The Dimensioning of Trunk Groups for Standard Gradings of the German GPO in Case of Finite Number of Traffic Sources

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1. Introduction

The dimensioning outlines of the German GPO include loss tables for the dimensioning of trunk groups with limited accessibility for offered PCT 1 (infinite number of traffic sources). With prescribed values of the probability of loss B, the offered traffic A and the accessibility k the corresponding number of trunks n can be read off the tables. These tables are valid for trunk groups for standard gradings of the German GPO.

If the traffic is offered to these trunk groups from a finite number s of traffic sources (PCT 2) the admissible offered traffic increases by an amount $d = f(s, n, k, B)$ with constant values n, k and B.

In section 2, gradings with offered PCT 1 are regarded. Subsequently, offered PCT 2 is considered in section 3. In this section, a simple formula for the calculation of d is introduced. Therewith, the PCT 1 – tables are also usable for PCT 2. Examples for the application of this d-formula are given in section 4. In section 5, results achieved by this d-formula are compared with artificial traffic trials performed on a digital computer.

2. Offered PCT 1 (Infinite Number of Traffic Sources)

2.1. *High Efficiency Gradings*

Trunk groups for high efficiency gradings (see *Fig. 1*) with skipping and slipping, i. e. with good traffic balance, can be calculated very accurately by means of the Modified Palm Jacobaeus Loss Formula (MPJ Formula) [1, 2, 3].

2.2. *Standard Gradings*

Standard Gradings (see *Fig. 2*), as generally used by the German GPO since many years, have a simplified structure which facilitates automatized design of gradings and remarkably less expenditure of work for the implementation and for eventual extensions [4].

The admissible offered PCT 1-traffic for prescribed parameters n, k and B is slightly smaller than for high efficiency gradings. The difference ΔA can be calculated according to [5, 6]. The above mentioned loss tables of the German GPO [7, 8] already take this difference ΔA into account.

3. Offered PCT 2 (Finite Number s of Traffic Sources)

If PCT 2 is offered to an one-stage connecting array with fixed parameters n, k and B, the admissible offered traffic $A_{PCT\,2}$ is greater than the corresponding tabulated value $A_{PCT\,1}$; [2, 3]. In [9] loss formulae for this case are derived.

The difference

$$d = A_{PCT\,2} - A_{PCT\,1} = f(s, n, k, B) \qquad (1)$$

can, however, for standard gradings of the German GPO, easily be calculated directly by means of the following formula

$$d = 0.3\,[n - 0.77 \cdot A_{PCT\,1}(1 - B)]\, n/(s + n) \qquad (2)$$

which is valid for $k \geqq 3$, $n \geqq 6$ and $A_{PCT\,1} \geqq 2.5$ Erlang. In Eqn. (2) the accessibility k is implicitely regarded by the values n, $A_{PCT\,1}$ and B.

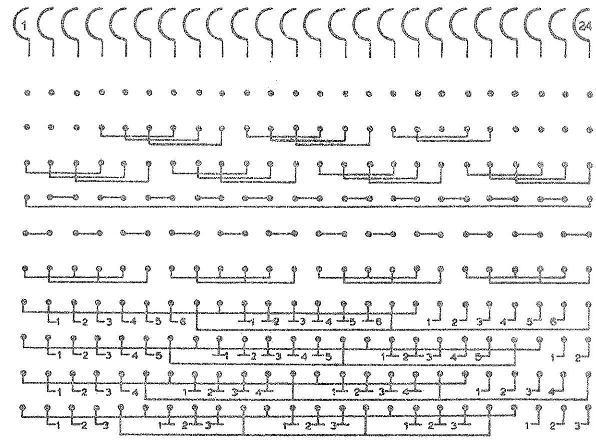

Fig. 1.
High efficiency grading with skipping and good traffic balance for the accessibility $k=10$ and $n=110$ trunks.

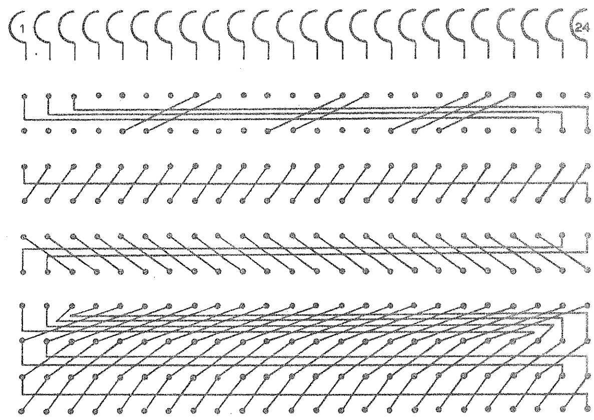

Fig. 2.
Standard grading of the German GPO for the accessibility $k=10$ and $n=110$ trunks.

