and the central control resp. (cf. *Fig. 2*). Then the calling subscriber dials the number of the called subscriber and the control interprets this number.

4.1.1. Internal Traffic

Then, considering an I-call, the internal connection is built up via the group selection network (GSN) and the SCN to the called subscriber. I.e. in case of I-calls the path-selection mode works as follows:

a) Attempt to connect the calling subscriber via the SCN with a free line (RS) of the outgoing trunk group (cf. Fig. 2). If all lines of this trunk group are busy or in case of internal blocking of the link system (SCN) the call gets lost.

b) Attempt to connect the occupied RS with the called subscriber via the GSN and SCN, but only if the called subscriber is free. In case of internal blocking of the GSN or SCN the call gets lost.

The following rates can be defined:

$_i\lambda_{tot}$:   total arrival rate of the I-traffic.

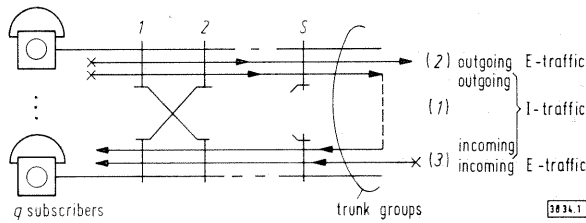$_{ig}\lambda_B$:   outgoing loss rate caused by internal blocking

Fig. 1.
The three types of traffic.

RS : Relay Set
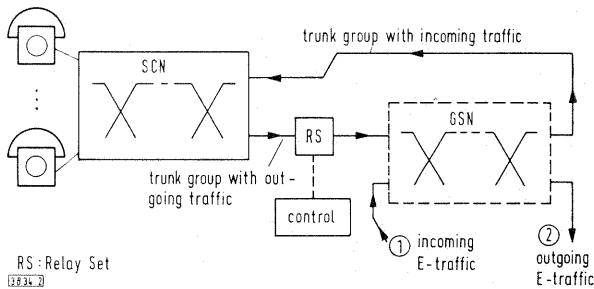
Fig. 2.
Speech path network of a telephone exchange (e.g. SCN with operation mode 2).

Table 1. The operation modes.

oper mode	structure	R	use of the trunk group	example of application
1		1	Bi-directional; incoming and out-going I-and E-traffic	PCM-ex-changes[8]
2		2	Uni-directional;. ①: outgoing I- and E-traffic ②: incoming I- and E-traffic	public ex-changes[9]
3		3	Uni-directional: ①: outgoing I-traffic, ②: incoming I-traffig; Bi-directional: ③: incoming and outgoing E-traffic	private ex-changes[10]

of the SCN or by full occupied outgoing trunk group.

$_i\lambda_{Bsub}$: loss rate caused by busy called subscribers.

$_{ic}\lambda$:   incoming arrival rate to the GSN and SCN resp. regarding that the called subscriber is free. This arrival rate $_{ic}\lambda$ is given by:

$$_{ic}\lambda = {_i\lambda_{tot}} - {_{ig}\lambda_B} - {_i\lambda_{Bsub}} . \qquad (4.1)$$

$_{ic}\lambda_B$:   incoming loss rate caused by internal blocking of the GSN or SCN.

With these arrival and loss rates the occupation rate $_i\lambda_Y$ yields:

$$_i\lambda_Y = {_{ic}\lambda} - {_{ic}\lambda_B} = {_i\lambda_{tot}} - {_{ig}\lambda_B} - {_i\lambda_{Bsub}} - {_{ic}\lambda_B} . \quad (4.2)$$

4.1.2. External Traffic

Analogously, the incoming E-traffic is handled. Arriving at point ① in Fig. 2, it will be tried to connect the incoming E-calls via the GSN and SCN with the called subscriber if the subscriber is free. The rates for the incoming E-traffic are:

$_{ec}\lambda_{tot}$:   total arrival rate,

$_{ec}\lambda_{Bsub}$:   loss rate caused by busy called subscribers,

$_{ec}\lambda_B$:   incoming loss rate caused by the GSN or SCN.

The incoming arrival rate $_{ec}\lambda$ is:

$$_{ec}\lambda = {_{ec}\lambda_{tot}} - {_{ec}\lambda_{Bsub}} . \qquad (4.3)$$

The occupation rate of the incoming E-traffic amounts to:

$$_{ec}\lambda_Y = {_{ec}\lambda} - {_{ec}\lambda_B} = {_{ec}\lambda_{tot}} - {_{ec}\lambda_{Bsub}} - {_{ec}\lambda_B} . \quad (4.4)$$

The outgoing E-traffic is handled as PCT 2, we get for the occupation rate $_{eg}\lambda_Y$:

$$_{eg}\lambda_Y = {_{eg}\lambda} - {_{eg}\lambda_B} \qquad (4.5)$$

with

$_{eg}\lambda$:   arrival rate of the outgoing E-traffic,

$_{eg}\lambda_B$: loss rate of the outgoing E-traffic.

4.2. Definition of the Characteristic Traffic Values

As shown in Section 4.1 two types of lost calls have to be distinguished:

1. losses caused by blocking of the link system: B_{link},
2. loss caused by busy called subscribers: B_{sub}.

Usual values for B_{sub} range from 10 % to 20 %. SCN's are normally dimensioned for values of B_{link} in the range of 0.1 % to 5 %. Calculating the probability of loss of a link system we have to consider only B_{link} but not B_{sub}. I.e. we consider only the loss rate caused by blocking of the link system. Therefore, it is appropriate to define the arrival rate which is relevant for the link system as the total arrival rate $_i\lambda_{tot}$ reduced by the loss rate $_i\lambda_{Bsub}$.

Thus, we get the offered I-traffic:

$$A_i = ({_i\lambda_{tot}} - {_i\lambda_{Bsub}}) \cdot h = ({_{ig}\lambda_B} + {_{ic}\lambda}) \cdot h \qquad (4.6)$$

with h = mean service time.

The probability of loss of the I-traffic gets:

$$B_i = \frac{{_{ig}\lambda_B} + {_{ic}\lambda_B}}{{_{ig}\lambda_B} + {_{ic}\lambda}} . \qquad (4.7)$$

Each successful I-call occupies two paths in the considered link system (SCN). Therefore, we get the internal carried traffic Y_i:

$$Y_i = 2 \cdot {_i\lambda_Y} \cdot h \qquad (4.8)$$

and with Eqns. (4.2), (4.6) and (4.7) it becomes:

$$Y_i = 2 \cdot A_i(1 - B_i) . \qquad (4.9)$$

Analogously, we get the characteristic traffic values for the incoming E-traffic (cf. Eqn. (4.3)):

$$A_{ec} = ({_{ec}\lambda_{tot}} - {_{ec}\lambda_{Bsub}}) h = {_{ec}\lambda} \cdot h \qquad (4.10)$$

$$B_{ec} = {_{ec}\lambda_B}/{_{ec}\lambda} \qquad (4.11)$$

$$Y_{ec} = {_{ec}\lambda_Y} \cdot h = A_{ec}(1 - B_{ec}) . \qquad (4.12)$$

The same equations are valid for the characteristic traffic values of the outgoing E-traffic, replacing the

indices of the incoming E-traffic by those of the outgoing E-traffic.

5. Calculation of the Probabilities of Loss and the Offered Traffics

5.1. *The Link System and the given Traffic Values*

5.1.1. The Link System

The parameters i_j, k_j, g_j $(j = 1, \ldots, S)$ and n_r $(r = 1, \ldots, R)$ of the link system are given (cf. *Fig. 3*). According to the three operation modes we have different numbers R of trunk groups (cf. Section 3, Table 1).

5.1.2. The given Traffic Values

The carried traffics of the three traffic types are prescribed:

— Y_i: for the I-traffic,
— Y_{eg}: for the outgoing E-traffic,
— Y_{ec}: for the incoming E-traffic.

Characterizing the carried traffic Y_{Sr} of a trunk group No. r $(r = 1, \ldots, R)$ we get according to the different operation modes:

—Operation mode 1:

$$Y_{S1} = Y_i + Y_{eg} + Y_{ec} \qquad (5.1)$$

— Operation mode 2:

$$Y_{S1} = Y_i/2 + Y_{eg}; \quad Y_{S2} = Y_i/2 + Y_{ec} \quad (5.2)$$

— Operation mode 3:

$$Y_{S1} = Y_{S2} = Y_i/2; \quad Y_{S3} = Y_{eg} + Y_{ec} . \quad (5.3)$$

The total carried traffic is:

$$Y_{tot} = Y_i + Y_{eg} + Y_{ec} \qquad (5.4)$$

and the total carried E-traffic is:

$$Y_e = Y_{eg} + Y_{ec} . \qquad (5.5)$$

From these given values we get the carried traffic of a multiple in stage No. j $(j = 1, \ldots, S)$:

$$Y_j = Y_{tot}/g_j . \qquad (5.6)$$

5.2. *Assumptions*

5.2.1. Assumption No. 1 (General)

The probabilities of state of the outlets of each multiple in stage No. 1 $w(x)$ and those of the link groups $p_j(x)$ and of the trunk groups $p_r(x)$ are independent of each other, but they depend on the given carried traffics, i.e. $\sum x\, p(x) = Y$ prescribed.

5.2.2. Assumption No. 2 (I-Traffic)

According to the path-selection mode described in Section 4.1 there exists a time delay between the path selection in outgoing and incoming direction of an I-call. This time delay is caused by the dialling of the telephone number and their interpretation in the control (cf. Fig. 2). In [11] it was shown that, with given carried traffic, this time delay does not influence the probability of loss considerably. Therefore, this time delay is neglected.