The corresponding admissible offered PCT 2-traffic is according to Eqn. (1):

$$A_{PCT\,2} = A_{PCT\,1} + d\,. \qquad (3)$$

Furthermore, we get the corresponding call rate per idle source (mean holding time $h=1$):

$$\alpha = \frac{A_{PCT\,2}}{s - A_{PCT\,2}(1 - B)}\,. \qquad (4)$$

k \ n	A_{PCT1} in Erlang ... 10 ...
.	.
.	.
.	.
29	... 14.5 ...
30	... 15.0 ...
31	... 15.6 ...
.	.
.	.
.	.

Fig. 3.
Excerpt from the PCT 1-loss tables for $B = 0.5\,\%$ [7, 8].

k \ n	A_{PCT1} in Erlang ... 10 ...
.	.
.	.
.	.
28	... 15.1 ...
29	... 15.7 ...
30	... 16.3 ...
31	... 16.9 ...
32	... 17.5 ...
.	.
.	.
.	.

Fig. 4.
Excerpt from the PCT 1-loss tables for $B = 1\,\%$ [7, 8].

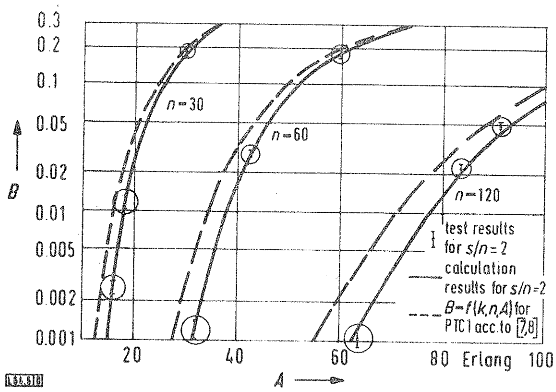

Fig. 5.
Comparison of the results, calculated with the d-formula, with test results. Accessibility $k = 10$, $s/n = 2$; mean interconnection number (grading ratio) $H = 2$.

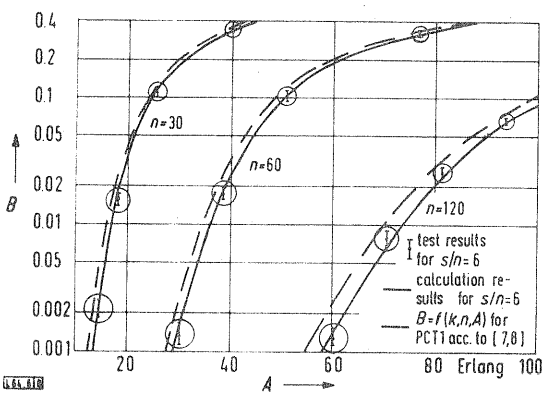

Fig. 6.
Comparison of the results, calculated with the d-formula with test results. Accessibility $k = 10$, $s/n = 6$; mean interconnection number $H = 2$.

4. Examples for the Application of the d-Formula

4.1. *Example 1*

Be given a standard grading with $k = 10$, $n = 30$, $s = 120$ and prescribed loss $B = 0.5\,\%$. The admissible offered traffic A_{PCT2} is wanted.

a) From the tables for PCT 1 [7, 8] we get for $k = 10$, $n = 30$ and $B = 0.5\,\%$ the offered PCT 1-traffic (see *Fig. 3*): $A_{PCT1} = 15.0$ Erlang.

b) By means of Eqn. (2) we get

$$d = 0.3\,[30 - 0.77 \cdot 15.0\,(1 - 0.005)]\,\frac{30}{120 + 30} = 1.11 \text{ Erlang}\,.$$

c) According to Eqn. (3) we get the admissible offered PCT 2-traffic:

$$A_{PCT2} = 15.0 + 1.11 = 16.11 \text{ Erlang}\,.$$

d) Finally, we can find the corresponding call rate per idle source from Eqn. (4)

$$\alpha = \frac{16.11}{120 - 16.11\,(1 - 0.005)} = 0.155\,.$$

4.2. *Example 2*

Be given $s = 40$, $k = 10$, $A_{PCT2} = 17.5$ Erlang and $B = 0.01 \mathrel{\hat{=}} 1\,\%$. The corresponding number n of trunks has to be determined. Since the offered PCT 2-traffic is prescribed, an iterative calculation is necessary.

a) The first step starts with the approximation

$$A^{(0)}_{PCT1} = A_{PCT2} = 17.5 \text{ Erlang}\,.$$

From the PCT 1-table (see Fig. 4, [7, 8]) for $k = 10$ we find: $n^{(0)} = 32$ as a first approximate value.