5.2.3. Assumption No. 3 (Incoming I- and E-traffic)

The incoming traffics are connected to the considered SCN via further selector stages (e.g. in Fig. 2 by the GSN). Losses caused by these selector stages are
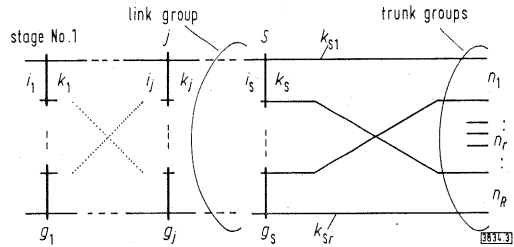

Fig. 3.
Link system with S stages.

neglected. Therefore, in case of incoming I- and E-traffic it is allowed to replace the point-to-point selection from a special inlet of these selector stages (GSN in Fig. 2) to the called subscriber (left side of the SCN) by the group selection from the called subscriber to the trunk group carrying the incoming traffic (cf. Fig. 2). (Of course, in case of outgoing I- and E-traffic group selection is considered.)

5.3. *The Principle of the Calculation Method*

The probabilities of loss for the three traffic types are divided each into different parts [3, 4, 5, 6, 7]:

1. The probability of loss, caused by blocking of the outlets of a multiple in stage No. 1.

2. The probability of loss, caused by trunk group blocking, i.e. by the limited access to the considered trunk group No. r or by the state "all lines of this trunk group No. r busy".

3. In case of $i_j > k_j$: the probabilities of loss, caused either by the limited access to the link group between stage No. j and stage No. $j + 1$ $(j = 2, \ldots, S - 1)$ or by the state "all lines of this link group busy".

The distribution function for fully available groups with I- and E-traffic and PCT 2 [7] is assumed for these probabilities of state. Generating arrival rates α_0 are introduced in such a way that with the distribution function of the fully available group the prescribed carried traffic results.

For the calculation of the probabilities of state and the probability of loss for the I-traffic, this traffic is divided into two parts

— Multiple-internal traffic (MI-traffic): The calling and the called subscriber are connected with the same multiple in stage No. 1, i.e. a MI-call occupies two outlets of the same multiple in stage No. 1.

— System-internal traffic (SI-traffic): The calling and the called subscriber are connected with different multiples in stage 1.

5.4. *The Calculation of the Probabilities of State*

5.4.1. The Probabilities of State $w(x)$

To calculate $w(x)$ we have to consider one multiple of stage No. 1 with its different traffic types (in the following we consider mean service time $h = 1$):

(1) Outgoing E-traffic with the generating arrival rate $_{eg}\alpha_{01}$ of an idle source. The carried traffic is:

$$_{eg}Y_1 = Y_{eg}/g_1 . \qquad (5.7)$$

Therefore, we get the arrival rate in state $\{x + 1\}$ of the considered multiple: $_{eg}\alpha_{01}(i_1 - (x + 1))$.

(2) Incoming E-traffic with the generating arrival rate $_{ec}\lambda_{01}$. The carried traffic is:

$$_{ec}Y_1 = Y_{ec}/g_1 . \qquad (5.8)$$

An incoming E-call is offered to the considered multiple if the called subscriber is idle. The probability that a special subscriber is idle in state $\{x + 1\}$ of the multiple is: $(i_1 - (x + 1))/i_1$.

(3) MI-traffic with the generating arrival rate $_m\alpha_{01}$ per idle source and the carried traffic $_mY_1$. With the assumption that all incoming calls are equally distributed among the g_1 multiples of stage No. 1 we get $_mY_1 = {}_iY_1/g_1$. With $_iY_1 = Y_i/g_1$ we get:

$$_mY_1 = \frac{Y_i}{g_1 \cdot g_1} . \qquad (5.9)$$

In state $\{x\}$ we have the arrival rate $_m\alpha_{01}(i_1 - x)$. A MI-connection is built up if the called subscriber is idle. A MI-call which is generated in state $\{x\}$ occupies the traffic source $x + 1$, therefore, the probability that the called subscriber (within the same multiple) is idle becomes: $(i_1 - (x + 1))/i_1$.

(4) Outgoing SI-traffic with the generating arrival rate $_s\alpha_{01}$ per idle source and the carried traffic $_{sg}Y_1$. A SI-call leads to a subscriber of one of the remaining $(g_1 - 1)$ multiples of the link system. In each of these multiples the mean number of busy subscribers is Y_1. Therefore, the mean value of the probability that the called subscriber is idle yields: $(i_1 - Y_1)/i_1$.

(5) Incoming SI-traffic with the generating arrival rate $_{sc}\alpha_{01}$ referred to the considered multiple and the carried traffic $_{sc}Y_1$. An incoming SI-call is generated by a subscriber of one of the remaining $(g_1 - 1)$ multiples. I.e. each idle source of these $(g_1 - 1)$ multiples has an arrival rate referred to the considered multiple of $_{sc}\alpha_{01} = {}_s\alpha_{01}/(g_1 - 1)$. The mean value of the idle sources of each of the $(g_1 - 1)$ multiples is: $(i_1 - Y_1)$. Therefore, we have the total incoming arrival rate: $_{sc}\alpha_{01}(i_1 - Y_1)(g_1 - 1) = {}_s\alpha_{01}(i_1 - Y_1)$. The probability that a special subscriber is idle in the considered multiple is $(i_1 - (x + 1))/i_1$, if the incoming SI-call is offered in state $\{x + 1\}$.

Therefore, we get the following recurrence formula [7]:

$$w(x + 2) = (_{eg}\alpha_{01} + {}_{ec}\alpha_{01}) \frac{i_1 - (x + 1)}{x + 2} w(x + 1) +$$
$$+ 2 {}_s\alpha_{01} \frac{i_1 - (x + 1)}{x + 2} w(x + 1) \frac{i_1 - Y_1}{i_1} +$$
$$+ {}_m\alpha_{01} \frac{i_1 - x}{x + 2} \cdot 2 \cdot w(x) \frac{i_1 - (x + 1)}{i_1} \qquad (5.10)$$

with

$$\sum_{x=0}^{a} w(x) = 1 .$$

The value a is:

$$\begin{aligned} a &= k_1 \quad \text{for} \quad i_1 \geq k_1 \\ a &= i_1 \quad \text{for} \quad i_1 < k_1 . \end{aligned} \qquad (5.11)$$

With the carried traffics $_{eg}Y_1$, $_{ec}Y_1$, $_sY_1 = {}_iY_1 - {}_mY_1$ and $_mY_1$ the quantities $_{eg}\alpha_{01}$, $_{ec}\alpha_{01}$, $_s\alpha_{01}$ and $_m\alpha_{01}$ have to be determined (by iteration) such that the following equations (5.12) to (5.18) are fulfilled [7].

Outgoing and Incoming E-traffic:

$$_{eg}Y_1 = {}_{eg}\alpha_{01}(i_1 - Y_1)\left(1 - \frac{i_1 - a}{i_1 - Y_1} w(a)\right). \qquad (5.12)$$

Caused by the suitable definition of the offered traffic according to Section 4 the carried traffic $_{ec}Y_1$ of the incoming E-traffic is also calculated by Eqn. (5.12), whereby $_{eg}\alpha_{01}$ is replaced by $_{ec}\alpha_{01}$.

SI-traffic:

The generating offered traffic $_{sg}A_{01}$ and $_{sc}A_{01}$ for the outgoing and incoming SI-traffic yields:

$$_{sg}A_{01} = {}_{sc}A_{01} = {}_sA_{01} = {}_s\alpha_{01}(i_1 - Y_1)\frac{i_1 - Y_1}{i_1} . \qquad (5.13)$$

The carried SI-traffic is:

$$_sY_1 = {}_{sg}Y_1 + {}_{sc}Y_1 = 2 \cdot {}_sA_{01}\left(1 - \frac{i_1 - a}{i_1 - Y_1} w(a)\right). \qquad (5.14)$$

MI-traffic:

The generating offered traffic $_mA_{01}$ is (cf. remark to traffic type (3)):

$$_mA_{01} = {}_m\alpha_{01} \sum_{x=0}^{a} (i_1 - x) w(x) \frac{i_1 - (x + 1)}{i_1} . \qquad (5.15)$$

An outgoing MI-call gets lost in state $\{a\}$. That call occupies the source $(a + 1)$; therefore, we get the probability $(i_1 - (a + 1))/i_1$ that the called subscriber is idle. Then, the probability of loss $_{mg}b_1$ in outgoing direction is (fully available trunk group):

$$_{mg}b_1 = \frac{(i_1 - a) w(a) (i_1 - (a + 1))}{\sum\limits_{x=0}^{a} (i_1 - x) w(x) (i_1 - (x + 1))} . \qquad (5.16)$$

An incoming MI-call gets lost in state $\{a - 1\}$. Therefore, we get analogously to Eqn. (5.16):

$$_{mc}b_1 = \frac{(i_1 - (a - 1)) w(a - 1) (i_1 - a)}{\sum\limits_{x=0}^{a} (i_1 - x) w(x) (i_1 - (x + 1))} . \qquad (5.17)$$

Acc. to Eqn. (4.9) the carried MI-traffic is:

$$_mY_1 = 2 \cdot {}_mA_{01}(1 - (_{mg}b_1 + {}_{mc}b_1)). \qquad (5.18)$$

5.4.2. The Probabilities of State $p_r(x)$

5.4.2.1. Operation Mode 1

Link systems with operation mode 1 have $R = 1$ trunk groups. The number of sources for this trunk group is:

$$q = i_1 \cdot g_1 . \qquad (5.19)$$

For this trunk group we have to distinguish between the three traffic types: outgoing E-, incoming E- and I-traffic. Therefore, we get the recurrence formula (acc. to Eqn. (5.10)):

$$p_1(x + 2) = (_{eg}\alpha_{0s} + {}_{ec}\alpha_{0s}) \frac{q - (x + 1)}{x + 2} p_1(x + 1) +$$
$$+ 2 \cdot {}_i\alpha_{0s} \frac{q - x}{x + 2} p_1(x) \frac{q - (x + 1)}{q}$$