b) With $n^{(0)} = 32$ and the above values for s, B and $A^{(0)}_{PCT1}$ we find by means of Eqn. (2):

$$d^{(0)} = 0.3\,[32 - 0.77 \cdot 17.5\,(1 - 0.01)]\,\frac{32}{40 + 32} = 2.48 \text{ Erlang}\,.$$

c) From $d^{(0)}$ we get:

$$A^{(1)}_{PCT1} = A_{PCT2} - d^{(0)} = 17.5 - 2.48 = 15.02 \text{ Erlang}\,.$$

d) The next reading off the table as close as possible to the value $A^{(1)}_{PCT1} = 15.02$ Erlang, yields (see *Fig. 4*):

$$n^{(1)} = 28 \quad \text{and} \quad A^{(1)}_{PCT1} = 15.1 \text{ Erlang}\,.$$

e) With $n^{(1)}$ and $A^{(1)}_{PCT1}$ we get from Eqn. (2):

$$d^{(1)} = 2.04 \text{ Erlang}\,.$$

f) With $d^{(1)}$ and $A^{(1)}_{PCT1}$ we get:

$$A_{PCT2} = A^{(1)}_{PCT1} + d^{(1)} = 15.1 + 2.04 = 17.14 \text{ Erlang}\,.$$

This calculated value is less than the prescribed offered traffic $A_{PCT2} = 17.5$ Erlang. Therefore, a further iteration step has to be done.

g) From $d^{(1)}$ we get:

$$A^{(2)}_{PCT1} = A_{PCT2} - d^{(1)} = 17.5 - 2.04 = 15.46 \text{ Erlang}\,.$$

h) The next reading off the table yields (see Fig. 4):

$$n^{(2)} = 29 \quad \text{and} \quad A^{(2)}_{PCT1} = 15.7 \text{ Erlang}\,.$$

i) With these values we get from Eqn. (2):

$$d^{(2)} = 2.14 \text{ Erlang}\,.$$

k) Now, the calculation

$$A_{PCT2} = A^{(2)}_{PCT1} + d^{(2)} = 17.85 \text{ Erlang}$$

yields a value greater than the prescribed offered PCT 2-traffic. Therefore, the number of trunk is:

$$n = 29\,.$$

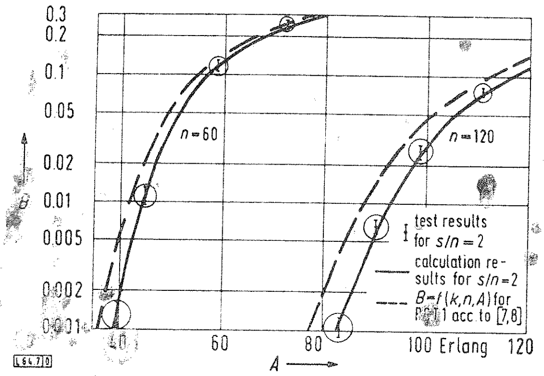

Fig. 7.
Comparison of the results, calculated with the d-formula, with test results. Accessibility $k=20$, $s/n=2$; mean interconnection number $H=2$.

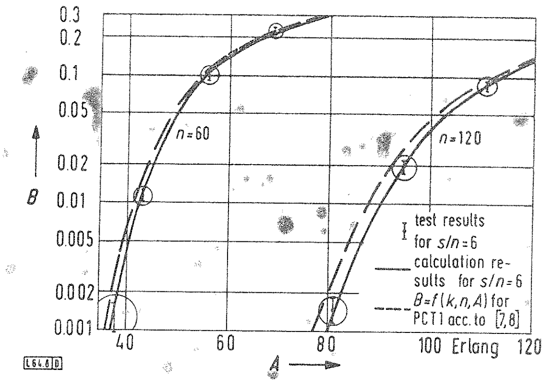

Fig. 8.
Comparison of the results, calculated with the d-formula, with test results. Accessibility $k=20$, $s/n=6$; mean interconnection number $H=2$.

Considering the finite number of sources the calculation yields in this example a saving of three trunks, in contrary to a dimensioning of the trunk group for the standard grading with the PCT 1-tables, only.

5. Comparison between Calculation and Artificial Traffic Trials

The function $B=f(s, n, k, A_{PCT\,2})$ for trunk groups with limited accessibility and with standard gradings was simulated on a digital computer checking the accuracy of the calculated $A_{PCT\,2}$-values. The results are shown in *Fig. 5 to 8* (grading schemes see e. g. in [6]).

6. Summary

A formula is presented which allows to calculate very easily trunk groups for standard gradings with offered PCT 2-traffic by means of the new dimensioning outlines of the German GPO.

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