with
$$\sum_{x=0}^{n_1} p_1(x) = 1 . \qquad (5.20)$$

Whereas, the carried traffics Y_{eg}, Y_{ec} and Y_i are prescribed, the quantities of the generating arrival rates $_{eg}\alpha_{0s}$, $_{ec}\alpha_{0s} = {}_{ec}\lambda_{0s}/q$ and $_i\alpha_{0s}$ have to be determined (by iteration) such that Eqns. (5.12) and (5.18)

are fulfilled. Hereby, the quantities referred to a multiple of stage No. 1 have to be replaced by the quantities referred to the trunk group.

5.4.2.2. Operation Mode 2

Link systems with operation mode 2 have $R = 2$ trunk groups. The mean number of sources per trunk group is:

$$\left. \begin{array}{l} q_1 = q - Y_{S2} \\ \text{and} \quad q_2 = q - Y_{S1} \end{array} \right\} . \qquad (5.21)$$

An internal call occupies in each trunk group one line. Therefore, the recurrence formula for the probabilities of state $p_r(x)$ is:

$$p_r(x + 1) = {}_{eg}\alpha_{0Sr} \frac{q_r - x}{x + 1} p_r(x) +$$

$$+ {}_{i}\alpha_{0Sr} \frac{q_r - x}{x + 1} p_r(x) \frac{q_r - (x + 1)}{q}$$

with $\qquad \sum\limits_{x=0}^{n_r} p_r(x) = 1$, $\qquad (5.22)$

where $(q_r - (x + 1))/q$ is the probability that the called subscriber is idle if an I-call is generated in state $\{x\}$. The generating arrival rate gets in case of:

— trunk group $r = 1$: ${}_{e}\alpha_{0S1} = {}_{eg}\alpha_{0S}$
— trunk group $r = 2$: ${}_{e}\alpha_{0S2} = {}_{ec}\alpha_{0S}$.

Again, with the prescribed carried traffics Y_{S1} and Y_{S2} (cf. Eqn. (5.2)) we get ${}_{e}\alpha_{0Sr}$ and ${}_{i}\alpha_{0Sr}$ by iteration such that the following equations are fulfilled:

$${}_{e}Y_{Sr} = {}_{e}\alpha_{0Sr}(q - Y_{tot}) \left(1 - \frac{q_r - n_r}{q - Y_{tot}} p_r(n_r)\right) \quad (5.23)$$

where ${}_{e}Y_{S1} = Y_{eg}$ and ${}_{e}Y_{S2} = Y_{ec}$.

$${}_{i}Y_{Sr} = \frac{Y_i}{2} = {}_{i}A_{0Sr}(1 - {}_{i}b_{Sr}) \qquad (5.24)$$

with the generating offered I-traffic-

$${}_{i}A_{0Sr} = {}_{i}\alpha_{0Sr} \sum\limits_{x=0}^{n_r} (q_r - x) p_r(x) \frac{q_r - (x + 1)}{q} \qquad (5.25)$$

and the probability of loss of a fully available group:

$${}_{i}b_{Sr} = \frac{(q_r - n_r) p_r(n_r) (q_r - (n_r + 1))}{\sum\limits_{x=0}^{n_r} (q_r - x) p_r(x) (q_r - (x + 1))} . \qquad (5.26)$$

5.4.2.3. Operation Mode 3

Link systems with operation mode 3 have $R = 3$ trunk groups. The trunk group No. 1 and No. 2 are only carrying the outgoing and incoming I-traffic resp. Therefore, the trunk groups No. 1 and No. 2 are always in the same state $\{x\}$ ($n_1 = n_2$). I.e. we get $p_1(x) = p_2(x) = p_{1,2}(x)$.

The mean number of sources for both of these trunk groups is:

$$q_{1,2} = q - Y_{S3} = q - Y_e . \qquad (5.27)$$

Calculating the probabilities of state $p_{1,2}(x)$ we have to consider that the internal arrival rate in state $\{x\}$ of the trunk group is ${}_{i}\alpha_{0S}(q_{1,2} - 2 \cdot x)$. The probability that the called subscriber is idle yields:

$$(q_{1,2} - (2 x + 1))/q .$$

Therefore, we get the recurrence formula:

$$p_{1,2}(x + 1) = {}_{i}\alpha_{0S} \frac{q_{1,2} - 2 x}{x + 1} p_{1,2}(x) \frac{q_{1,2} - (2 x + 1)}{q}$$

with $\qquad \sum\limits_{x=0}^{n_1} p_{1,2}(x) = 1 . \qquad (5.28)$

Furthermore, the Eqns. (5.24), (5.25) and (5.26) are valid, whereby $(q_r - x)$ has to be replaced by $(q_{1,2} - 2 x)$ and $(q_r - (x + 1))$ has to replaced by $(q_{1,2} - (2 x + 1))$.

The probabilities of state $p_3(x)$ are calculated acc. to Eqns. (5.22) and (5.23) with:

$$q_3 = q - Y_i, \quad {}_{e}Y_{S3} = Y_e \quad \text{and} \quad {}_{i}\alpha_{0S3} = 0.$$

5.4.3. The Probabilities of State $p_j(x)$

The link group between stage No. j and stage No. $j + 1$ of the link system ($j = 2, \ldots, S - 1$) has $n_j = g_j \cdot k_j$ link lines (Fig. 3). The number of sources for this link group is $q = i_1 \cdot g_1$.

In the link group we have to distinguish the three traffic types outgoing and incoming E-traffic and I-traffic. Therefore, the recurrence formula acc. to Eqn. (5.20) is valid. Hereby, the quantities referred to the trunk group No. 1 have to be replaced by the quantities referred to the link group.

5.5. *The Trunk Group Blocking and the Link Group Blocking*

Acc. to [3, 4, 5, 6, 7] we have to calculate the trunk group and link group blocking. These are denoted with $P\{k_{mr}\}$ for trunk group No. r and with $P\{k_{mj}\}$ for the link group between stage No. j and stage No. $j + 1$. Hereby, k_{mr} and k_{mj} is the Mean Accessibility from the inlet of stage No. 1 to the considered trunk or link group resp.

k_{mr} and k_{mj} are calculated according to [3, 5, 7] or [12] resp.

Group blocking is calculated as for one stage arrangements with constant accessibility k_m [3, 4]. Therefore, we need the blocking probability $c(x)$, which is calculated acc. to [3].

5.5.1. E-Traffics

In case of the E-traffics we get:

$$_{eg}P\{k_{mr}\} = {}_{ec}P\{k_{mr}\} = \sum\limits_{x=k_{mr}}^{n_r} p_r(x) \cdot c_r(x) . \quad (5.29)$$

The Index r depends on the operation mode and represents the number of that trunk group on which the outgoing and incoming E-traffic resp. is carried.

Analogously, we get:

$$_{eg}P\{k_{mj}\} = {}_{ec}P\{k_{mj}\} = \sum\limits_{x=k_{mj}}^{n_j} p_j(x) c_j(x) . \quad (5.30)$$

5.5.2. I-Traffic

5.5.2.1. Link Group Blocking

For outgoing I-traffic we get acc. to Eqn. (5.30):

$$_{ig}P\{k_{mj}\} = {}_{eg}P\{k_{mj}\} . \qquad (5.31)$$

For incoming I-traffic we get [2, 7]:

$$_{ic}P\{k_{mj}\} = \sum_{x=k_{mj}}^{n_j} p_j(x)(1-c_j(x))\cdot c_j(x+1). \qquad (5.32)$$

In Eqn. (5.32) link group blocking in outgoing direction is excluded.

Therefore, we get the total internal link group blocking:

$$_iP\{k_{mj}\} = {}_{ig}P\{k_{mj}\} + {}_{ic}P\{k_{mj}\}. \qquad (5.33)$$

5.5.2.2. Trunk Group Blocking

Operation mode 1: For outgoing I-traffic we get:

$$_{ig}P\{k_{mr}\} = \sum_{x=k_{m1}}^{n_1} p_1(x)\,c_1(x). \qquad (5.34)$$

The incoming I-traffic is also carried on trunk group No. 1. Therefore, we get acc. to Eqn. (5.32):

$$_{ic}P\{k_{mr}\} = \sum_{x=k_{m1}}^{n_1} p_1(x)(1-c_1(x))\cdot c_1(x+1). \qquad (5.35)$$

Operation mode 2: For outgoing I-traffic we get $_{ig}P\{k_{mr}\}$ acc. to Eqn. (5.34).

The incoming I-traffic is carried on trunk group No. 2. The probabilities of state $p_1(x)$ are assumed to be independent of $p_2(x)$. Therefore, we get the trunk group blocking for the incoming I-traffic, excluding the blocking in outgoing direction:

$$_{ic}P\{k_{mr}\} = \sum_{x=k_{m2}}^{n_2} p_2(x)\,c_2(x)(1-{}_{ig}P\{k_{mr}\}). \quad (5.36)$$

Operation mode 3: For outgoing I-traffic we get Eqn. (5.34) again. Excluding the blocking in outgoing direction in the trunk group No. 1 we get [7]:

$$_{ic}P\{k_{mr}\} = \sum_{x=k_{m2}}^{n_2} p_{1,2}(x)(1-c_1(x))\cdot c_2(x). \quad (5.37)$$

For the three operation modes the total trunk group blocking $_iP\{k_{mr}\}$ is calculated acc. to Eqn. (5.33).

5.6. The Probabilities of Loss and the Offered Traffics

5.6.1. E-Traffic

The arrival rate of the outgoing E-traffic is:

$$_{eg}\lambda = {}_{eg}\alpha\,(i_1-Y_1). \qquad (5.38)$$

With the total arrival rate $_{ec}\lambda_{tot}$ of the incoming E-traffic we get according to the definition in Section 4:

$$_{ec}\lambda = {}_{ec}\lambda_{tot}\frac{i_1-Y_1}{i_1}.$$

With $_{ec}\alpha = {}_{ec}\alpha_{tot}/i_1$ we get:

$$_{ec}\lambda = {}_{ec}\alpha\,(i_1-Y_1). \qquad (5.39)$$

Acc. to the principle described in Section 5.3 the loss rate for outgoing E-traffic yields [3, 4, 7]:

— In state No. 1:

$$_{eg}\lambda_B^{(1)} = {}_{eg}\alpha\,(i_1-a)\,w(a). \qquad (5.40)$$

— In the following stages:

$$_{eg}\lambda_B^{(j)} = ({}_{eg}\lambda - {}_{eg}\lambda_B^{(1)})\times$$
$$\times\left(1-\prod_{j=2}^{S-1}(1-{}_{eg}P\{k_{mj}\})(1-{}_{eg}P\{k_{mr}\})\right). \quad (5.41)$$

Analogously, we get the loss rates of the incoming E-traffic.

With Eqns. (5.38) to (5.41) we get:

$$B_{eg} = \frac{{}_{eg}\lambda_B^{(1)} + {}_{eg}\lambda_B^{(j)}}{{}_{eg}\lambda}\quad\text{and}\quad B_{ec}=B_{eg}. \quad (5.42)$$

The offered traffics yield:

$$A_{eg} = \frac{Y_{eg}}{1-B_{eg}}\;;\qquad A_{ec} = \frac{Y_{ec}}{1-B_{ec}}. \quad (5.43)$$

5.6.2. I-Traffic

The probability of loss B_i of the I-traffic is divided in B_m caused by the MI-traffic and B_S caused by the SI-traffic.

The part $1/g_1$ of the I-traffic is MI-traffic, the other part is SI-traffic. Therefore, we get the probability of loss referred to the offered I-traffic:

$$B_i = \frac{1}{g_1}B_m + \left(1-\frac{1}{g_1}\right)B_S. \qquad (5.44)$$

MI-traffic:

Acc. to the definition Eqn. (4.7) we get:

$$B_m = \frac{{}_{mg}\lambda_B + {}_{mc}\lambda_B}{{}_{mg}\lambda_B + {}_{mc}\lambda}. \qquad (5.45)$$

With the arrival rate $_m\alpha$ of the MI-traffic the loss rate of the outgoing MI-traffic yields (analogously to Eqns. (5.40) and (5 41)):

$$_{mg}\lambda_B = {}_m\alpha\Big[(i_1-a)\,w(a) + ((i_1-Y_1)-(i_1-a)w(a))\times$$
$$\times\left(1-\prod_{j=2}^{S-1}(1-{}_{ig}P\{k_{mj}\})(1-{}_{ig}P\{k_{mr}\})\right)\Big]. \quad (5.46)$$

The incoming arrival rate yields:

$$_{mc}\lambda = {}_m\alpha\sum_{x=0}^{a-1}(i_1-x)\,w(x)\frac{i_1-(x+1)}{i_1}\times$$
$$\times\prod_{j=2}^{S-1}(1-{}_{ig}P\{k_{mj}\})(1-{}_{ig}P\{k_{mr}\}). \quad (5.47)$$

The loss rate of the incoming MI-traffic is calculated in three steps:

— Blocking in stage No. 1:

$$_{mc}\lambda_B^{(1)} = {}_m\alpha\,(i_1-(a-1))\,w(a-1)\frac{i_1-a}{i_1}\times$$
$$\times\prod_{j=2}^{S-1}(1-{}_{ig}P\{k_{mj}\})(1-{}_{ig}P\{k_{mr}\}). \quad (5.48)$$

— Blocking in the link groups:

Firstly, we calculate the occupation rate in stage No. 1:

$$_m\lambda_Y^{(1)} = {}_m\alpha\sum_{x=0}^{a-2}(i_1-x)\,w(x)\frac{i_1-(x+1)}{i_1}. \quad (5.49)$$

(Calls generated in state $\{a\}$ or $\{a-1\}$ get lost). Therewith, we get:

$$_{mc}\lambda_B^{(j)} = {}_m\lambda_Y^{(1)}\sum_{j=2}^{S-1}{}_{ic}P\{k_{mj}\}\prod_{\xi=2}^{j-1}(1-{}_iP\{k_{m\xi}\})\times$$
$$\times\prod_{\nu=\xi+1}^{S-1}(1-{}_{ig}P\{k_{m\nu}\})(1-{}_{ig}P\{k_{mr}\}). \quad (5.50)$$

— Blocking of a trunk group:

$$_{mc}\lambda_B^{(r)} = {}_m\lambda_Y^{(1)}\prod_{j=2}^{S-1}(1-{}_iP\{k_{mj}\})\cdot {}_{ic}P\{k_{mr}\}. \quad (5.51)$$

The loss rate of the incoming MI-traffic yields:

$$_{mc}\lambda_B = {}_{mc}\lambda_B^{(1)} + {}_{mc}\lambda_B^{(j)} + {}_{mc}\lambda_B^{(r)} . \quad (5.52)$$

SI-traffic:
The probability of loss of the SI-traffic is defined by:

$$B_s = \frac{_{sg}\lambda_B + {}_{sc}\lambda_B}{_{sg}\lambda_B + {}_{sc}\lambda} . \quad (5.53)$$

The loss rate $_{sg}\lambda_B$ of the outgoing SI-traffic is calculated acc. to Eqn. (5.46), where $_m\alpha$ is replaced by the arrival rate $_s\alpha$ of the SI-traffic. With

$$_{sg}\lambda_Y^{(1)} = {}_s\alpha \left[(i_1 - Y_1) - (i_1 - a)\, w(a) \right] \quad (5.54)$$

we get the incoming arrival rate:

$$_{sc}\lambda = {}_{sg}\lambda_Y^{(1)} \cdot \frac{i_1 - Y_1}{i_1} \prod_{j=2}^{S-1} (1 - {}_{ig}P\{k_{mj}\})(1 - {}_{ig}P\{k_{mr}\}) . \quad (5.55)$$

The loss rate of the incoming SI-traffic is calculated in three steps:

— The loss rate in stage No. 1:

$$_{sc}\lambda_B^{(1)} = {}_{sg}\lambda_Y^{(1)} \cdot w(a) \frac{i_1 - a}{i_1} \prod_{j=2}^{S-1} (1 - {}_{ig}P\{k_{mj}\}) \times$$
$$\times (1 - {}_{ig}P\{k_{mr}\}) . \quad (5.56)$$

— The rate $_{sc}\lambda_B^{(j)}$, caused by blocking of a link group:
With

$$_s\lambda_Y^{(1)} = {}_{sg}\lambda_Y^{(1)} \cdot \left[\frac{i_1 - Y_1}{i_1} - \frac{i_1 - a}{i_1} w(a) \right]$$

we get Eqn. (5.50), where $_m\lambda_Y^{(1)}$ has to be replaced by $_s\lambda_Y^{(1)}$.

— The loss rate $_{sc}\lambda_B^{(r)}$, caused by blocking of a trunk group:

$_{sc}\lambda_B^{(r)}$ is calculated acc. to Eqn. (5.51), where $_m\lambda_Y^{(1)}$ has to be replaced by $_s\lambda_Y^{(1)}$.

The incoming loss rate yields:

$$_{sc}\lambda_B = {}_{sc}\lambda_B^{(1)} + {}_{sc}\lambda_B^{(j)} + {}_{sc}\lambda_B^{(r)} . \quad (5.57)$$

Finally, we calculate the offered traffic:

$$A_i = \frac{Y_j}{2(1 - B_i)} . \quad (5.58)$$

Dimensioning a link system with I- and E-traffic it is of interest to consider the total probability of loss. We get:

$$B_{tot} = \frac{A_{eg} \cdot B_{eg} + A_{ec}\, B_{ec} + A_i\, B_i}{A_{eg} + A_{ec} + A_i} . \quad (5.59)$$

6. Comparison between Calculation and Artificial Traffic Trials

In *Diagram 1* one example is shown. Hereby, � means the test results with a confidence interval of 95 %. The solid lines are obtained with the presented calculation method. Further examples are shown in [7].

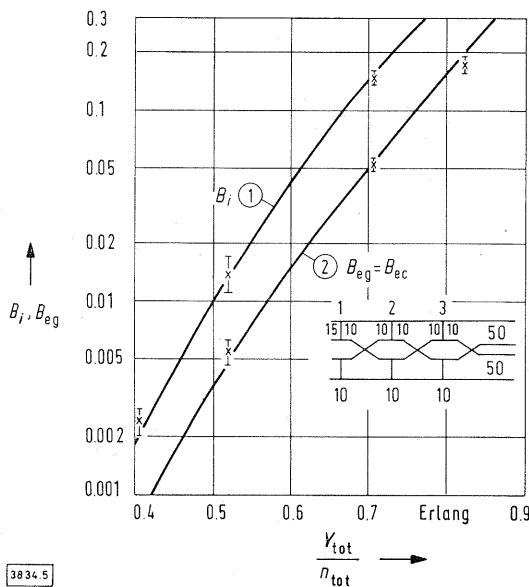The comparison shows the good accordance between approximate and test results.

Diagram 1.
Three stage link system with operation mode 2.
Curve ① Probability of loss of the internal traffic,
Curve ② Probability of loss of the outgoing and incoming external traffic resp. as function of the carried traffic per outlet $Y_{tot}/n_{tot} = Y_{tot}/(n_1 + n_2) = Y_{S1}/n_1 = Y_{S2}/n_2$.
Parameter $Y_1/Y_{tot} = 0.5$.

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